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The Hawking effect. (English, French summaries)
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To derive his famous discovery that a black hole of mass m behaves like a black body at the Hawking temperature $T_H = (8\pi m)^{-1}$ [Comm. Math. Phys. 43 (1975), no. 3, 199–220; MR 52#2517], S. W. Hawking considered a free, massless, scalar quantum field on a space-time containing a spherically symmetric collapsing star. His basic result was that the in-vacuum state $\omega_{\rm in}$ of the field will look like a thermal state at T_H at late times and large distances. The analysis behind this conclusion basically requires the study of an initial value problem for the Klein-Gordon equation on the vacuum region of the space-time (i.e. outside the star), with boundary values on the star surface. In his first paper, however, Hawking circumvented exact calculations of this kind by using a geometrical optics approximation. In later papers the problem is bypassed by the study of quantum fields in the exterior Schwarzschild solution (M_S, g_S). In particular, in [W. G. Unruh, Phys. Rev. D (3) 14 (1976), no. 4, 870–892; S. W. Hawking, Phys. Rev. D. (3) 14 (1976), no. 10, 2460–2473; MR 57#8892] an idealized state—the Unruh state ω_U—is studied on (M_S, g_S) which, it is argued, should have properties similar to $\omega_{\rm in}$ for late times and large distances, while being independent of the details of the collapse. In particular, ω_U is characterized by a natural boundary condition on the past horizon. Later these considerations were made more rigorous by J. Dimock and B. S. Kay [Ann. Physics 175 (1987), no. 2, 366–426; MR 88h:83043] using scattering theory for the Klein-Gordon equation on (M_S, g_S). However, asymptotic completeness was proven only for massless fields. A completely different approach was published by K. Fredenhagen and R. Haag [Comm. Math. Phys. 127 (1990), no. 2, 273–284; MR 90m:83057]. In their paper a collapsing star is considered as in Hawking's original work but the analysis is carried out by scaling limit techniques, which avoid the study of the scattering theory completely.

In the present paper the analysis of Dimock and Kay is extended significantly in two different directions: First of all the author considers the original problem, i.e. scattering outside a spherically symmetric collapsing star, stationary in the past, with Dirichlet boundary conditions on the star surface, and second the massive case is covered as well. The main result of the paper describes the behavior of the solutions of this initial and boundary value problem in the limit $t \to \infty$ (in Regge-Wheeler coordinates). This is used together with CCR-algebra methods (especially generating functionals for quasi-free states) to confirm rigorously that the field becomes thermal with temperature T_H on future null infinity. Due to the general assumptions made about the history of the star surface this property does not depend on the details of the collapse (as long as the star is spherically symmetric and stationary in the past, of course).

This initial and boundary value problem, which is essential for the original derivation of the Hawking effect, is treated in this paper and some previous publications of the author [J. Math. Pures Appl. (9) 76 (1997), no. 2, 155–210; MR 98a:83068; Ann. Inst. H. Poincare Phys. Theor. 67 (1997), no. 2, 181–222; MR 98i:83046] for the first time in a very detailed, mathematically rigorous way. This includes in particular the existence and asymptotic completeness of the wave operators. The latter is based essentially on the use of Dollard-modified wave operators for long range potentials, which were introduced in classical wave scattering outside a static black hole by the author [Ann. Inst. H. Poincare Phys. Theor. 61 (1994), no. 4, 411–441; MR 96a:58190].

Although there was not really any doubt about the result, this paper is a valuable contribution to the study of the Hawking effect. First of all it clarifies some open questions about Hawking's original arguments and secondly its results about the Klein-Gordon equation outside a collapsing star are interesting in their own right. Potential readers should be warned, however: the presentation is quite technical and in many places it is somewhat hard to follow the arguments.

Reviewed by *Michael Keyl*