On the power required to control the circular cylinder wake by rotary oscillations

Michel Bergmann,* Laurent Cordier, and Jean-Pierre Brancher

LEMTA, UMR 7563 (CNRS - INPL - UHP)

ENSEM - 2, avenue de la forêt de Haye

BP 160 - 54504 Vandœuvre cedex, France

(Dated: July 18, 2006)

Abstract

In this Brief Communication, we determine an approximate relation which gives the mean time power required to control the wake flow downstream from a circular cylinder. The control law is the sinusoidal tangential velocity imposed on whole or part of the cylinder surface. The mean control power thus depends on four parameters: the amplitude and the Strouhal number of forcing, the control angle which defines the controlled upstream part of the cylinder and the Reynolds number. This relation indicates that the control power grows like the square of the forcing amplitude, like the square root of the forcing Strouhal number, linearly with the control angle and varies like the inverse of the square root of the Reynolds number. We show that the values obtained with this approximate relation are in very good agreement with the corresponding values given numerically. Finally, the energetic efficiency of the control is discussed. We claimed that the most energetically efficient control law corresponds *a priori* to low forcing amplitudes applied to a restricted upstream part of the cylinder for relatively high values of the Reynolds number.

Keywords: circular cylinder ; rotary oscillations ; power control ; energetic efficiency.

The control of the cylinder wake flow by rotary oscillations has been intensively studied over the last decade. The majority of these studies were motivated by the experiments of Tokumaru and Dimotakis[1] where 80% of relative mean drag reduction was empirically found at $Re = 15,000$. Due to the rapid progress achieved in computational fluid dynamics and the remarkable developments in modern control theory (optimal and robust control mainly), the active control of wake flows was intensively explored numerically[2–4]. All these studies gave place to convincing results for the different values of Reynolds number considered. However, in most of the cases, the cost of the control was not considered, except in Refs. 2,5. Consequently, the energetic efficiency of a given technique of control is difficult to evaluate and comparing the various control methods is even more delicate. It is thus of interest to analyze the influence of the control parameters on the control power evolution. Hence, the objective of this Brief Communication is to determine, and validate numerically, an approximate relation giving the control power according to the different control parameters of the cylinder wake and the Reynolds number.

Here, the incompressible and viscous flow around a circular cylinder of diameter D (radius R) is considered in a two-dimensional domain Ω. The cylinder boundary is divided in two regions: an upstream part animated with an unsteady tangential velocity V_T, and the remainder of the cylinder which is not controlled. Hereafter, the upstream controlled region is geometrically defined by $-\theta_c < \theta < \theta_c$ (see Fig. 1) where θ and θ_c, initialized by convention at the front stagnation point of the cylinder, are respectively the curvilinear coordinates of a point on Γ_c and the control angle. Equivalently, the controlled boundary of the cylinder is characterized by the control surface $S_c = 2\theta_c\, R\, \ell_z$ where ℓ_z is the spanwise length of the cylinder (for two dimensional configurations, $\ell_z = 1$). As it is generally the case[1–8], the control law is selected to be harmonic $V_T(t) = A_0 \sin(\omega\, t)$ where A_0 and ω respectively represent the dimensional amplitude and pulsation of forcing. For convenience, the dimensionless velocity $\gamma(t) = V_T(t)/U_\infty$ is introduced, where U_∞ denotes the inflow velocity. Then, the control law writes equivalently $\gamma(t) = A\sin(2\pi St_f t)$ with $A = A_0/U_\infty$ and $St_f = R\omega/(\pi U_\infty)$. Here, A and St_f denote respectively the nondimensionalized forcing amplitude and Strouhal numbers. Moreover, since the control is applied only on a restricted part of the cylinder surface, the physical problem depends implicitly on four control parameters, namely A, St_f, θ_c and the Reynolds number $Re = \rho U_\infty D/\mu$, where ρ and μ are respectively the density and the dynamic viscosity of the fluid. Finally, for all the numerical simulations used for the validation, the Navier-Stokes equations are solved in the pressure-velocity formulation with a fractional step method in time and a finite element method in space[4].

Now, we examine the power budget related to flow control *i.e.* the sum of the work that has to be done against the drag force and the work needed to control the flow. By definition, the force which is acting on the cylinder can be expressed as $\boldsymbol{F} = -\int_{\Gamma_c} \overline{\overline{\sigma}}\,\boldsymbol{n}\,d\Gamma$ where $\overline{\overline{\sigma}}$ is the stress tensor and Γ_c is the cylinder boundary. Let F_D be the total drag force obtained by projecting $\boldsymbol{F}$ on the direction $\boldsymbol{e_x}$, the power spending related to this effort is thus given by the relation $P_D(t) = F_D(t)U_\infty$.

We then focus on the power used to control the cylinder flow. If we do not account for inertial effects related to rotating the cylinder, the control power can be evaluated by $P_c(t) = \mathcal{M}_z(t)\dot{\theta}$ where $\dot{\theta} = \gamma\, U_\infty/R$ is the angular velocity of the cylinder and $\mathcal{M}_z$ is the component on the direction $\boldsymbol{e_z} = \boldsymbol{e_x} \wedge \boldsymbol{e_y}$ of the moment of forces, or torque $\mathcal{M}$, applied to the obstacle. By definition, the moment of forces on a body placed in a flow is given by $\mathcal{M} = -\int_{\Gamma_c} \boldsymbol{r} \wedge \overline{\overline{\sigma}}\,\boldsymbol{n}\,d\Gamma$ where $\boldsymbol{r} = x\boldsymbol{e_x} + y\boldsymbol{e_y}$ defines the coordinates of any point on Γ_c. For reasons of simplicity, we introduce the coefficient of moment defined as $C_{\mathcal{M}_z} = \frac{\mathcal{M}_z}{1/2M_r}$, where

M_r is a reference moment given by $M_r = F_r \times L_r$ with F_r and L_r, a reference force and a reference length, respectively. The reference force is now determined as $F_r = p_r \times S_r$ where p_r and S_r are the pressure reference and surface reference, respectively. Usually $p_r = \rho\, U_\infty^2$ and $S_r = 2R\,\ell_z$. Here, the reference length L_r is selected equal to the diameter of the cylinder $D = 2R$.

After some algebraic manipulations, the instantaneous coefficient of moment can be evaluated numerically from:

$$C_{\mathcal{M}_z}(t) = \frac{2}{Re}\int_0^{2\pi}\Big(-\frac{\partial v}{\partial x}x^2 - 2\frac{\partial v}{\partial y}xy + 2\frac{\partial u}{\partial x}xy + \frac{\partial u}{\partial y}y^2 - \frac{\partial u}{\partial y}x^2 + \frac{\partial v}{\partial x}y^2\Big)\,d\theta,$$

where all the variables are nondimensionalized with respect to the oncoming velocity U_∞ and the cylinder diameter D. Finally, since $\mathcal{M}_z = \frac{1}{2}M_r\,C_{\mathcal{M}_z}$ with $M_r = F_r L_r = 4\rho U_\infty^2 R^2 \ell_z$, the instantaneous power useful for rotation merely writes as $P_C(t) = \rho U_\infty^3 R\ell_z\ 2\gamma(t)C_{\mathcal{M}_z}(t)$. To simplify the future comparisons of the control power, let us define the time average over a finite horizon T, corresponding to a few cylinder oscillation periods, as $\langle\cdot\rangle_T = \frac{1}{T}\int_{t_0}^{t_0+T}\cdot\,dt$. The analytic formulation of the mean control power is then

$$\mathcal{P}_C = \langle P_C\rangle_T = \rho U_\infty^3 R\ell_z\ \langle 2\gamma(t)C_{\mathcal{M}_z}(t)\rangle_T. \tag{1}$$

Unfortunately this relation does not depend explicitly on the control parameters A, St_f, θ_c and Re. Hence, more precise developments seem necessary.

The mean time power necessary to ensure the oscillatory rotation of the cylinder can be approximated as follows:

$$\widetilde{\mathcal{P}_C} = -\langle S_c\,\tau_w\,A_0\sin(\omega\,t)\rangle_T,$$

where τ_w is the wall shear stress. To determine an expression of τ_w which depend on the control parameters and the Reynolds number, we now make the following assumptions:

H1 the oscillations of the cylinder do not affect the external flow,

H2 the effects of curvature can be neglected ($\frac{\delta}{R} \ll 1$).

The H1 assumption is equivalent to saying that the oscillations do not diffuse outside the presumedly laminar boundary layer of characteristic thickness[9] $\delta = \mathcal{O}\left(\sqrt{\frac{\nu R}{U_\infty}}\right)$. This assumption implies immediately that the Reynolds number must be sufficiently large ($Re \gg 1$). In addition, the H1 assumption implies that we must satisfy the condition $\frac{\delta_0}{\delta_a} \ll 1$ where $\delta_0 = \mathcal{O}\left(\sqrt{\frac{\nu}{\omega}}\right)$ is the thickness of the diffusion Stokes layer induced by the oscillations and δ_a is the thickness of the boundary layer at the front stagnation point. Using the H1 assumption *a priori* (what was checked a posteriori), we can consider that the front stagnation point does not move. One can thus calculate the flow in the vicinity of the stagnation point starting from the complex potential without circulation. In this case, the stream function $\psi = Im\{f(z)\}$ simply writes $\psi(X,Y) = U_\infty^2\left(Y - \frac{R^2 Y}{(R+X)^2+Y^2}\right)$ with $X = x - R$ and $Y = y$. Near the stagnation point, *i.e.* for $(X, Y) \to (0, 0)$, the stream function behaves like $\frac{2U_\infty}{R}XY$. Therefore, $\delta_a = \mathcal{O}\left(\sqrt{\frac{\nu R}{2U_\infty}}\right)$ and the condition $\frac{\delta_0}{\delta_a} \ll 1$ writes:

$$\frac{\delta_0}{\delta_a} = \mathcal{O}\left(\sqrt{\frac{2U_\infty}{R\omega}}\right) = \mathcal{O}\left(\sqrt{\frac{2}{\pi St_f}}\right) \ll 1. \tag{2}$$

Since the velocity field is tangent to the cylinder surface near the front stagnation point, the velocity component in the direction $\boldsymbol{e_y}$ is written as:

$$v(r^*, t) = A_0 \exp\left(-\sqrt{\frac{\omega}{2\nu}}\, r^*\right) \sin\left(\omega t - \sqrt{\frac{\omega}{2\nu}}\, r^*\right),$$

where $r^* = r - R$ (here, r represents the radial coordinate). One can then easily deduce the expression of the wall shear stress:

$$\tau_w = \mu \left.\frac{\partial v}{\partial r^*}\right|_{r^*=0} = -\mu A_0 \sqrt{\frac{\omega}{2\nu}} \left[sin(\omega\, t) + cos(\omega\, t)\right].$$

Since we have $\langle cos(\omega\, t) sin(\omega\, t)\rangle_T = 0$ and $\langle sin^2(\omega\, t)\rangle_T = \frac{1}{2}$ for $T = 2k\pi/\omega$, $k \in \mathbb{N}$, the mean control power becomes $\widetilde{\mathcal{P}_C} = S_c \mu \frac{A_0^2}{2} \sqrt{\omega/2\nu}$. Finally, after some manipulations, the expression of the power needed to apply the rotary oscillations to the cylinder can be simplified as

$$\widetilde{\mathcal{P}_C} = \rho\, U_\infty^3\, R\, \ell_z\, \sqrt{\pi}\, \frac{\theta_c A^2 \sqrt{St_f}}{\sqrt{Re}}. \tag{3}$$

This approximate expression now depends on the four control parameters. For the numerical validation, we chose to compare directly the mean control power coefficient obtained respectively from Eq. (1) and Eq. (3). Let $P_r = F_r U_\infty = \rho U_\infty^3 2R\ell_z$ be the reference control power, by definition, the mean control power coefficient is $C_P = \langle P\rangle_T / \frac{1}{2} P_r$. Applying this relation to the drag power $P_D(t) = F_D(t)\, U_\infty$ we can deduce an expression for the mean power drag coefficient:

$$C_P^D = \langle C_D\rangle_T, \tag{4}$$

where $C_D(t) = F_D(t)/\rho U_\infty^2 R\ell_z$ is the instantaneous drag coefficient, given for example in Ref. 8. Similarly, the expression of the mean control power coefficient based on Eq. (1) is

$$C_P^C = \langle 2\, C_{\mathcal{M}_z}(t)\, \gamma(t)\rangle_T, \tag{5}$$

and the corresponding expression for the mean control power coefficient obtained with the approximate relation (3) is written

$$C_P^C = \sqrt{\pi}\, \frac{\theta_c A^2 \sqrt{St_f}}{\sqrt{Re}}. \tag{6}$$

The control is said energetically efficient (*resp.* inefficient), if the value of the total mean power coefficient $C_P = C_P^D + C_P^C$ (where C_P^C can be evaluated either by Eq. (5), or by Eq. (6)) is lower (*resp.* greater) than the value obtained in the uncontrolled case (where naturally $C_P^C = 0$).

Now, we will compare the results given by the approximate relation (6) to the numerical ones given by Eq. (5). For that, we will successively represent the evolutions of the control powers according to one of the four control parameters of the flow, the others being fixed. For these comparisons, the parameters maintained fixed are selected among $A = 5$, $St_f = 1$, $\theta_c = 180°$ and $Re = 200$. Figures 2, 3, 4 and 5 show respectively the evolutions of the mean control power coefficient versus the amplitude A, the Strouhal number St_f, the control angle θ_c and the Reynolds number. Figure 2 shows an excellent agreement between the values obtained numerically and those determined with the approximate relation. One can thus claim that the power required to control the cylinder wake grows effectively like the square of the forcing amplitude as the expression (3) predicted it. As it is noticeable in Fig. 3, the agreement between the numerical and approximate expressions of the control power according to the forcing Strouhal number is still very good, and that even if the Strouhal number is of order unit. The small differences observed for the smallest values of the Strouhal number can be explained by the assumptions made to derive the approximate relation, and more particularly that given by Eq. (2). In spite of that, the values obtained numerically exceed hardly the values found by the approximate relation. The same behavior is clearly visible in Fig. 4 where the linear dependence of the control power as a function of the angle θ_c is quite well verified. As we can see in Fig. 5, the approximate relation does not suitably predict the variation, according to the Reynolds number, of the control power obtained numerically. Indeed, in one hand, for Reynolds number lower than 200, the numerical values exceed those predicted by the approximate relation. This result can be explained easily by the assumption of laminar boundary layer ($Re >> 1$) carried out to determine the approximate relation. In the other hand, for Reynolds number greater than 200, the numerical values are slightly lower than those determined with the approximate relation. This time, these differences can be explained by the fact that for a value of the Reynolds number close to 200, where a good agreement is observed, a three-dimensional Hopf bifurcation occurs[10]. As a normal consequence, our two-dimensional numerical simulation cannot capture accurately three-dimensional dynamic phenomena. However, from a qualitative point of view, the control power seems to evolve like the reverse of the square root of the Reynolds number as it was predicted by Eq. (3).

In this Brief Communication, we determined an approximate relation which gives, according to the 4 parameters of controlled flow A, St_f, θ_c and Re, the mean power which is necessary to control by oscillatory rotation the cylinder wake. Even if all the assumptions used to determine this approximation are not verified exactly, we showed that this relation was in very good agreement with the numerically given corresponding values. Consequently, the power required for the cylinder oscillations is almost completely dissipated in the Stokes layer. Since the control power grows with the square of the forcing amplitude, it is nearly impossible to obtain an energetically efficient control for high amplitudes. Protas and Styczek[5] had numerically shown this result besides while considering as cost function the total power in an optimal control approach. In addition, the approximate relation shows that the control power is much less penalized by the Strouhal number than it is by the amplitude of forcing. It is also remarkable that the control power decreases linearly with the control angle θ_c. Indeed, since a gain of reduction of drag is obtained by controlling only a well defined upstream part of the cylinder[8] (and not all the surface of the cylinder as it is usually the case), the maximum of energetic efficiency is awaited for θ_c lower than 180°. Finally, the approximate relation indicates that the control power decreases when the Reynolds number increases. Moreover, Protas and Wesfreid[7] showed that the potential gain of drag increased with the Reynolds number. Consequently, the most energetically efficient controls should be obtained *a priori* for relatively high values of the Reynolds number.

---

* Electronic address: Michel.Bergmann@ensem.inpl-nancy.fr

[1] P. Tokumaru and P. Dimotakis, “Rotary oscillatory control of a cylinder wake,” J. Fluid Mech., **224**, 77 (1991).

[2] J.-W. He, R. Glowinski, R. Metcalfe, A. Nordlander, and J. Périaux, “Active control and drag optimization for flow past a circular cylinder. Part 1. Oscillatory cylinder rotation,” J. Comp. Phys., **163**, 83 (2000).

[3] C. Homescu, I. Navon, and Z. Li, “Suppression of vortex shedding for flow around a circular cylinder using optimal control,” Int. J. Numer. Meth. Fluids, **38**, 43 (2002).

[4] M. Bergmann, L. Cordier, and J.-P. Brancher, “Optimal rotary control of the cylinder wake using POD Reduced Order Model,” Phys. Fluids, **17**, 097101 (2005).

[5] B. Protas and A. Styczek, “Optimal rotary control of the cylinder wake in the laminar regime,” Phys. Fluids, **14**, 2073 (2002).

[6] C. Min and H. Choi, “Suboptimal feedback control of vortex shedding at low Reynolds numbers,” J. Fluid Mech., **401**, 123 (1999).

[7] B. Protas and J. Wesfreid, “Drag force in the open-loop control of the cylinder wake in the laminar regime,” Phys. Fluids, **14**, 810 (2002).

[8] M. Bergmann, L. Cordier, and J.-P. Brancher, “On the generation of a reverse Von Kármán street for the controlled cylinder wake in the laminar regime,” Phys. Fluids, **18**, 028101 (2006).

[9] H. Schlichting and K. Gersten, *Boundary-Layer Theory* (Springer, 2000).

[10] C. Williamson, “Vortex dynamics in the cylinder wake,” Ann. Rev. Fluid. Mech., **28**, 477 (1996).

List of figures

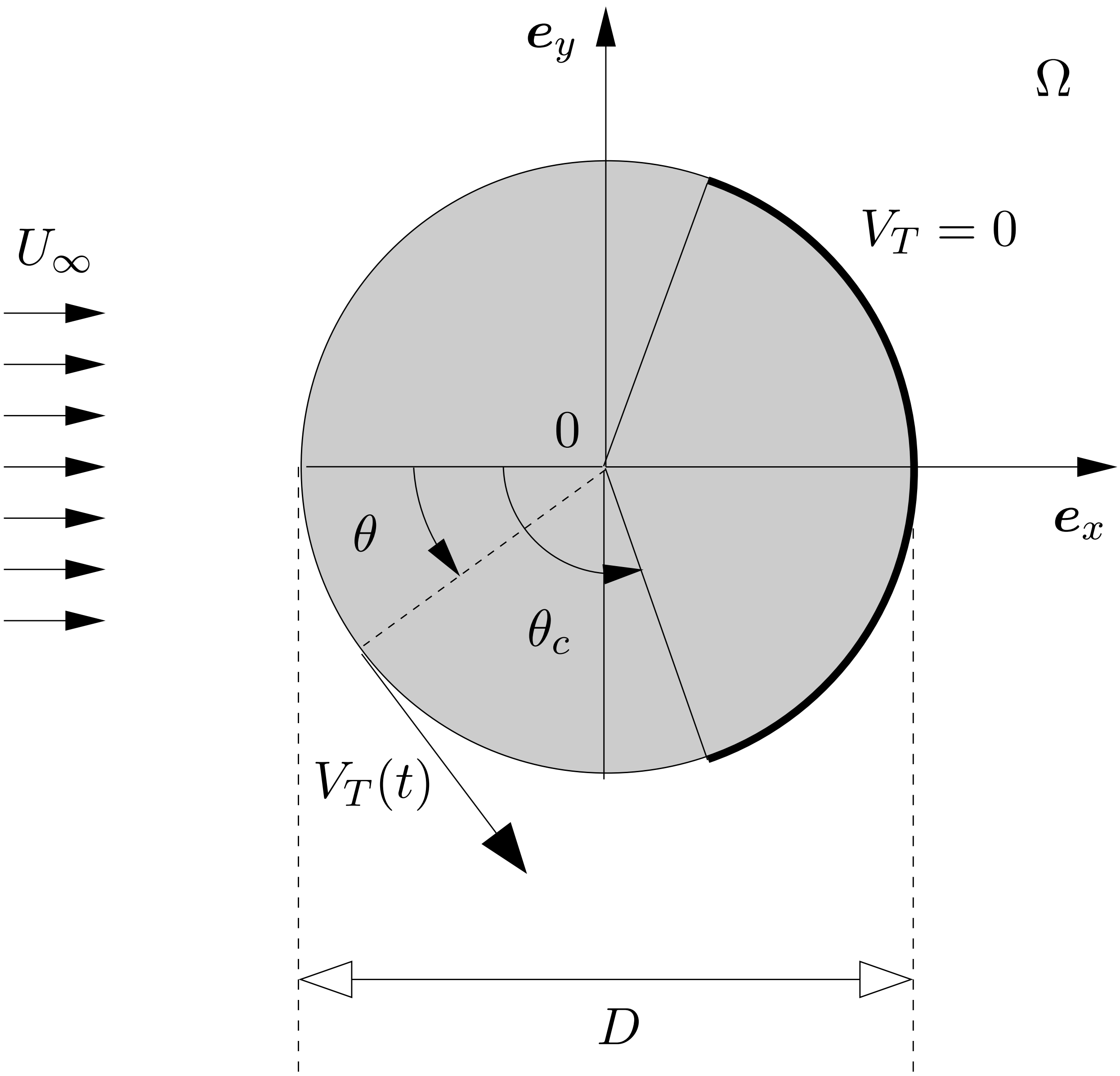

FIG. 1:

Bergmann, Cordier and Brancher, Physics of Fluids

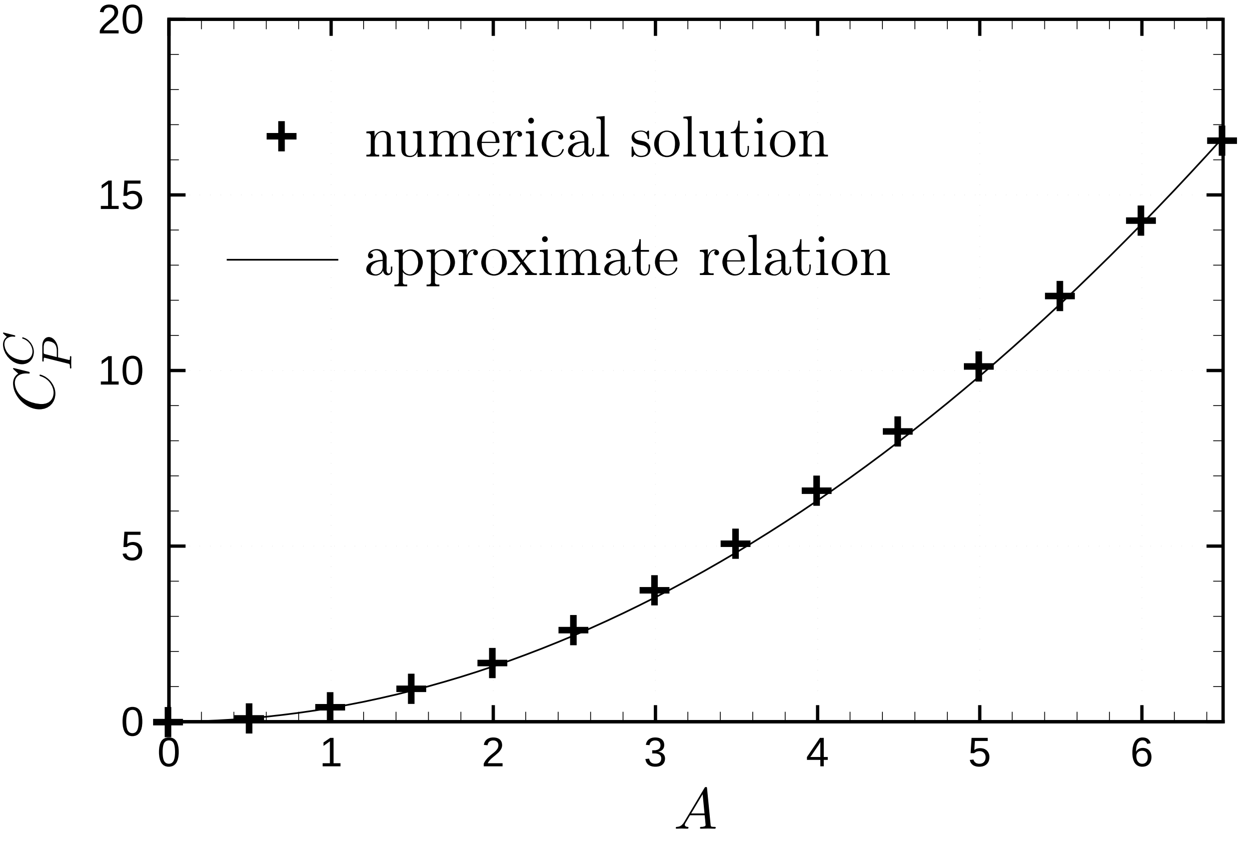

FIG. 2:

Bergmann, Cordier and Brancher, Physics of Fluids

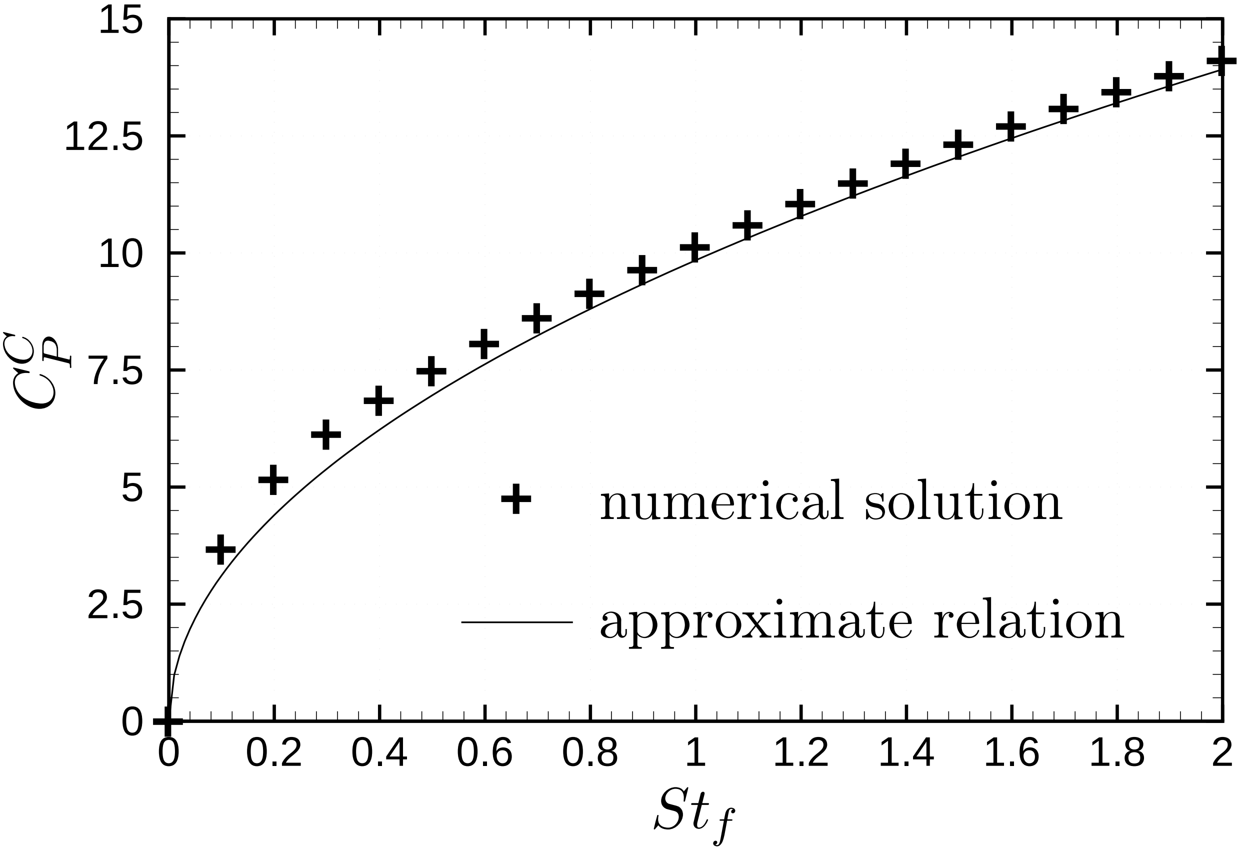
15
12.5
10
7.5
5
2.5
0
C_P^C
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
St_f
numerical solution
approximate relation

FIG. 3:

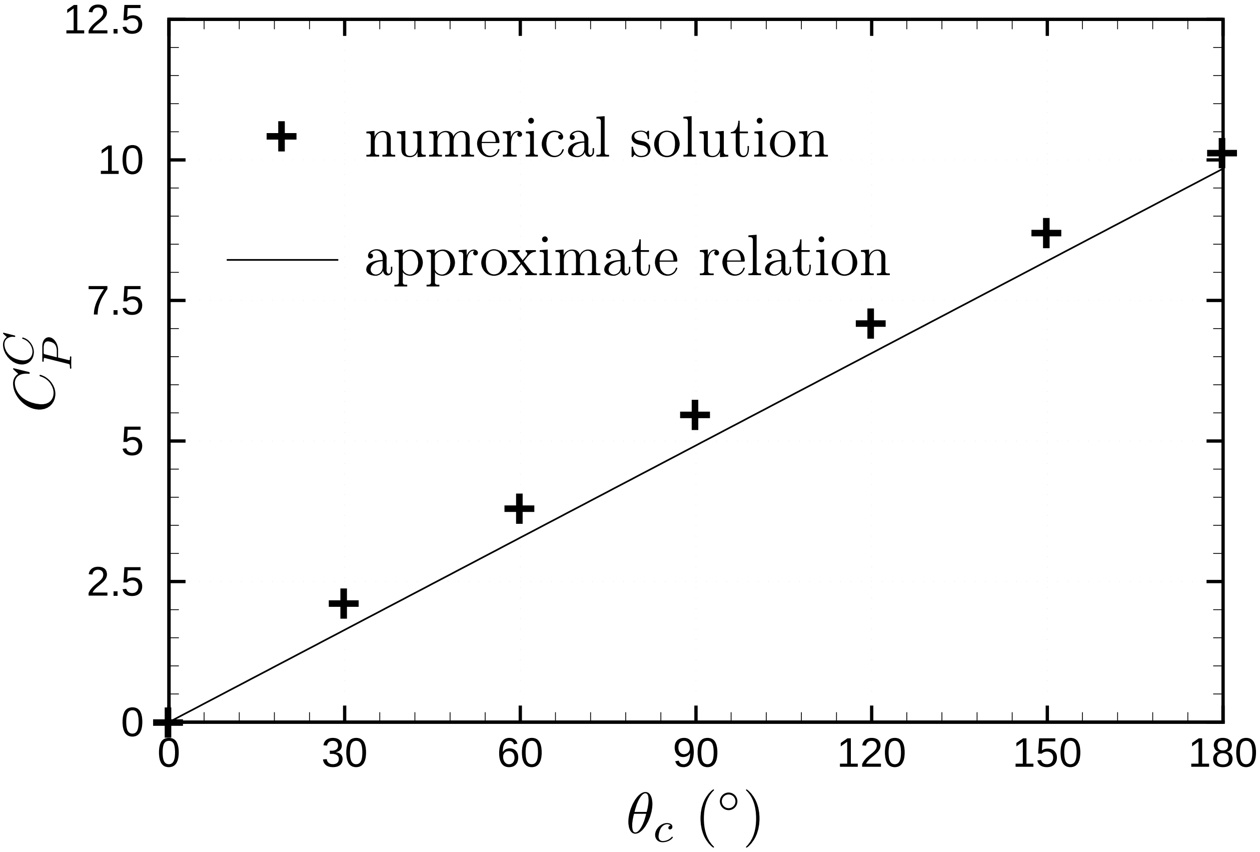

FIG. 4:

Bergmann, Cordier and Brancher, Physics of Fluids

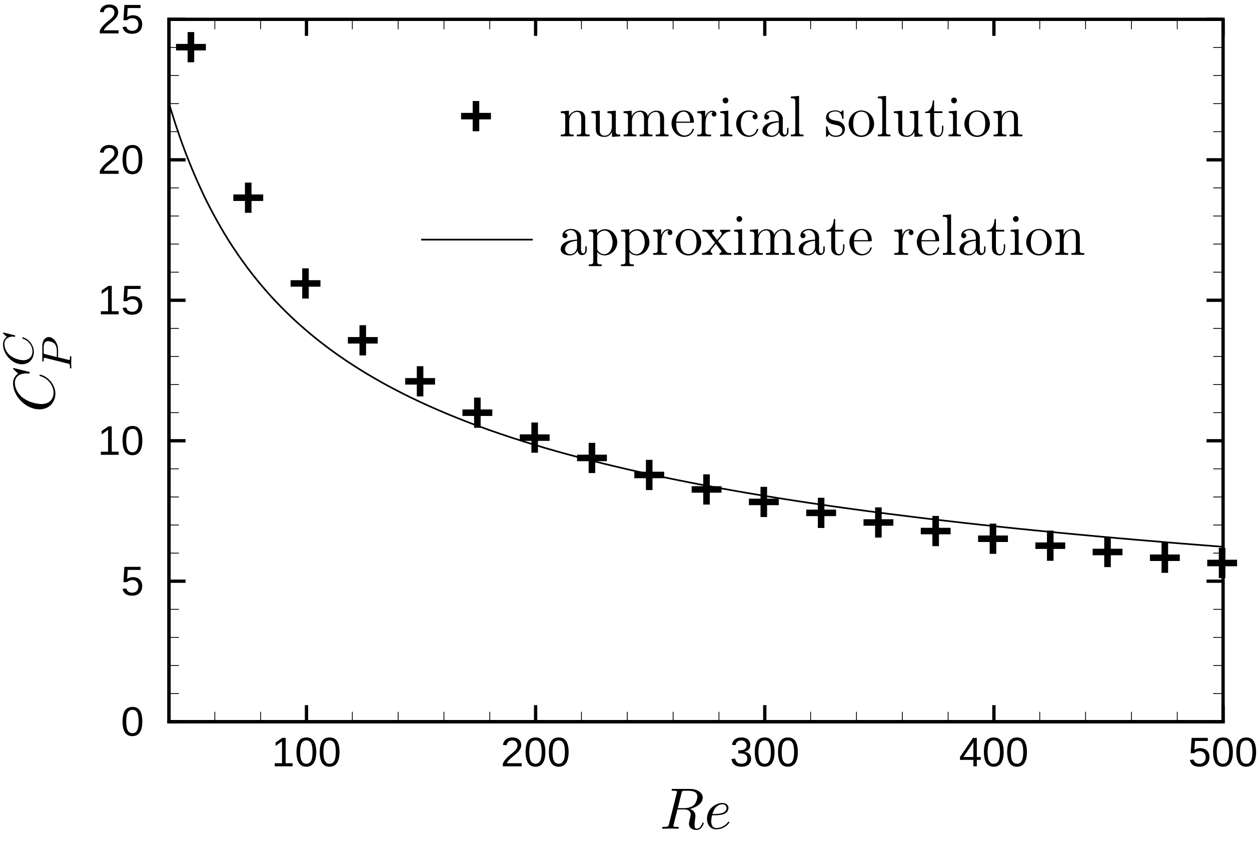

FIG. 5:

Bergmann, Cordier and Brancher, Physics of Fluids