Corrigendum

Professor Dongho Byeon (Seoul National University) pointed out a mistake in the statement of Theorem 1.2 in my paper [Bel99]. The mean value 1/2 should be replaced by 1/4, there and in the last formula p. 8; the proof is otherwise correct. Explicitly, we have

$$\sum_{\Delta \in \Delta^+_{fund}(X)} \frac{3^{r_3(-3\Delta)} - 3^{r_3(\Delta)}}{2} \sim \frac{(H^- - H^+)X}{\zeta^2(2)} = \frac{X}{\pi^2}$$

$$\sum_{\Delta \in \Delta^+_{fund}(X)} 3^{r_3(\Delta)} \sim \frac{4}{3} \sum_{\Delta \in \Delta^+_{fund}(X)} 1 \sim \frac{4}{3} \times \frac{3X}{\pi^2}$$

where the first line appears on p. 8, and the second is the Davenport-Heilbronn theorem. Dividing the first relation by the second yields the result.

Using properly Cohen-Lenstra heuristics and Dutarte's conjecture, we are also led to this value 1/4 as follows:

Let $q := 1/3$ in the sequel. For $|q| < 1$ and $n \geq 0$, one defines

$$(q)_n = \prod_{i=1}^{n} (1 - q^i).$$

Summing heuristically over all possible 3-rank r, we get

$$\sum_{\substack{\Delta \in \Delta^+_{fund}(X) \\ \delta(\Delta)=0}} 3^{r_3(\Delta)} \Big/ \sum_{\Delta \in \Delta^+_{fund}(X)} 1 \quad \overset{?}{\longrightarrow} \quad \sum_{r \geq 0} q^{-r} P(\delta = 0 \mid r_3 = r) P(r_3 = r)$$

$$\overset{?}{=} \quad \sum_{r \geq 0} q^{-r} \times q^{r+1} \times \frac{q^{r(r+1)} (q)_\infty}{(q)_r (q)_{r+1}}$$

$$= \quad q$$

where we use Conjecture (3.2) and [Coh84, Conjecture C 9] in the second line, and a well-known q-identity in the last, see e.g [Coh84, Corollary 6.7]. Applying once again Davenport-Heilbronn's result, we expect a weighted mean value of $3q/4 = 1/4$, in accordance with the corrected theorem.

References

[Bel99] K. Belabas, On the mean 3-rank of quadratic fields, *Compositio Mathematica* **118** (1999), pp. 1–9.

[Coh84] H. Cohen & H. W. Lenstra, Jr., Heuristics on class groups of number fields, in *Number theory, Noordwijkerhout 1983* (Berlin), Lecture Notes in Math., vol. 1068, Springer, Berlin, 1984, pp. 33–62.