Contrôle optimal par réduction de modèle POD et méthode à région de confiance du sillage laminaire d'un cylindre circulaire.

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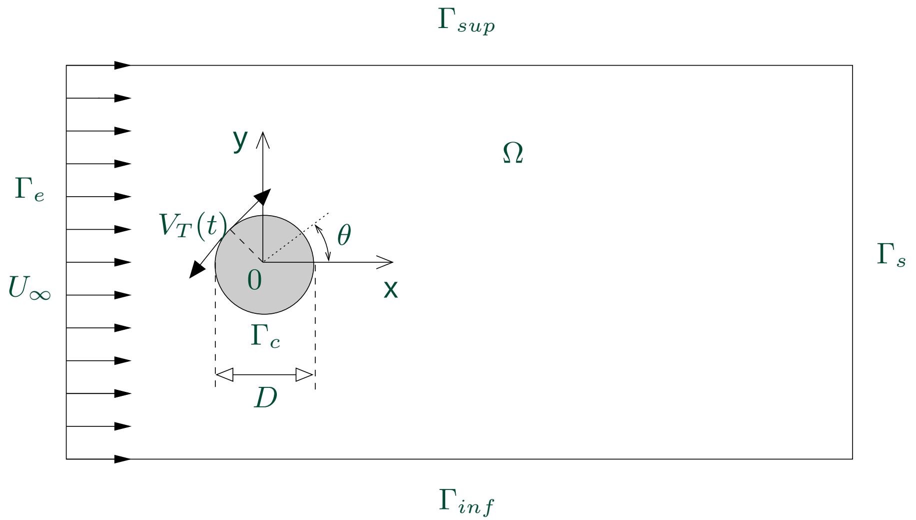
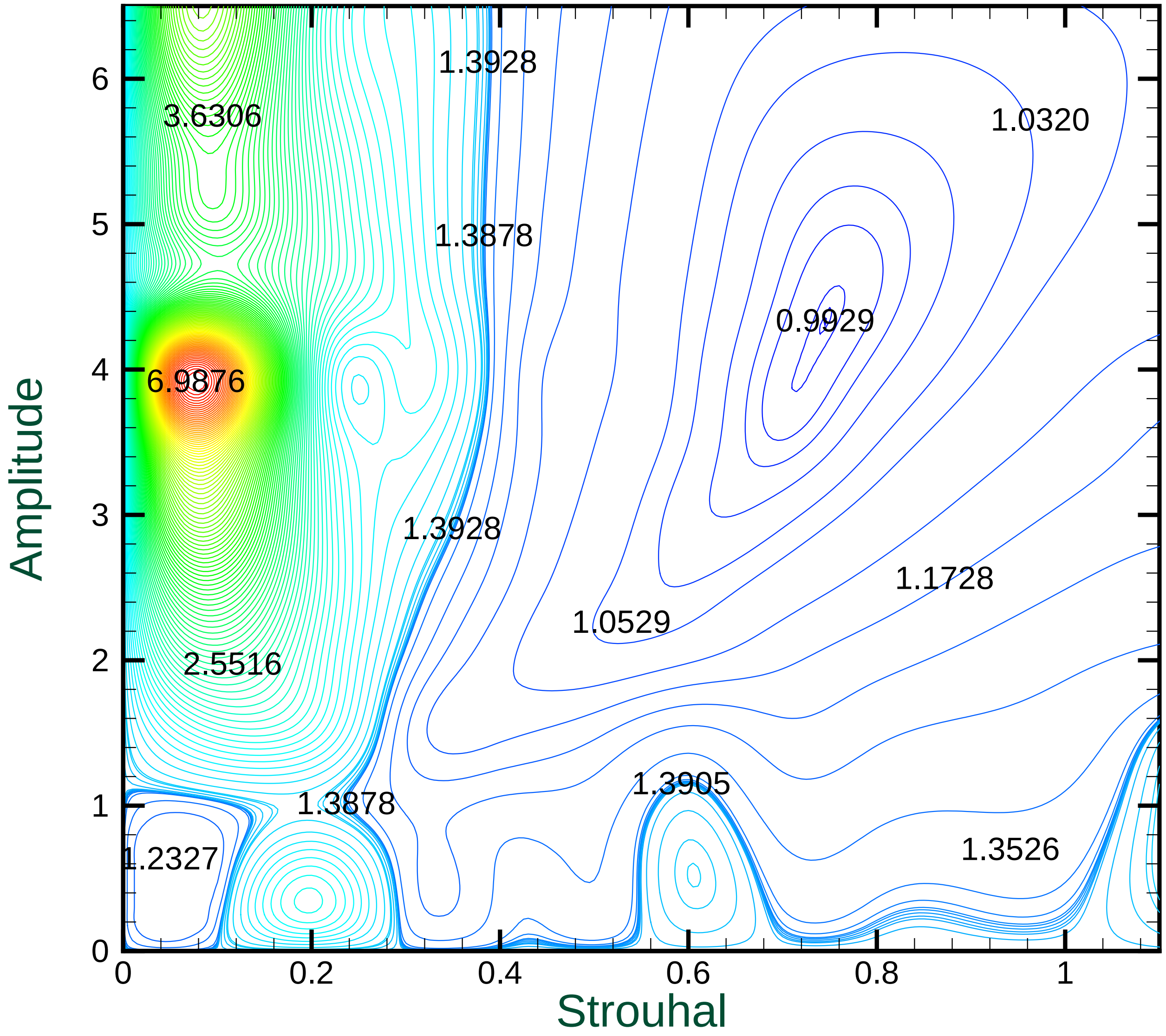
- Two dimensional flow around a circular cylinder at $Re = 200$
- Viscous, incompressible and Newtonian fluid
- Cylinder oscillation with a tangential sinusoidal velocity $\gamma(t)$

$$\gamma(t) = \frac{V_T}{U_\infty} = A \sin(2\pi St_f t)$$



Find the control parameters $c = (A,\ St_f)^T$ such that the mean drag coefficient is minimized

$$\langle C_D \rangle_T = \frac{1}{T}\int_0^T \int_0^{2\pi} 2\, p\, n_x\, R\, d\theta\, dt - \frac{1}{T}\int_0^T \int_0^{2\pi} \frac{2}{Re}\left(\frac{\partial u}{\partial x} n_x + \frac{\partial u}{\partial y} n_y\right) R\, d\theta\, dt,$$



Variation of the mean drag coefficient with A and St_f.

Numerical minimum $(A_{min}, St_{f_{min}}) = (4.3, 0.74)$.

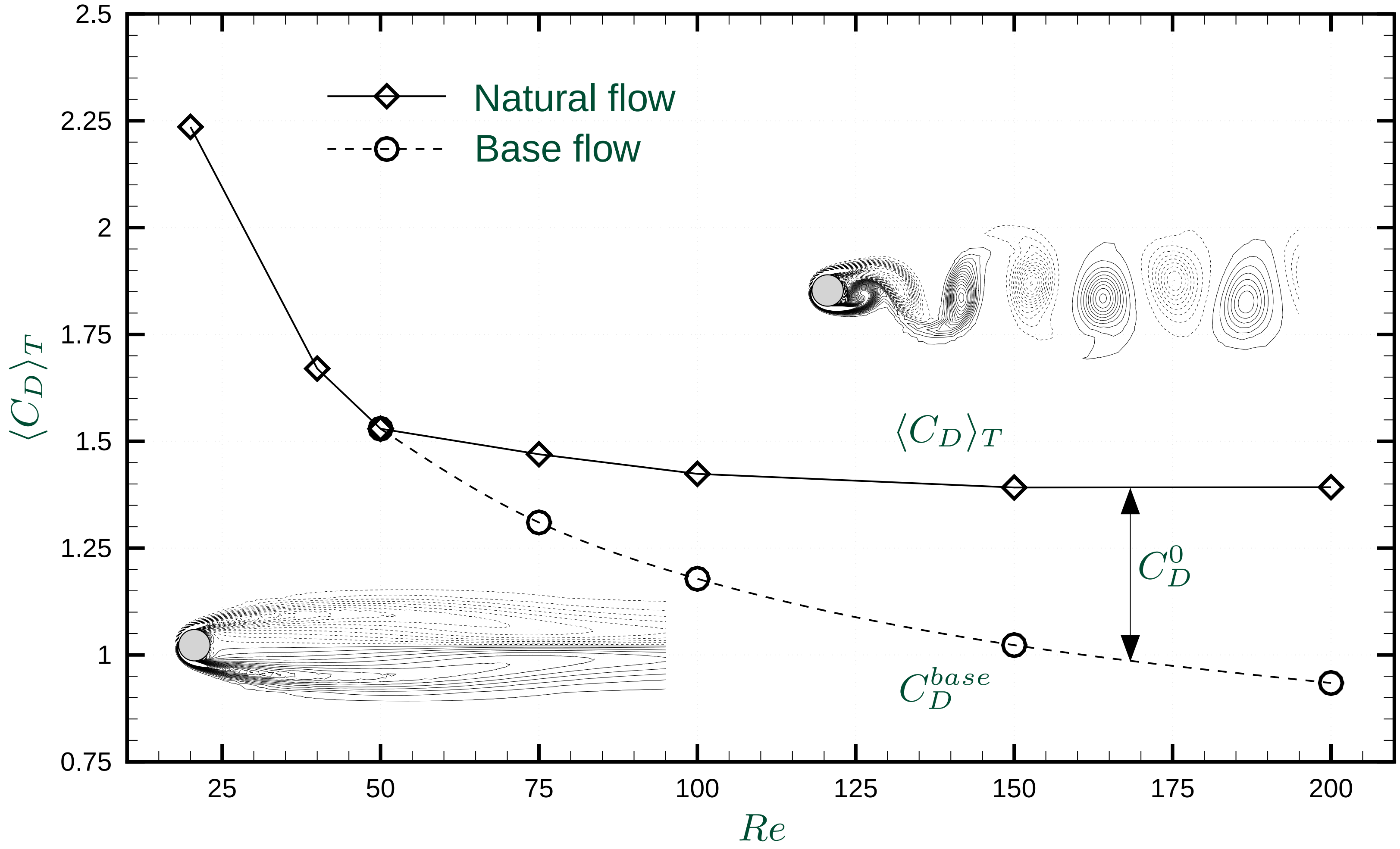
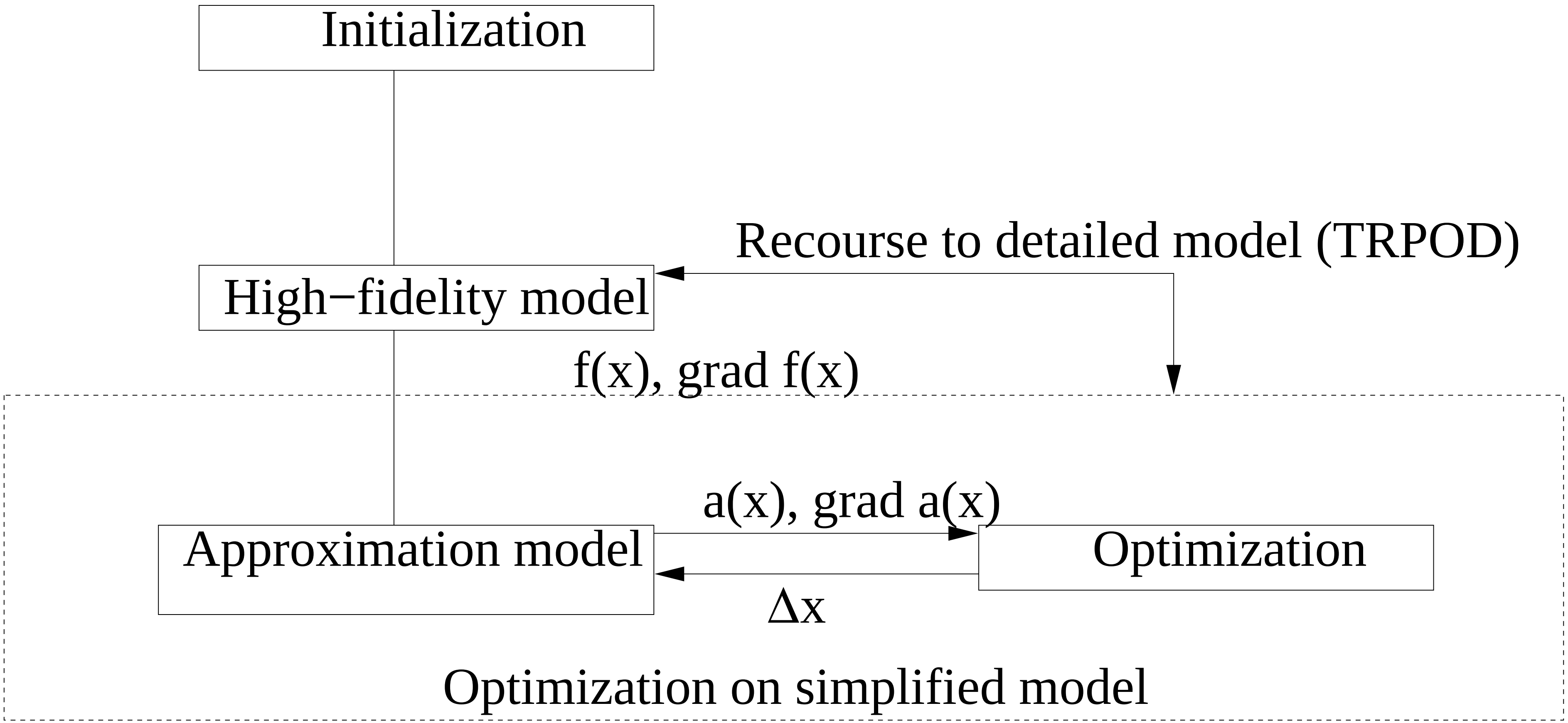


Fig. : *Variation with the Reynolds number of the mean drag coefficient. Contributions and corresponding flow patterns of the base flow and unsteady flow.*

Protas, B. et Wesfreid, J.E. (2002) : Drag force in the open-loop control of the cylinder wake in the laminar regime. *Phys. Fluids*, **14**(2), pp. 810-826.

Reduced Order Model (ROM) and optimization problems



- Introduced in turbulence by Lumley (1967).
- Method of information compression
- Look for a realization $\boldsymbol{\Phi}(\boldsymbol{X})$ which is closer, in an average sense, to realizations $\boldsymbol{u}(\boldsymbol{X})$. ($\boldsymbol{X} = (\boldsymbol{x}, t) \in \mathcal{D} = \Omega \times \mathbb{R}^+$)
- $\boldsymbol{\Phi}(\boldsymbol{X})$ solution of the problem :

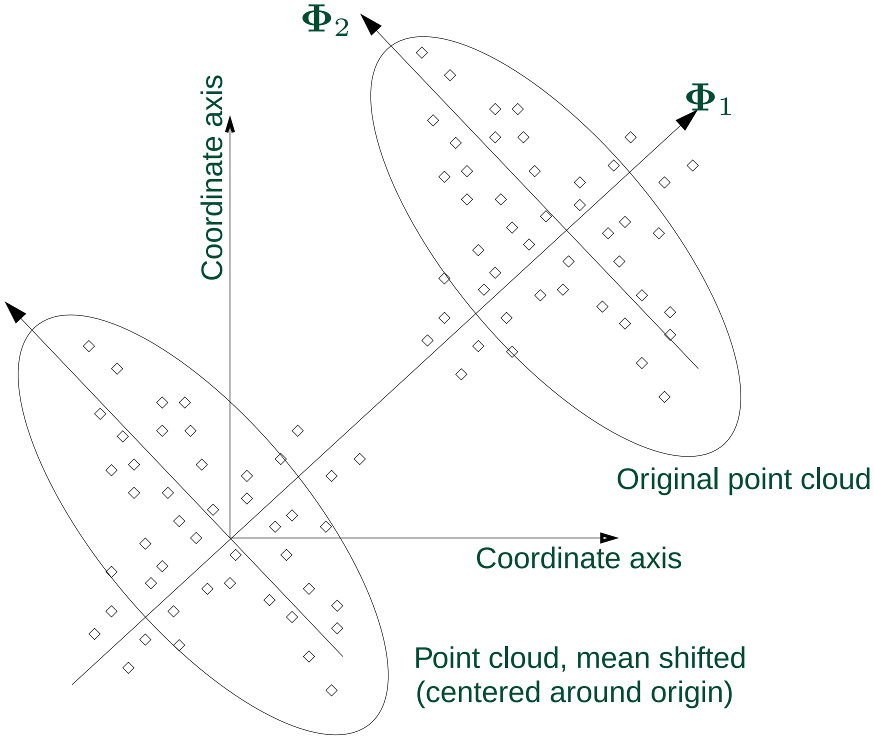
$$\max_{\boldsymbol{\Phi}} \langle |(\boldsymbol{u}, \boldsymbol{\Phi})|^2 \rangle \quad \text{s.t.} \quad \|\boldsymbol{\Phi}\|^2 = 1.$$

- Snapshots method, Sirovich (1987) :

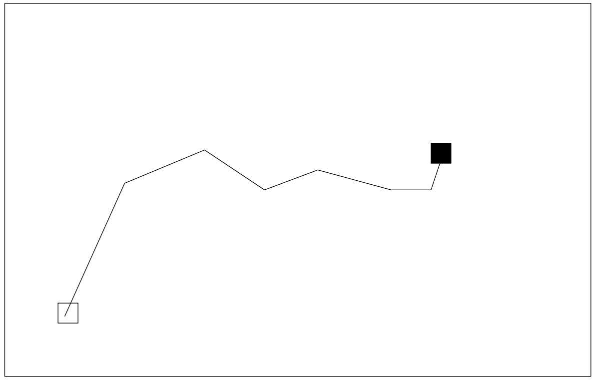
$$\int_T C(t, t') a^{(n)}(t')\, dt' = \lambda^{(n)} a^{(n)}(t).$$

- Optimal convergence *in L^2 norm* (energy)of $\boldsymbol{\Phi}(\boldsymbol{X})$

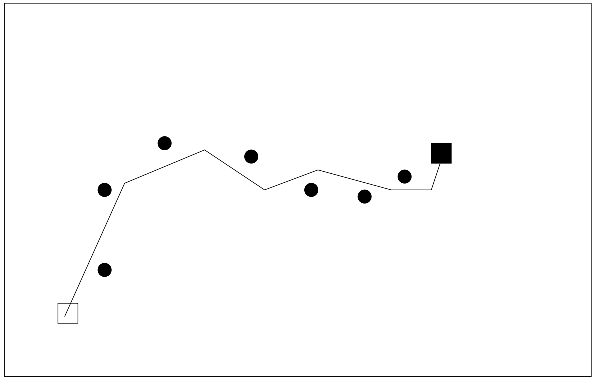
$\Rightarrow$ Dynamical order reduction is possible.



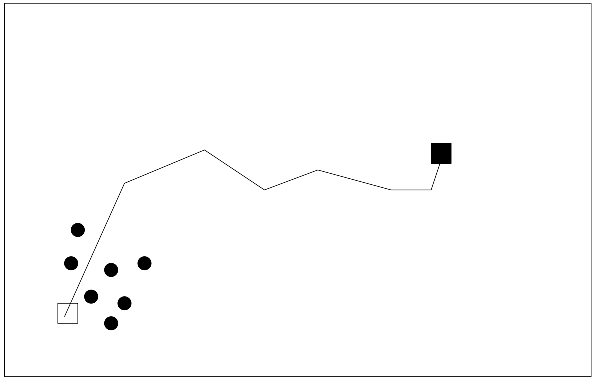
ROM *Parameter sampling in an optimization setting*



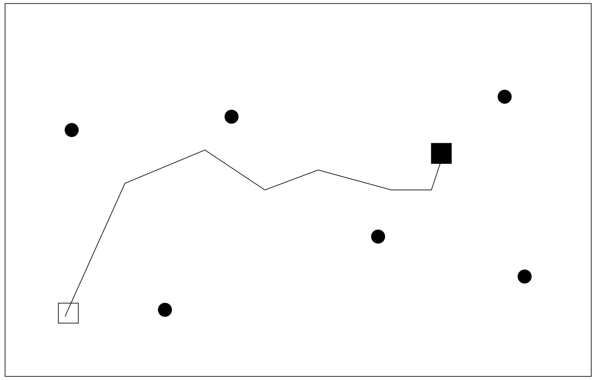
General configuration.



Ideal sampling.



Unsuitable sampling.



Unsuitable sampling.

Discussion of parameter sampling in an optimization setting (from Gunzburger, 2004). —— path to optimizer using full system, □ initial values, ■ optimal values, and • parameter values used for snapshot generation.

ROM *A simple configuration, a rich dynamical behavior*

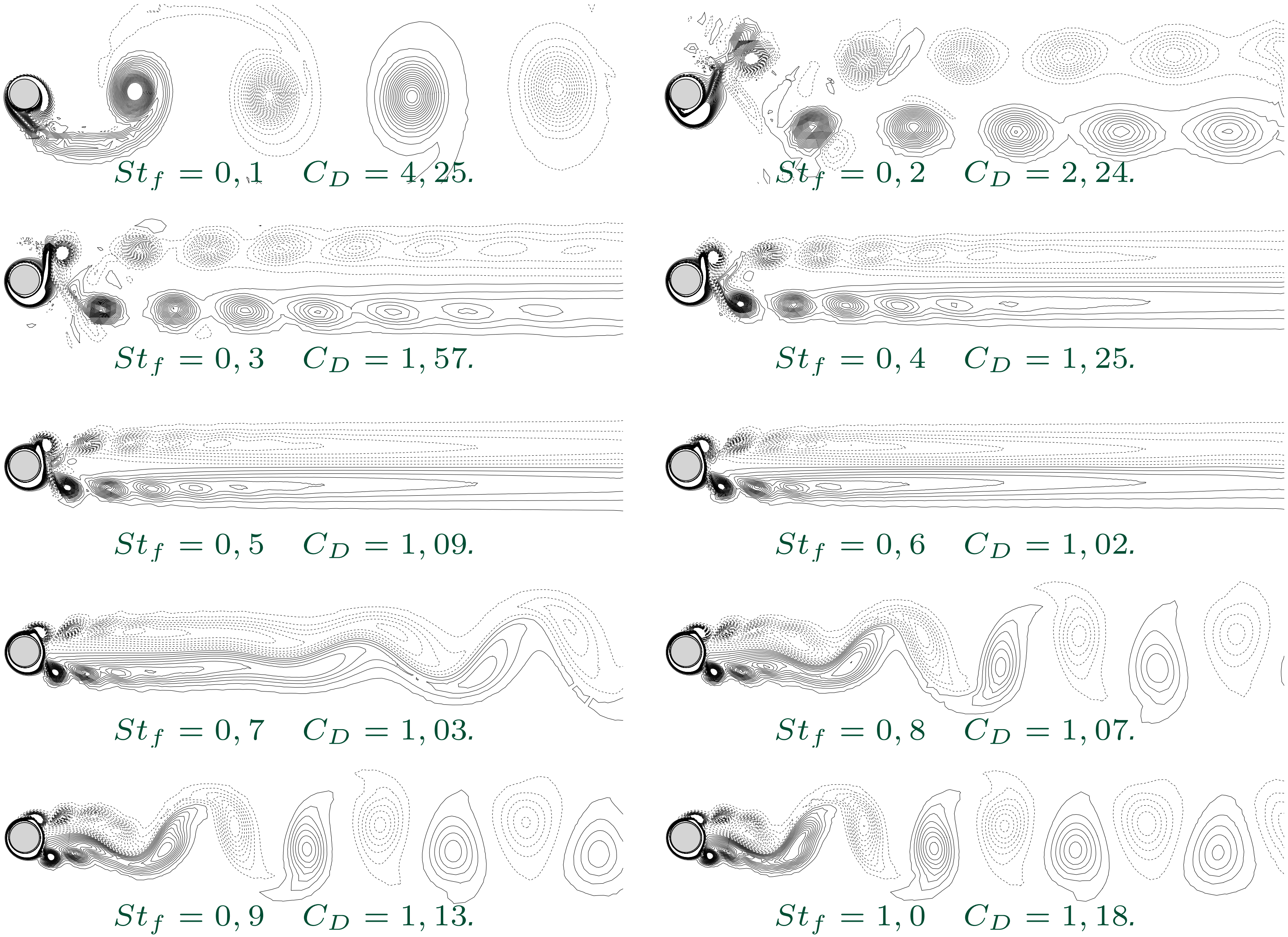


Fig. : *Iso-values of the vorticity fields ω_z for $A = 3$*

- Necessity for a given reference flow to introduce new modes : either new operating conditions or shift-modes

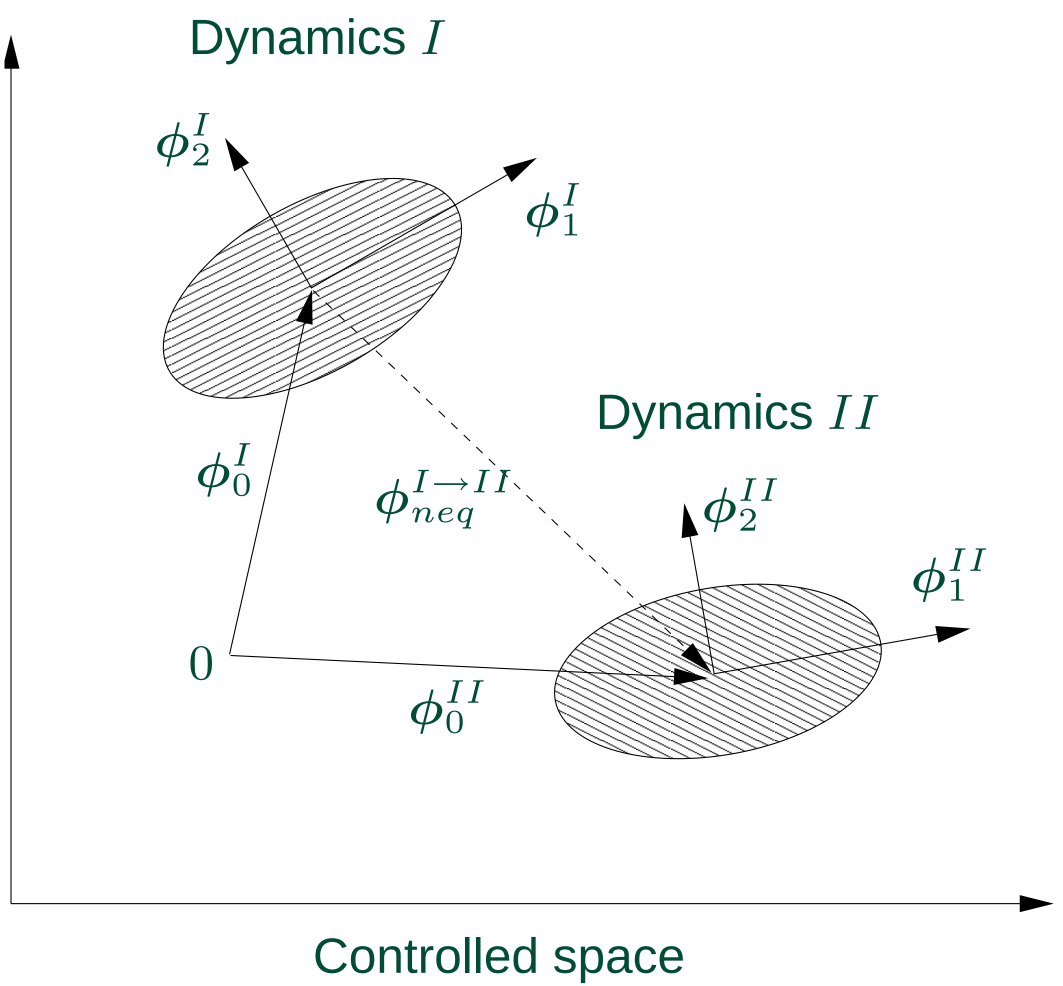


Fig. : *Schematic representation of a dynamical transition with a non-equilibrium mode*

- POD approximations consistent with our approach :

$$\boldsymbol{U}(\boldsymbol{x},\, t) = (u, v, p)^T = \underbrace{\sum_{i=0}^{N} a_i(t)\, \boldsymbol{\phi}_i(\boldsymbol{x})}_{\text{Galerkin modes}} + \underbrace{\sum_{i=N+1}^{N+M} a_i(t)\, \boldsymbol{\phi}_i(\boldsymbol{x})}_{\text{non-equilibrium modes}} + \underbrace{\gamma(\boldsymbol{c},\, t)\, \boldsymbol{U}_c(\boldsymbol{x})}_{\text{control function}}$$

Physical aspects	Modes	Dynamical aspects
actuation mode	$\boldsymbol{U}_c$	predetermined dynamics
mean flow mode	$\boldsymbol{U}_m,\, i = 0$	$a_0 = 1$
Galerkin modes Dynamics of the reference flow I	$i = 1$	**POD ROM** Temporal dynamics of the modes (eventually, the mode $i = 0$ is solved then $a_0 \equiv a_0(t)$)
	$i = 2$	
	$\cdots$	
	$i = N$	
non-equilibrium modes Inclusion of new operating conditions $II, III, IV, \cdots$	$i = N + 1$	
	$\cdots$	
	$i = N + M$	

ROM *Galerkin projection*

▶ Galerkin projection of *NSE* onto the POD basis :

$$\left(\boldsymbol{\phi}_i,\, \frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{\nabla}\cdot(\boldsymbol{u}\otimes\boldsymbol{u})\right) = \left(\boldsymbol{\phi}_i,\, -\boldsymbol{\nabla} p + \frac{1}{Re}\Delta\boldsymbol{u}\right).$$

▶ Reduced order dynamical system where only $(N+M+1)$ $(\ll N_{POD})$ modes are retained (state equations) :

$$\begin{aligned}\frac{d\,a_i(t)}{d\,t} &= \sum_{j=0}^{N+M}\mathcal{B}_{ij}\,a_j(t) + \sum_{j=0}^{N+M}\sum_{k=0}^{N+M}\mathcal{C}_{ijk}\,a_j(t)a_k(t)\\ &\quad + \mathcal{D}_i\,\frac{d\,\gamma}{d\,t} + \left(\mathcal{E}_i + \sum_{j=0}^{N+M}\mathcal{F}_{ij}\,a_j(t)\right)\gamma(\boldsymbol{c},\,t) + \mathcal{G}_i\gamma^2(\boldsymbol{c},\,t),\\ a_i(0) &= (\boldsymbol{U}(\boldsymbol{x},\,0),\,\boldsymbol{\phi}_i(\boldsymbol{x})).\end{aligned}$$

$\mathcal{B}_{ij}$, $\mathcal{C}_{ijk}$, $\mathcal{D}_i$, $\mathcal{E}_i$, $\mathcal{F}_{ij}$ and $\mathcal{G}_i$ depend on $\boldsymbol{\phi}_i$, $\boldsymbol{U}_c$ and Re.

Surrogate drag function and model objective function *Generalities*

- Drag operator :

$$
\begin{aligned}
\mathcal{C}_{\mathcal{D}} &: \mathbb{R}^3 \longrightarrow \mathbb{R} \\
\boldsymbol{u} &\mapsto 2\int_0^{2\pi} \left(u_3 n_x - \frac{1}{Re}\frac{\partial u_1}{\partial x} n_x - \frac{1}{Re}\frac{\partial u_1}{\partial y} n_y \right) R\, d\theta,
\end{aligned} \tag{1}
$$

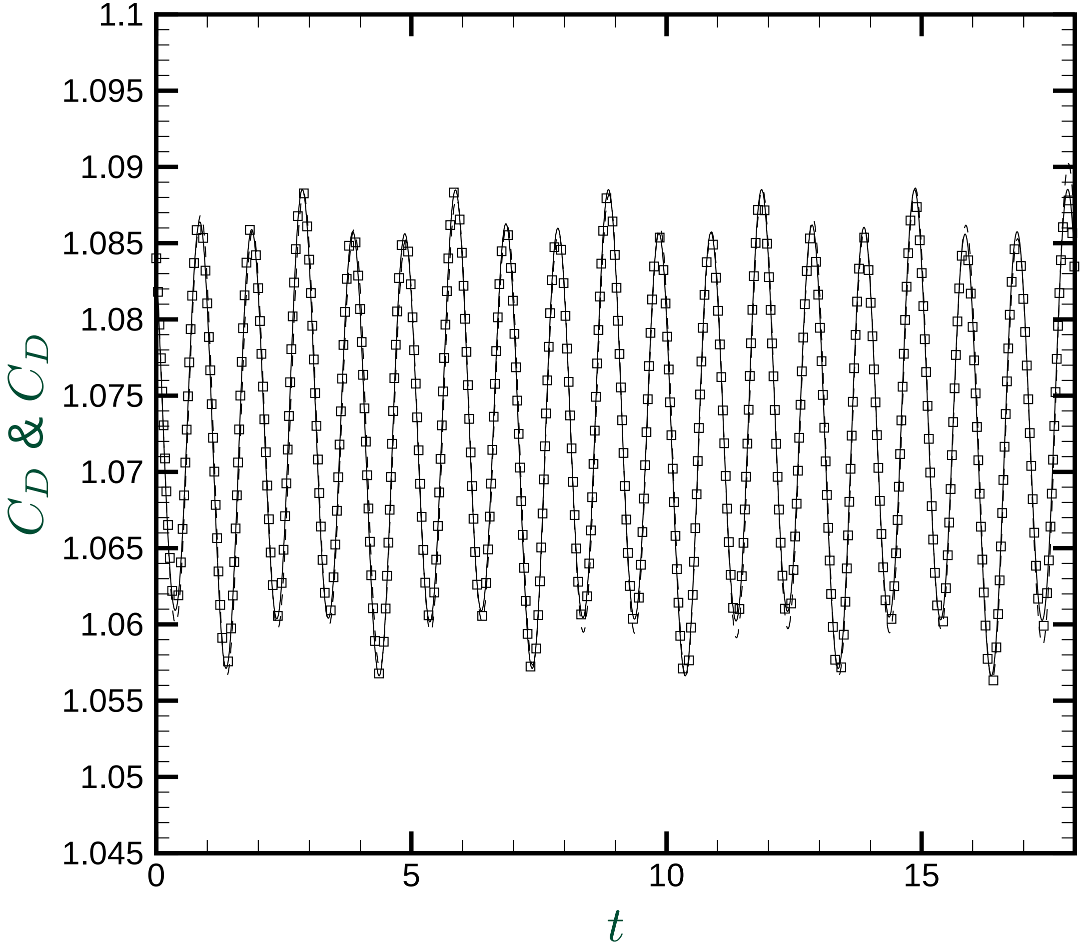
- Surrogate drag function :

$$
\widetilde{C_D}(t) = \underbrace{a_0(t)N_0 + \sum_{i=N+1}^{N+M} a_i(t)N_i}_{\text{evolution of the mean drag}} + \underbrace{\sum_{i=1}^{N} a_i(t)N_i}_{\text{fluctuations } C'_D(t)} \qquad \text{with } N_i = \mathcal{C}_{\mathcal{D}}(\boldsymbol{\phi}_i).
$$

- Model objective function :

$$
m = \langle \widetilde{C_D}(t) \rangle_T = \frac{1}{T}\int_0^T \left(a_0(t)N_0 + \sum_{i=N+1}^{N+M} a_i(t)N_i \right) dt.
$$

- Comparison of real drag coefficient C_D (symbols) and model function $\widetilde{C_D}$ (lines) at the design parameters.



Robustness of the model objective function *Test case $A = 2$ and $St = 0.5$*

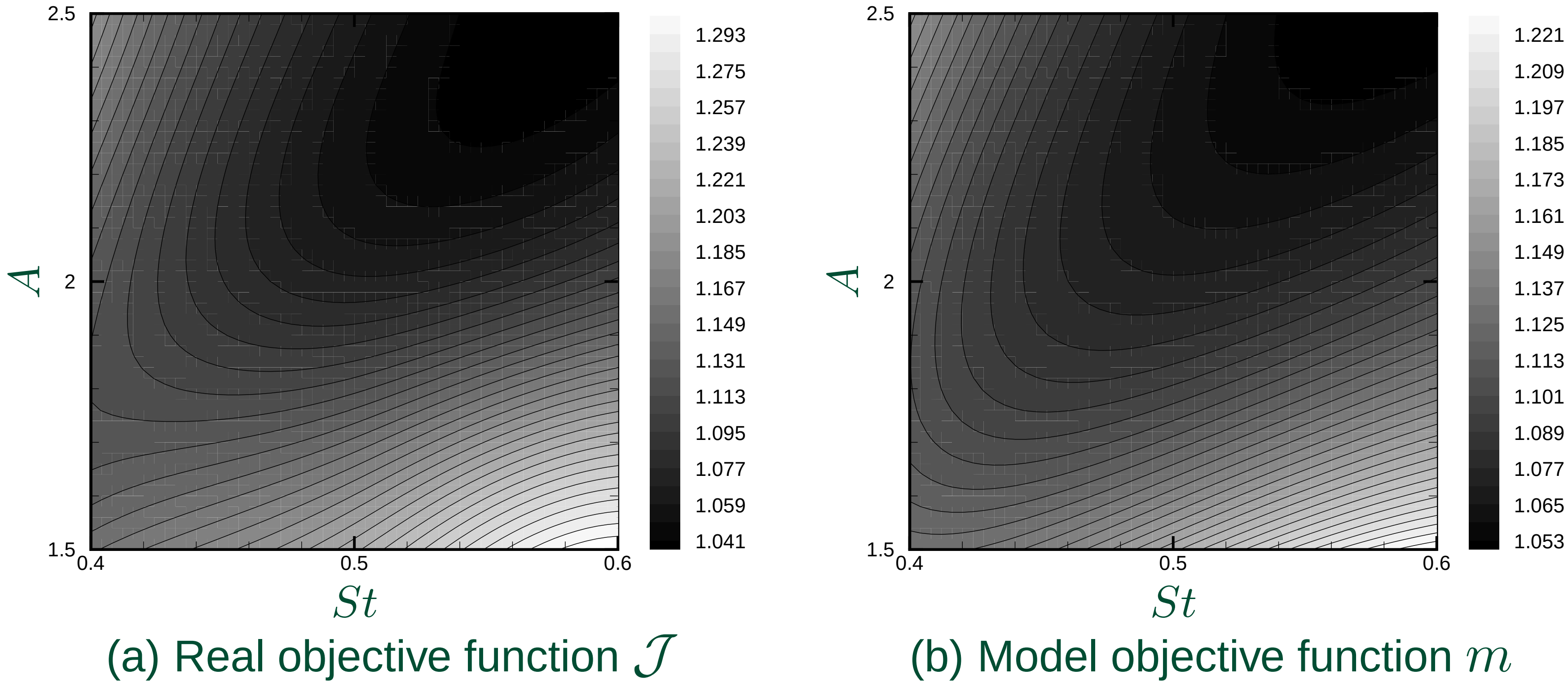


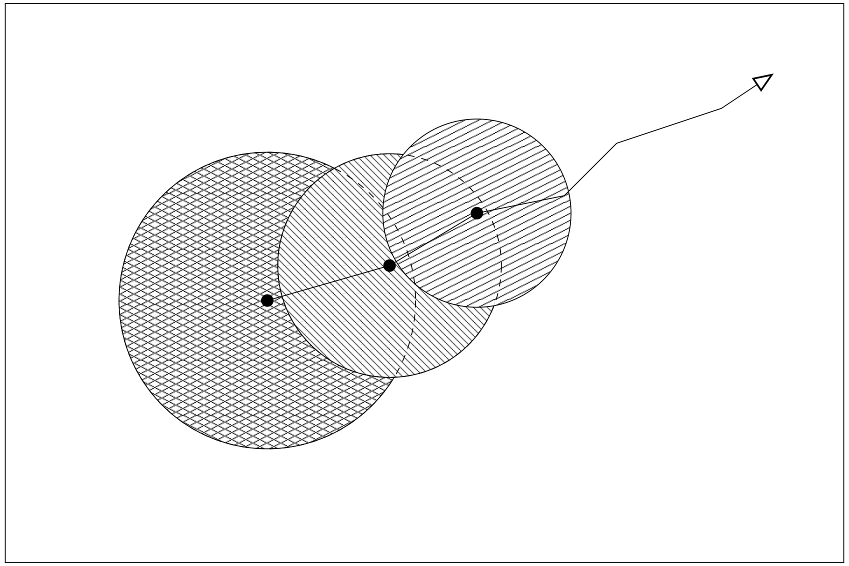
Fig. : *Comparison of the real and the model objective functions associated to the mean drag coefficient.*

Range of validity of the POD ROM restricted to a vicinity of the design parameters

Objective : Use ROMs to solve large-scale optimization problems with assurance of :

1. Automatic restriction of the range of validity

2. Global convergence



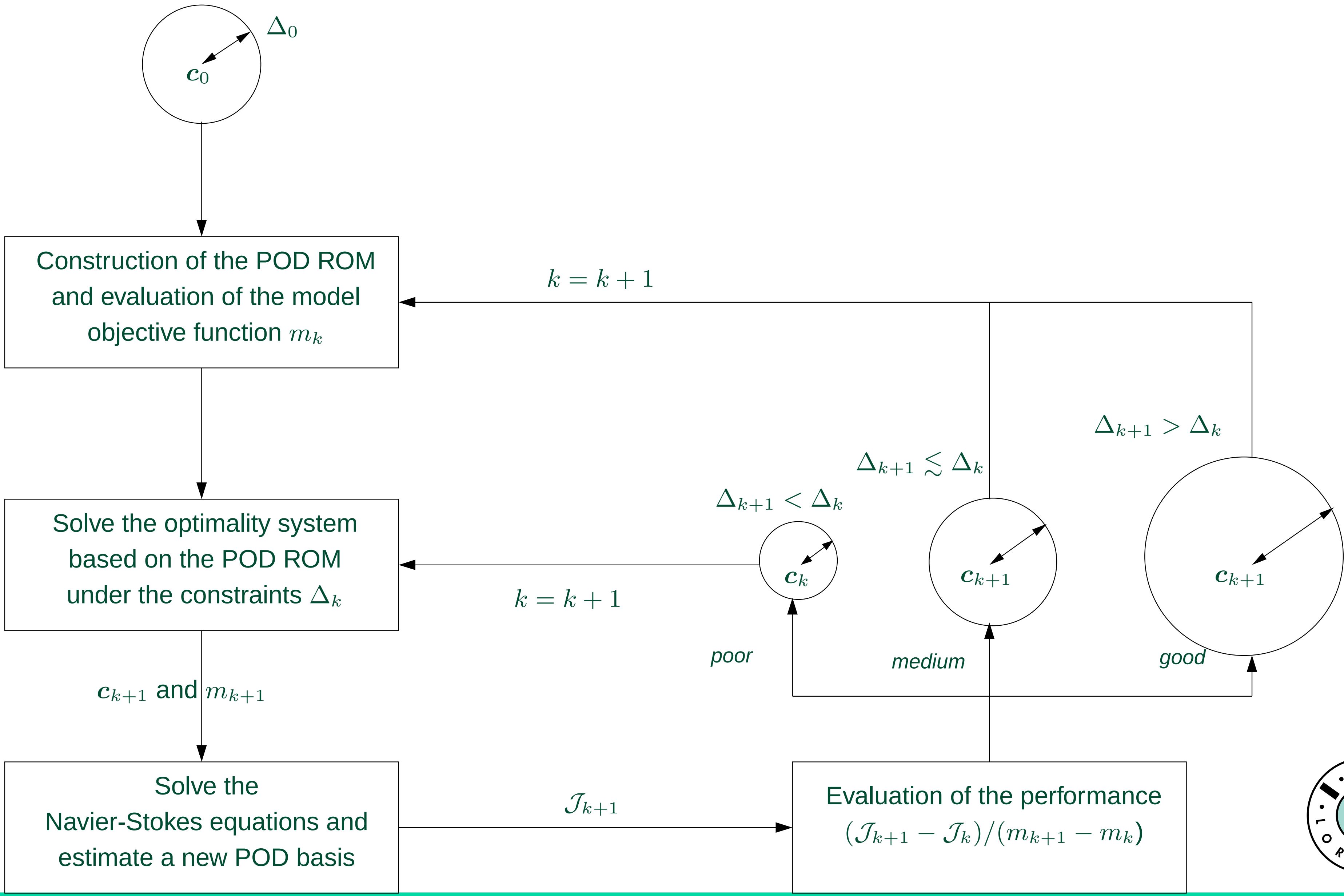
Solution

- Embed the POD technique into the concept of trust-region methods :
  Trust-Region Proper Orthogonal Decomposition (Fahl, 2000)

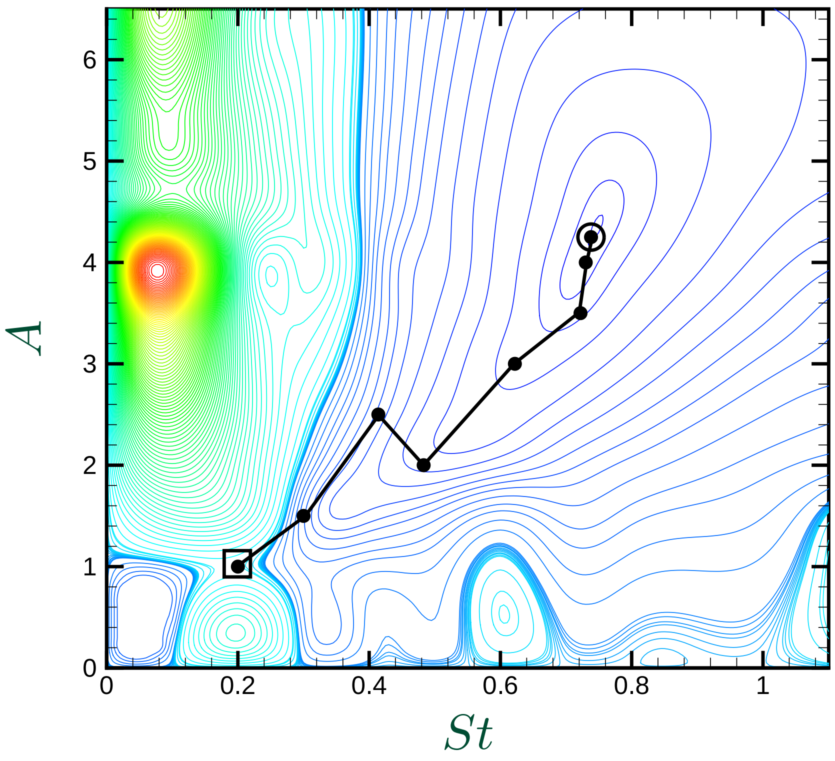
Conn, A.R., Gould, N.I.M. et Toint, P.L. (2000) : Trust-region methods. *SIAM, Philadelphia.*

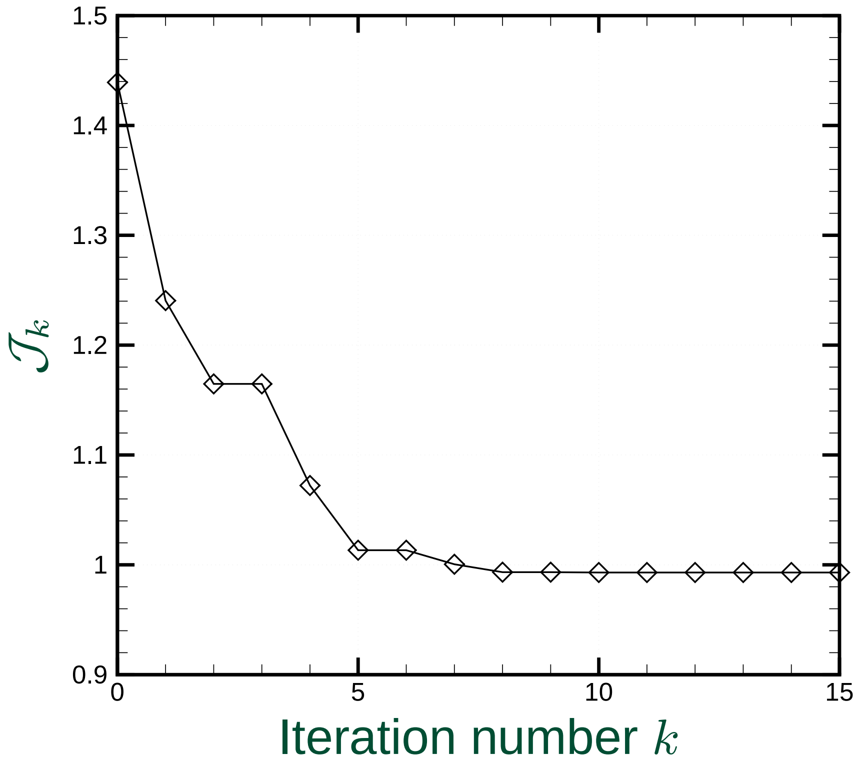
Trust-Region Proper Orthogonal Decomposition (TRPOD) *Algorithm*

Initialization : $\boldsymbol{c}_0$, Navier-Stokes resolution, $\mathcal{J}_0$. $k = 0$.



Initial control parameters : $A = 1.0$ et $St = 0.2$



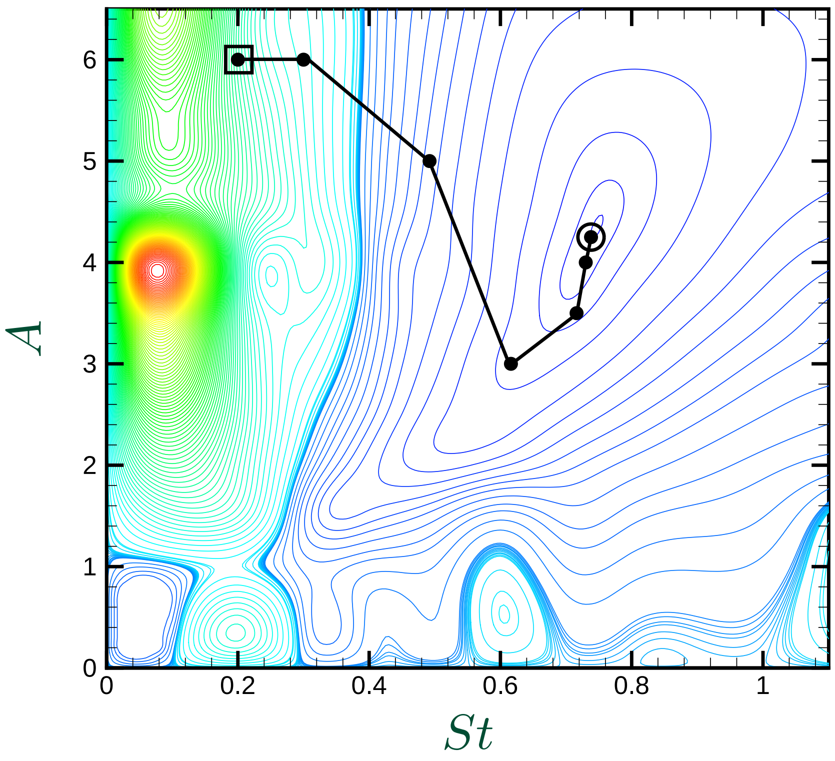


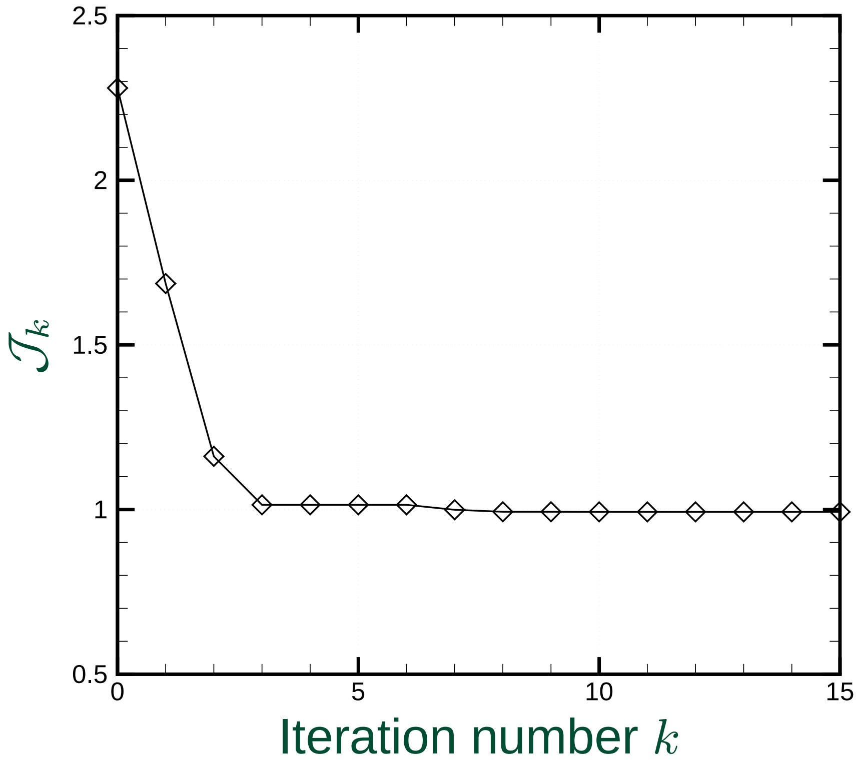
Optimal control parameters : $A = 4.25$ et $St = 0.74$

Mean drag coefficient : $\mathcal{J} = 0.993$

8 resolutions of the Navier-Stokes equations

Initial control parameters : $A = 6.0$ et $St = 0.2$



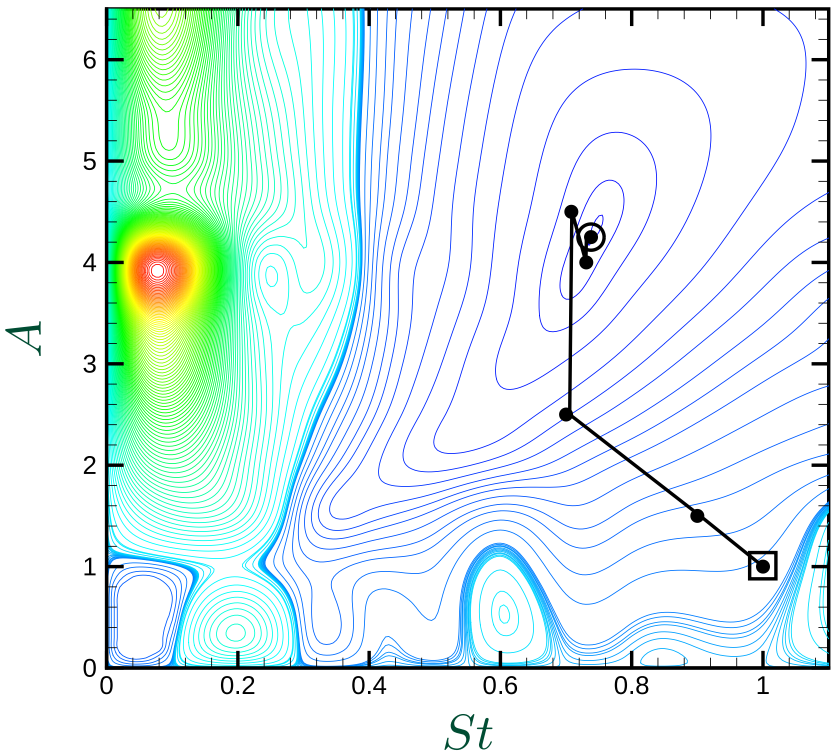


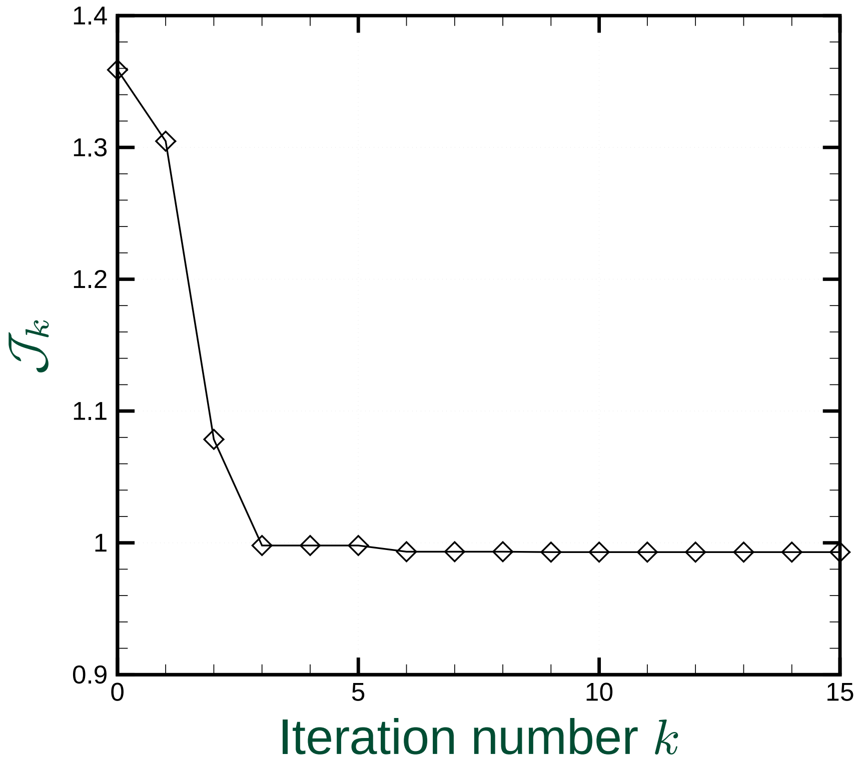
Optimal control parameters : $A = 4.25$ and $St = 0.74$

Mean drag coefficient : $\mathcal{J} = 0.993$

6 resolutions of the Navier-Stokes equations

Initial control parameters : $A = 1.0$ et $St = 1.0$



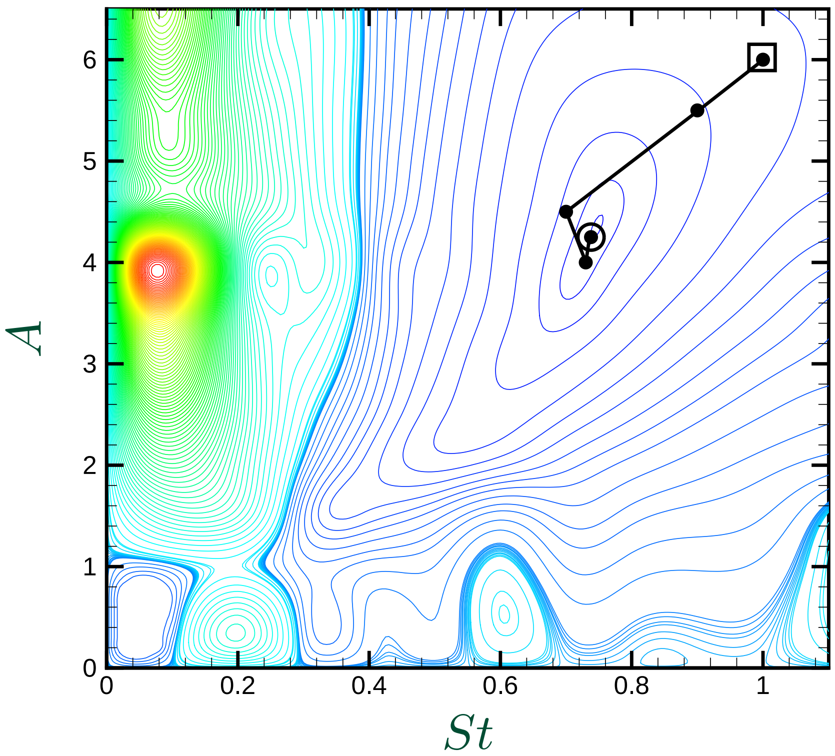


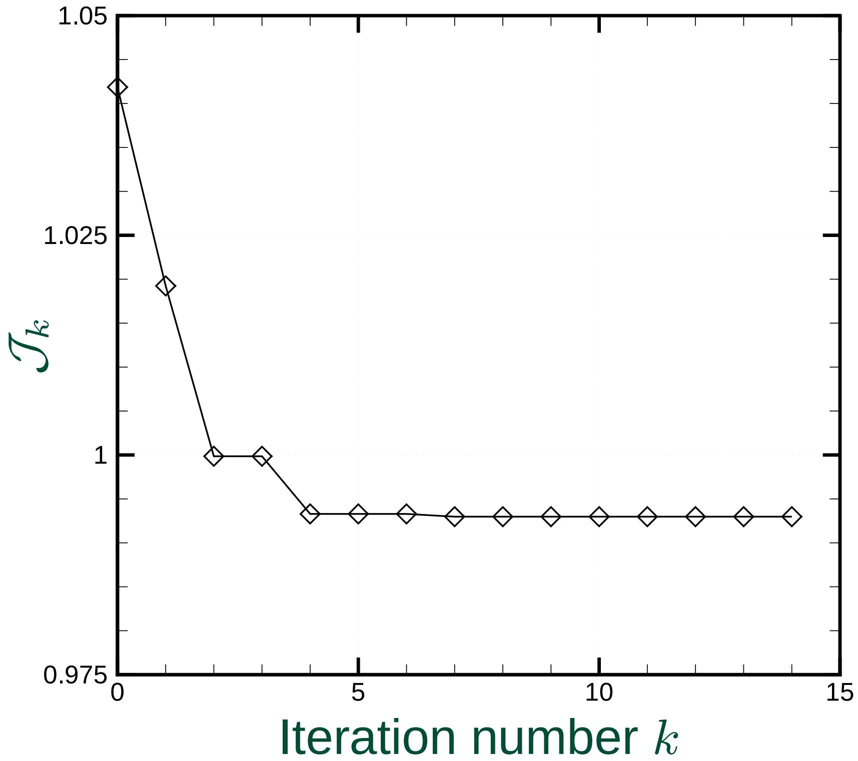
Optimal control parameters : $A = 4.25$ and $St = 0.74$

Mean drag coefficient : $\mathcal{J} = 0.993$

5 resolutions of the Navier-Stokes equations

Initial control parameters : $A = 6.0$ et $St = 1.0$



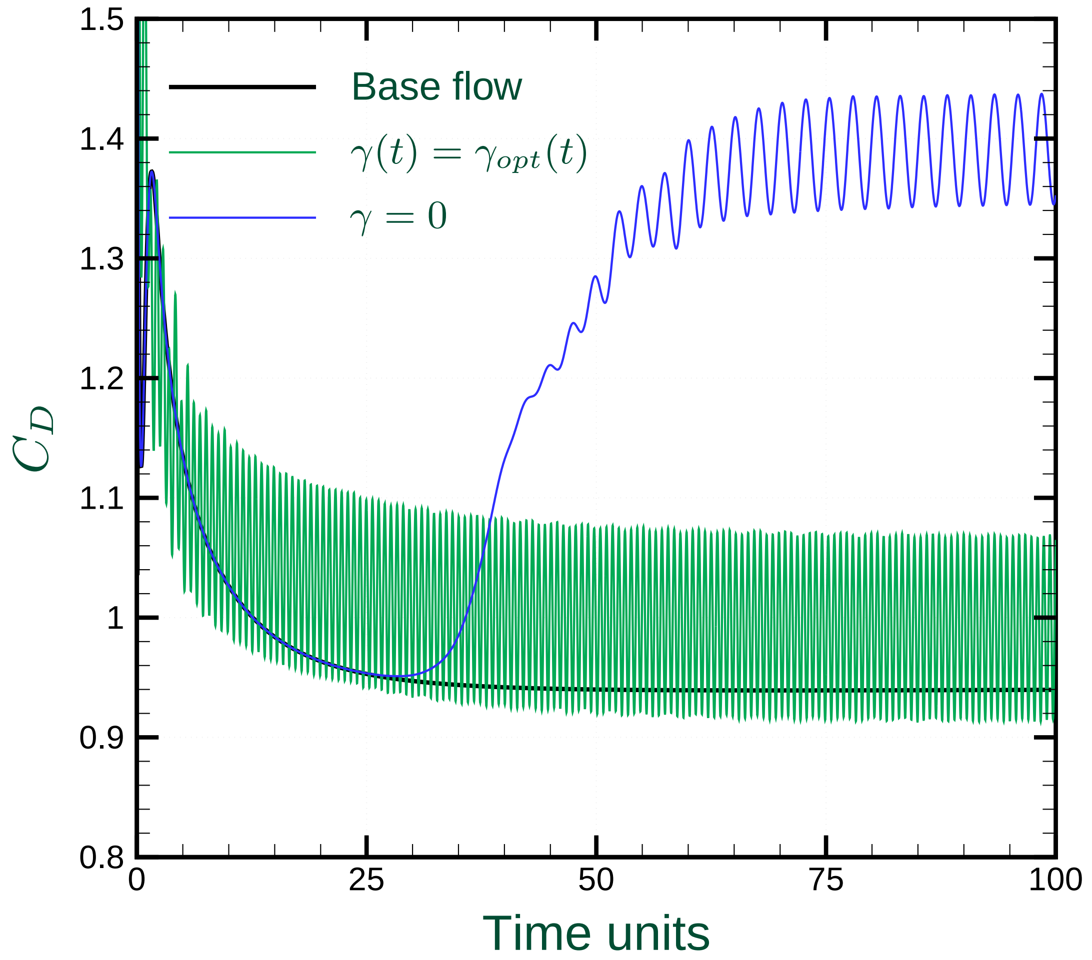


Optimal control parameters : $A = 4.25$ and $St = 0.74$

Mean drag coefficient : $\mathcal{J} = 0.993$

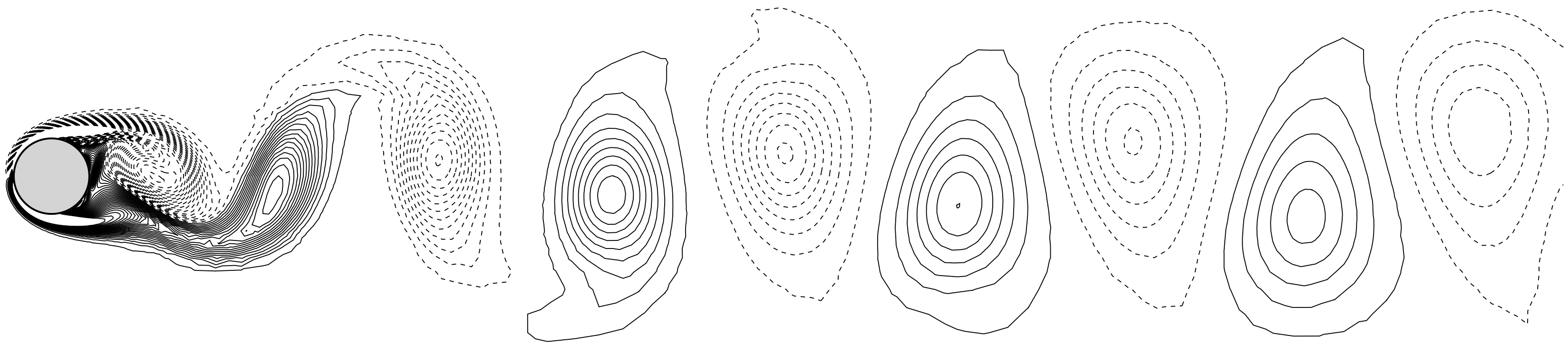
4 resolutions of the Navier-Stokes equations

- Optimal control law : $\gamma_{opt}(t) = A\sin(2\pi St\, t)$ avec $A = 4.25$ et $St = 0.74$

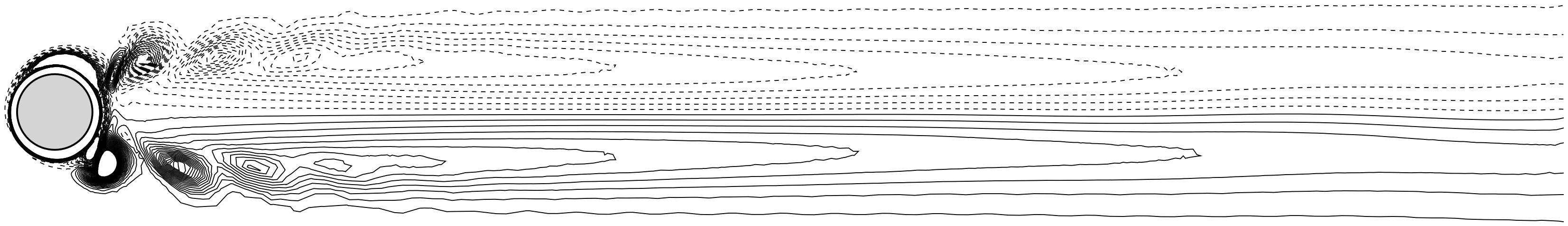


- Relative drag reduction of 30% ($\mathcal{J}_0 = 1,4 \;\Rightarrow\; \mathcal{J}_{opt} = 0,99$)

TRPOD *Vorticity contour plots*



Uncontrolled flow, $\gamma = 0$.



Controlled flow, $\gamma = \gamma_{opt}$.

Fig. : *Iso-values of vorticity* ω_z.

Controlled flow : near wake strongly unsteady, far wake (after 5 diameters) steady and symmetric $\rightarrow$ steady unstable base flow

Numerical costs *Discussion*

- Optimal control of NSE by He *et al.* (2000) :

  $\Rightarrow 30\%$ drag reduction for $A = 3$ and $S_t = 0.75$.

- Optimal control POD ROM by Bergmann *et al.* (2005) with no reactualization of the POD ROM :

  $\Rightarrow 25\%$ drag reduction for $A = 2.2$ and $S_t = 0.53$.

  - Reduction costs compared to NSE :
    - CPU time : 100
    - Memory storage : 600

  but no mathematical proof concerning the Navier-Stokes optimality.

- TRPOD (this study) :

  $\Rightarrow$ More than 30% of drag reduction for $A = 4.25$ and $S_t = 0.738$.

  - Reduction costs compared to NSE :
    - CPU time : 4
    - Memory storage : 400

  but global convergence.

$\hookrightarrow$ "Optimal" control of 3D flows becomes possible !

Conclusions and perspectives

- Conclusions on TRPOD
  - Important relative drag reduction : more than 30% of relative drag reduction
  - Global convergence : mathematical assurance that the solution is identical to the one of the high-fidelity model
  - TRPOD compared to NSE $\Rightarrow$ important reduction of numerical costs :
    $\hookrightarrow$ Reduction factor of the CPU time : 4
    $\hookrightarrow$ Reduction factor of the memory storage : 400

  "OPTIMAL" CONTROL OF 3D FLOWS POSSIBLE BY POD ROM

- Perspectives
  - Optimal control of the channel flow at $Re_\tau = 180$
  - Test other reduced basis method than classical POD
    - Centroidal Voronoi Tessellations (Gunzburguer, 2004) : "intelligent" sampling in the control parameter space
    - Balanced POD (Rowley, 2004)
    - Model-based POD (Willcox, CDC-ECC 2005) : modify the definitions of the POD modes

Reverse von Karman flow

- Contrôle partiel (3 paramètres de contrôle)

  $\Rightarrow$ Effet propulsif, signe écoulement moyen inversé

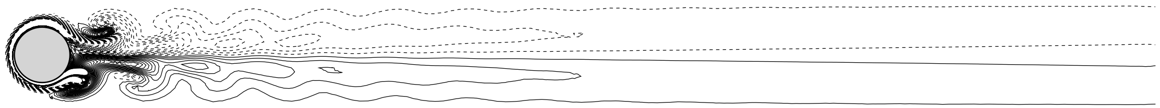


Fig. : *Contrôle sur la partie amont :* $-120^{\circ} \leq \theta \leq 120^{\circ}$