Control of the cylinder wake in the laminar regime by Trust-Region Proper Orthogonal Decomposition.

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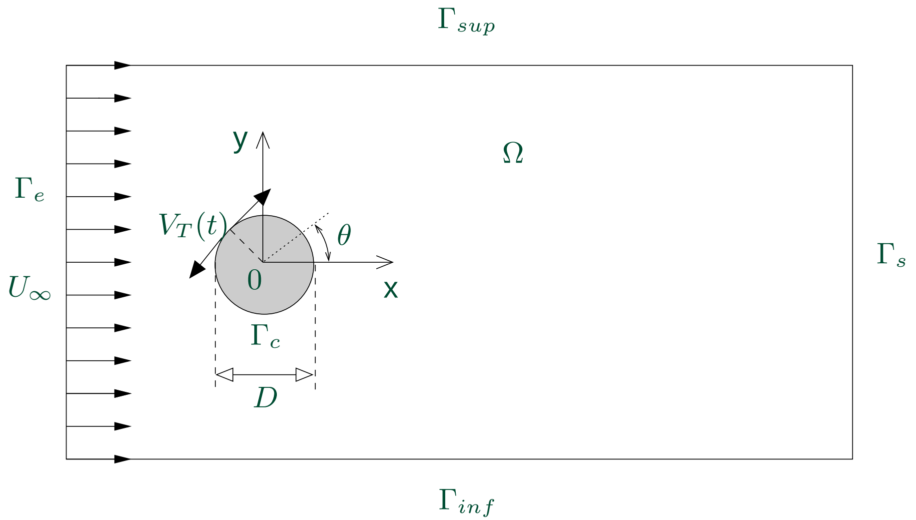
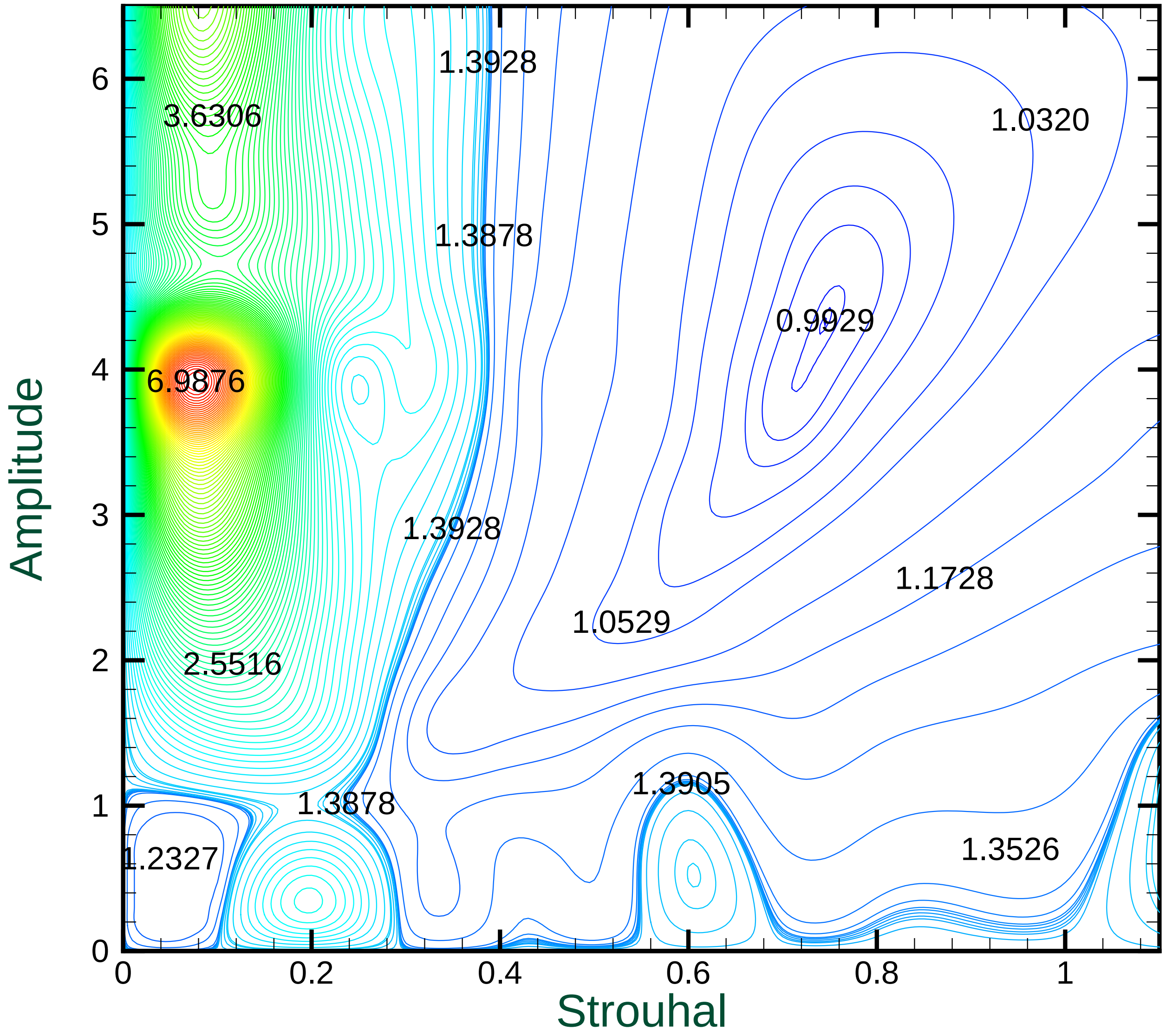
- Two dimensional flow around a circular cylinder at $Re = 200$
- Viscous, incompressible and Newtonian fluid
- Cylinder oscillation with a tangential sinusoidal velocity $\gamma(t)$

$$\gamma(t) = \frac{V_T}{U_\infty} = A \sin(2\pi St_f t)$$



Find the control parameters $c = (A,\ St_f)^T$ such that the mean drag coefficient is minimized

$$\langle C_D \rangle_T = \frac{1}{T}\int_0^T \int_0^{2\pi} 2\, p\, n_x\, R\, d\theta\, dt - \frac{1}{T}\int_0^T \int_0^{2\pi} \frac{2}{Re}\left(\frac{\partial u}{\partial x} n_x + \frac{\partial u}{\partial y} n_y\right) R\, d\theta\, dt,$$



Variation of the mean drag coefficient with A and St_f.
Numerical minimum $(A_{min}, St_{f_{min}}) = (4.3, 0.74)$.

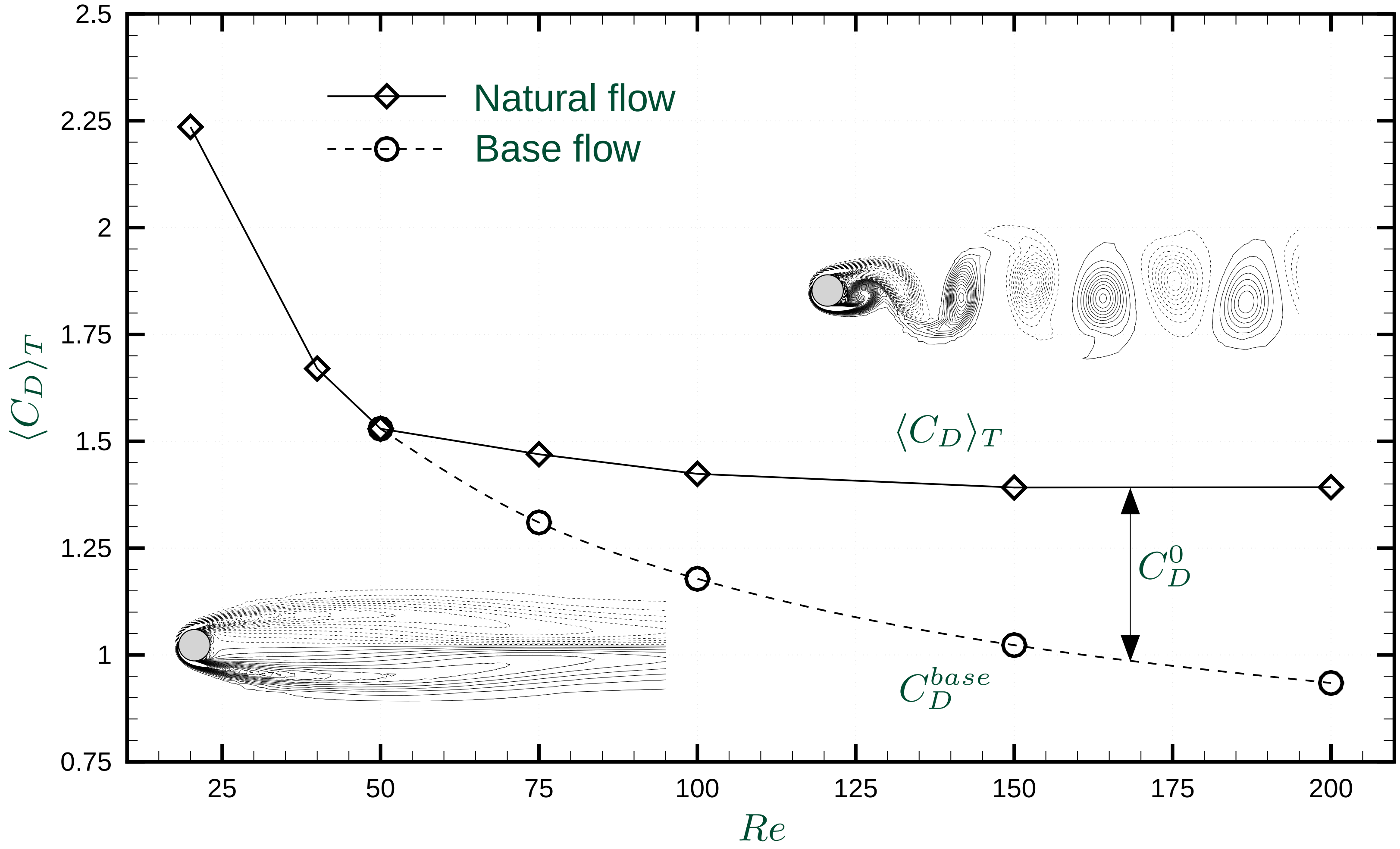
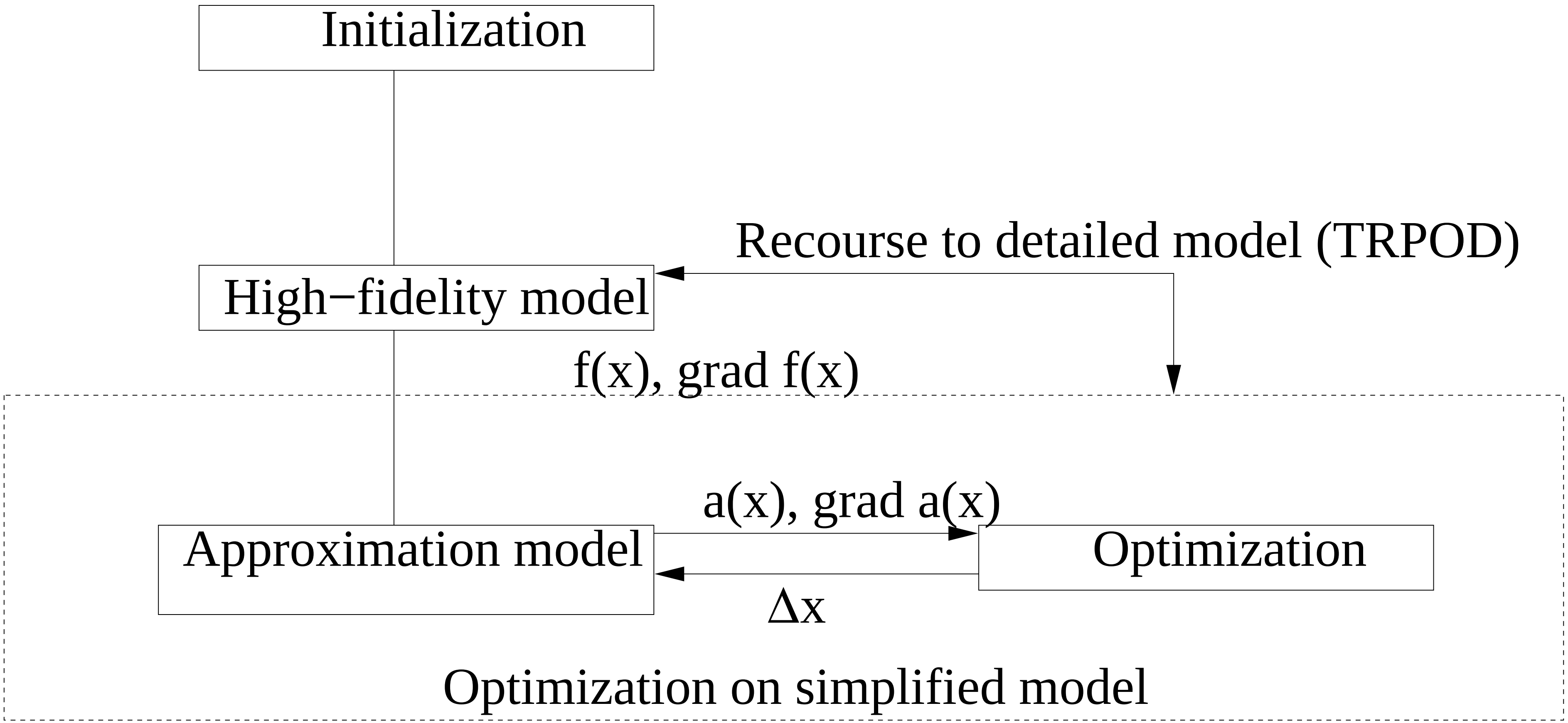


Fig. : *Variation with the Reynolds number of the mean drag coefficient. Contributions and corresponding flow patterns of the base flow and unsteady flow.*

Protas, B. et Wesfreid, J.E. (2002) : Drag force in the open-loop control of the cylinder wake in the laminar regime. *Phys. Fluids*, **14**(2), pp. 810-826.

Reduced Order Model (ROM) and optimization problems



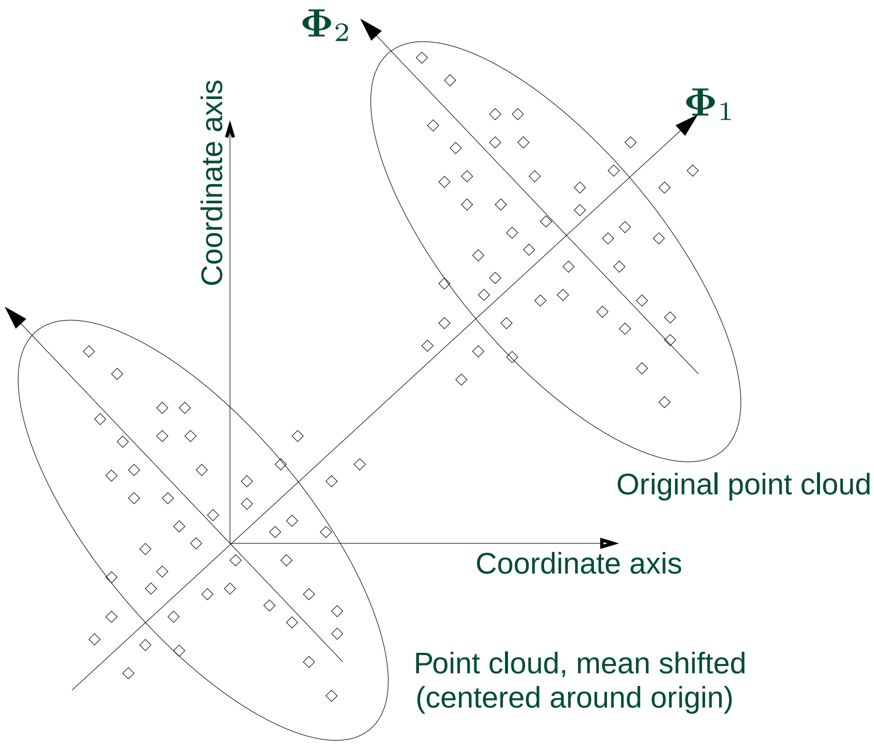
- Introduced in turbulence by Lumley (1967).
- Method of information compression
- Look for a realization $\mathbf{\Phi}(\boldsymbol{X})$ which is closer, in an average sense, to realizations $\boldsymbol{u}(\boldsymbol{X})$. ($\boldsymbol{X} = (\boldsymbol{x}, t) \in \mathcal{D} = \Omega \times \mathbb{R}^+$)
- $\mathbf{\Phi}(\boldsymbol{X})$ solution of the problem :

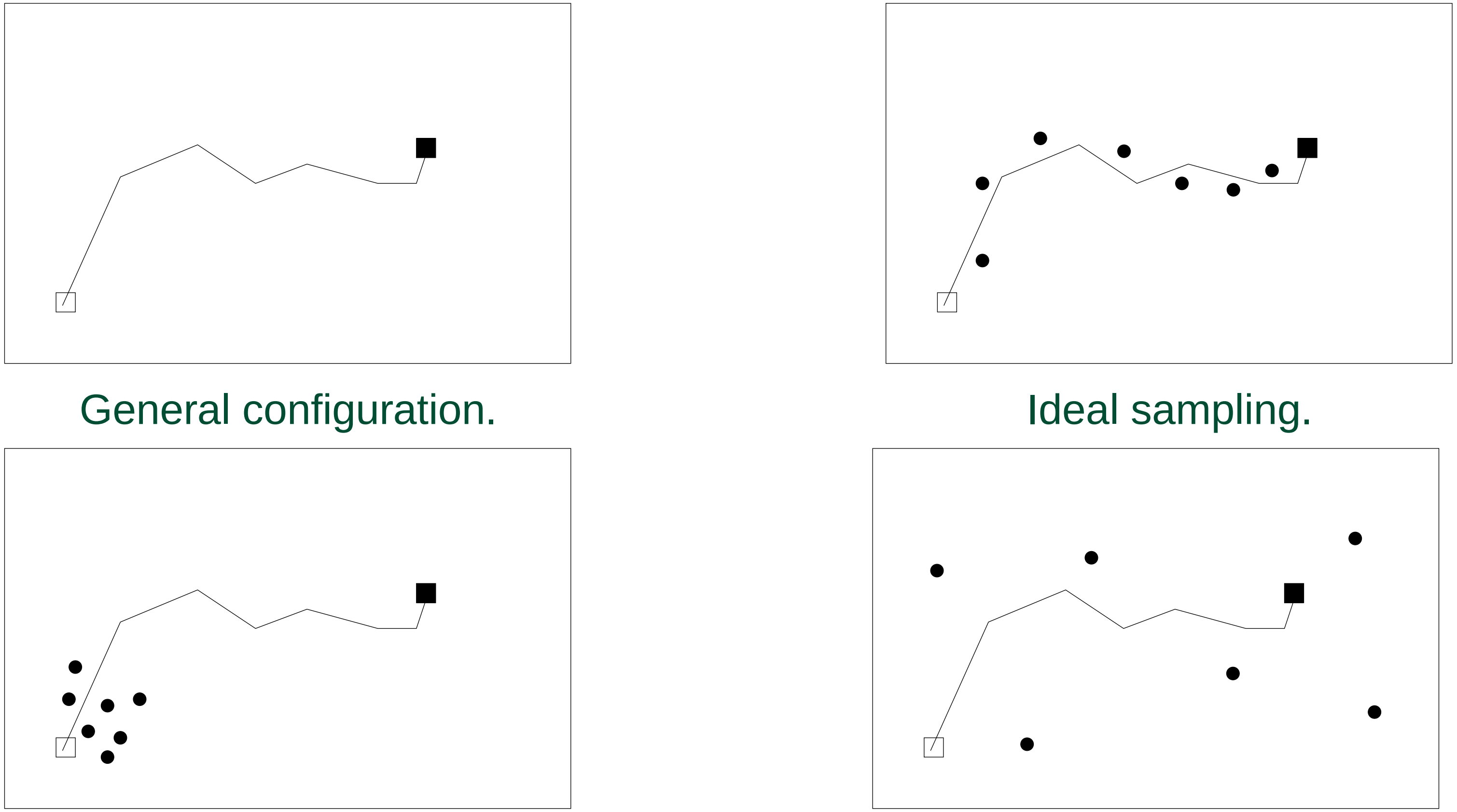
$$\max_{\mathbf{\Phi}} \langle |(\boldsymbol{u}, \mathbf{\Phi})|^2 \rangle \quad \text{s.t.} \quad \|\mathbf{\Phi}\|^2 = 1.$$

- Snapshots method, Sirovich (1987) :

$$\int_T C(t, t') a^{(n)}(t')\, dt' = \lambda^{(n)} a^{(n)}(t).$$

- Optimal convergence *in* L^2 *norm* (energy)of $\mathbf{\Phi}(\boldsymbol{X})$
$\Rightarrow$ Dynamical order reduction is possible.





General configuration. Ideal sampling.

Unsuitable sampling. Unsuitable sampling.

Discussion of parameter sampling in an optimization setting (from Gunzburger, 2004).
—— path to optimizer using full system, □ initial values, ■ optimal values, and ●
parameter values used for snapshot generation.

► Necessity for a given reference flow to introduce new modes : either new operating conditions or shift-modes

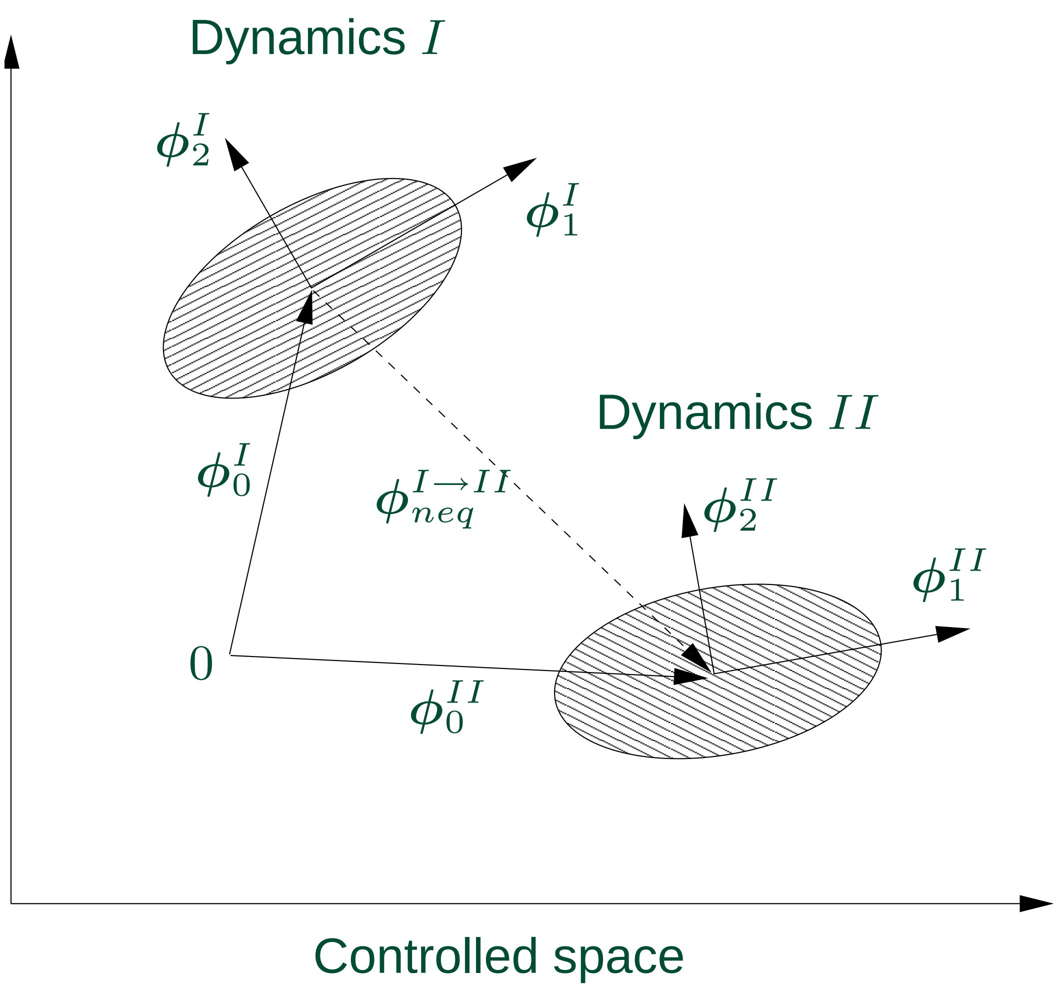


Fig. : *Schematic representation of a dynamical transition with a non-equilibrium mode*

- POD approximations consistent with our approach :

$$\boldsymbol{U}(\boldsymbol{x},\,t) = (u,v,p)^T = \underbrace{\sum_{i=0}^{N} a_i(t)\,\boldsymbol{\phi}_i(\boldsymbol{x})}_{\text{Galerkin modes}} + \underbrace{\sum_{i=N+1}^{N+M} a_i(t)\,\boldsymbol{\phi}_i(\boldsymbol{x})}_{\text{non-equilibrium modes}} + \underbrace{\gamma(\boldsymbol{c},\,t)\,\boldsymbol{U}_c(\boldsymbol{x})}_{\text{control function}}$$

Physical aspects	**Modes**	**Dynamical aspects**
actuation mode	$\boldsymbol{U}_c$	predetermined dynamics
mean flow mode	$\boldsymbol{U}_m,\, i=0$	$a_0 = 1$
Galerkin modes Dynamics of the reference flow I	$i=1$	**POD ROM** Temporal dynamics of the modes (eventually, the mode $i=0$ is solved then $a_0 \equiv a_0(t)$)
	$i=2$	
	$\cdots$	
	$i=N$	
non-equilibrium modes Inclusion of new operating conditions $II, III, IV, \cdots$	$i=N+1$	
	$\cdots$	
	$i=N+M$	

▶ Galerkin projection of *NSE* onto the POD basis :

$$\left(\boldsymbol{\phi}_i,\, \frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{\nabla}\cdot(\boldsymbol{u}\otimes\boldsymbol{u})\right) = \left(\boldsymbol{\phi}_i,\, -\boldsymbol{\nabla} p + \frac{1}{Re}\Delta \boldsymbol{u}\right).$$

▶ Reduced order dynamical system where only $(N+M+1)$ $(\ll N_{POD})$ modes are retained (state equations) :

$$\frac{d\,a_i(t)}{d\,t} = \sum_{j=0}^{N+M} \mathcal{B}_{ij}\,a_j(t) + \sum_{j=0}^{N+M}\sum_{k=0}^{N+M} \mathcal{C}_{ijk}\,a_j(t)a_k(t)$$
$$+\,\mathcal{D}_i\,\frac{d\,\gamma}{d\,t} + \left(\mathcal{E}_i + \sum_{j=0}^{N+M} \mathcal{F}_{ij}\,a_j(t)\right)\gamma(\boldsymbol{c},\,t) + \mathcal{G}_i\gamma^2(\boldsymbol{c},\,t),$$
$$a_i(0) = (\boldsymbol{U}(\boldsymbol{x},\,0),\,\boldsymbol{\phi}_i(\boldsymbol{x})).$$

$\mathcal{B}_{ij}$, $\mathcal{C}_{ijk}$, $\mathcal{D}_i$, $\mathcal{E}_i$, $\mathcal{F}_{ij}$ and $\mathcal{G}_i$ depend on $\boldsymbol{\phi}_i$, $\boldsymbol{U}_c$ and Re.

Surrogate drag function and model objective function

- Drag operator :

$$\mathcal{C}_{\mathcal{D}} : \mathbb{R}^3 \longrightarrow \mathbb{R}$$
$$\boldsymbol{u} \longmapsto 2\int_0^{2\pi} \left(u_3 n_x - \frac{1}{Re}\frac{\partial u_1}{\partial x} n_x - \frac{1}{Re}\frac{\partial u_1}{\partial y} n_y \right) R\, d\theta, \qquad (1)$$

- Surrogate drag function :

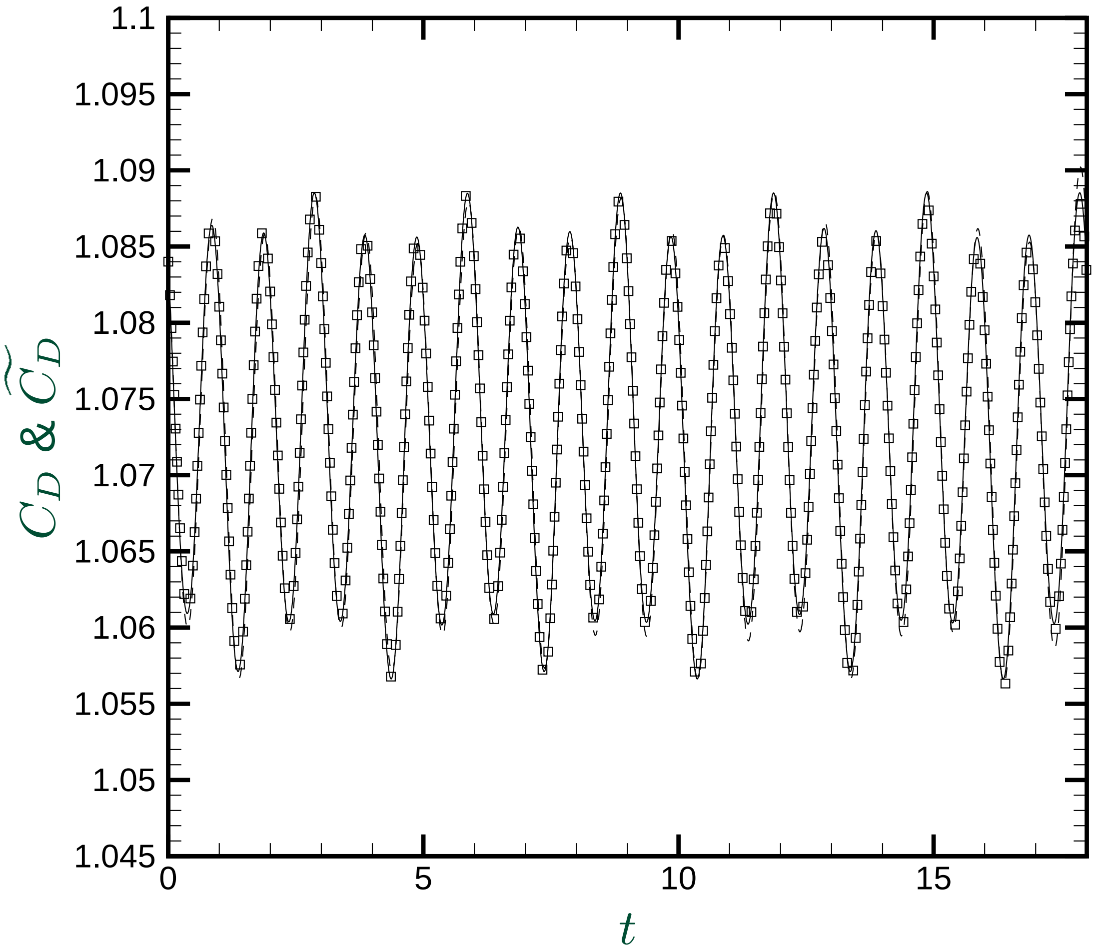
$$\widetilde{C_D}(t) = \underbrace{a_0(t)N_0 + \sum_{i=N+1}^{N+M} a_i(t)N_i}_{\text{evolution of the mean drag}} + \underbrace{\sum_{i=1}^{N} a_i(t)N_i}_{\text{fluctuations } C'_D(t)} \qquad \text{with } N_i = \mathcal{C}_{\mathcal{D}}(\boldsymbol{\phi}_i).$$

- Model objective function :

$$m = \langle \widetilde{C_D}(t) \rangle_T = \frac{1}{T}\int_0^T \left(a_0(t)N_0 + \sum_{i=N+1}^{N+M} a_i(t)N_i \right) dt.$$

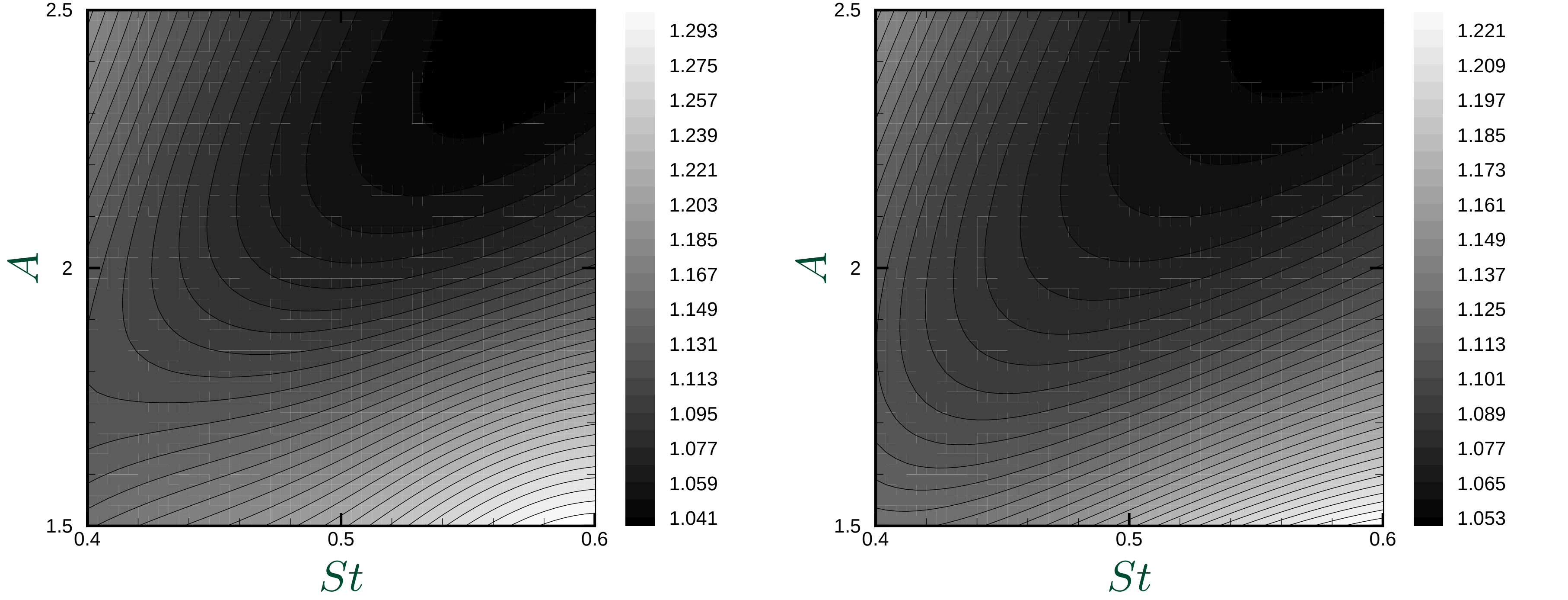
Surrogate drag function

- Comparison of real drag coefficient C_D (symbols) and model function $\widetilde{C_D}$ (lines) at the design parameters.



Robustness of the model objective function

Test case $A = 2$ and $St = 0.5$



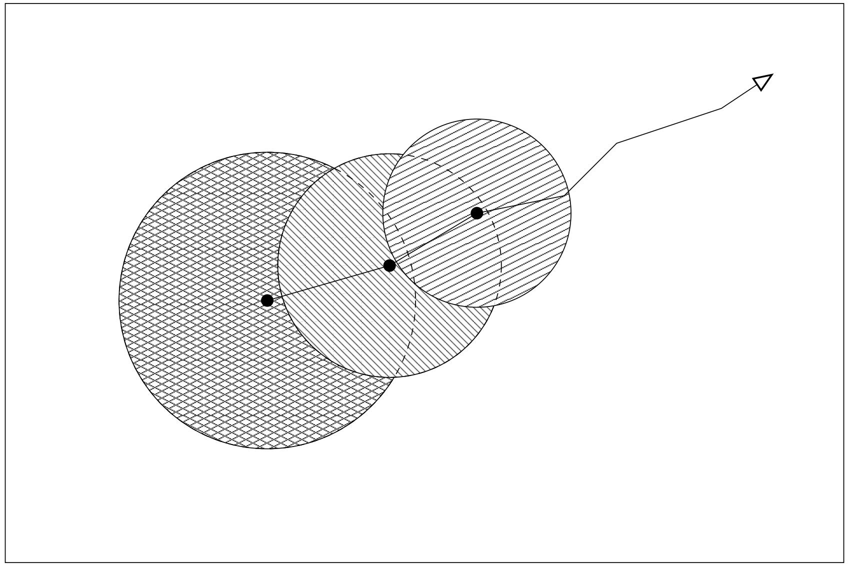
(a) Real objective function $\mathcal{J}$ (b) Model objective function m

Fig. : *Comparison of the real and the model objective functions associated to the mean drag coefficient.*

Range of validity of the POD ROM restricted to a vicinity of the design parameters

Objective : Use ROMs to solve large-scale optimization problems with assurance of :

1. Automatic restriction of the range of validity
2. Global convergence

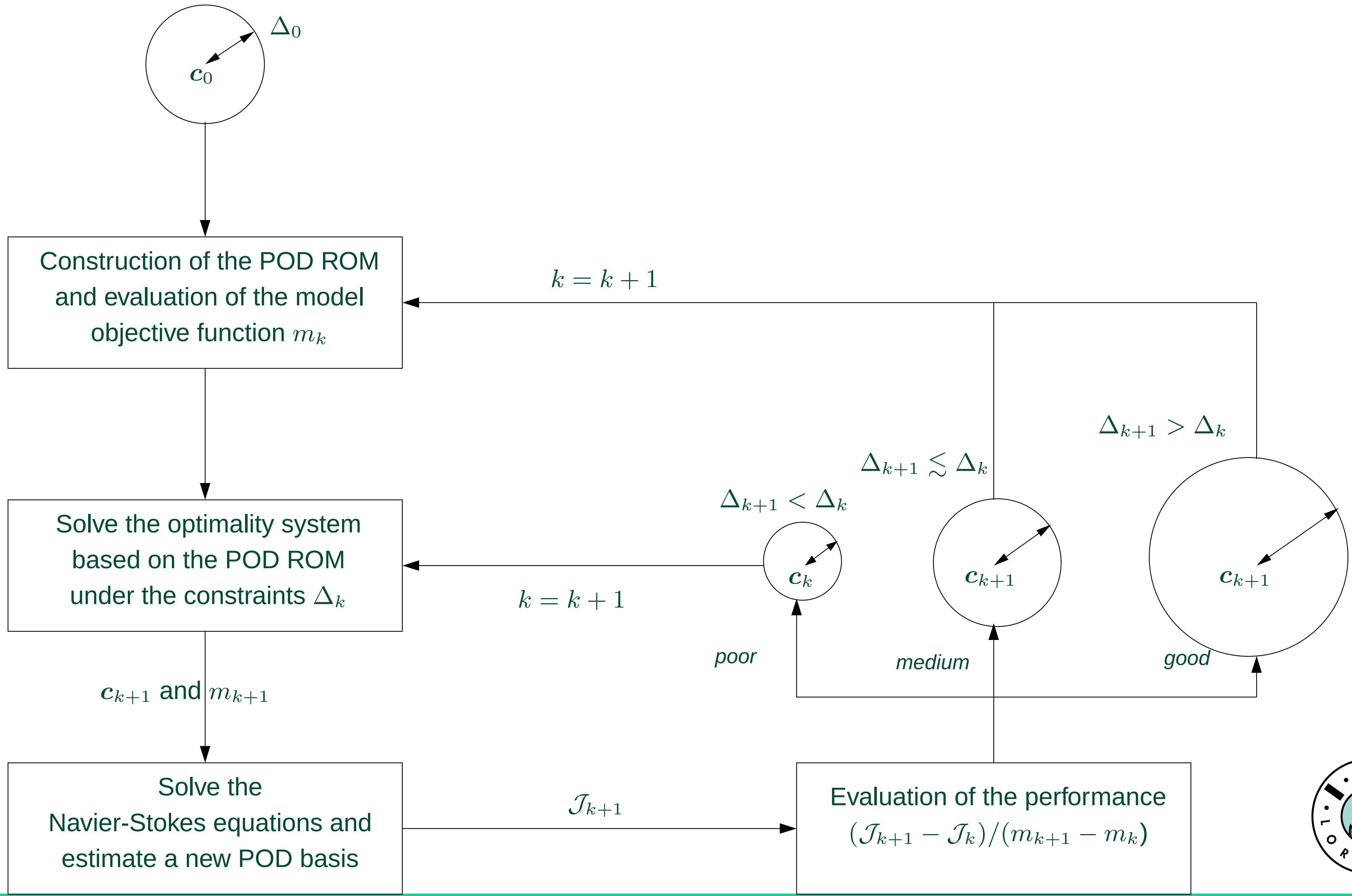


Solution

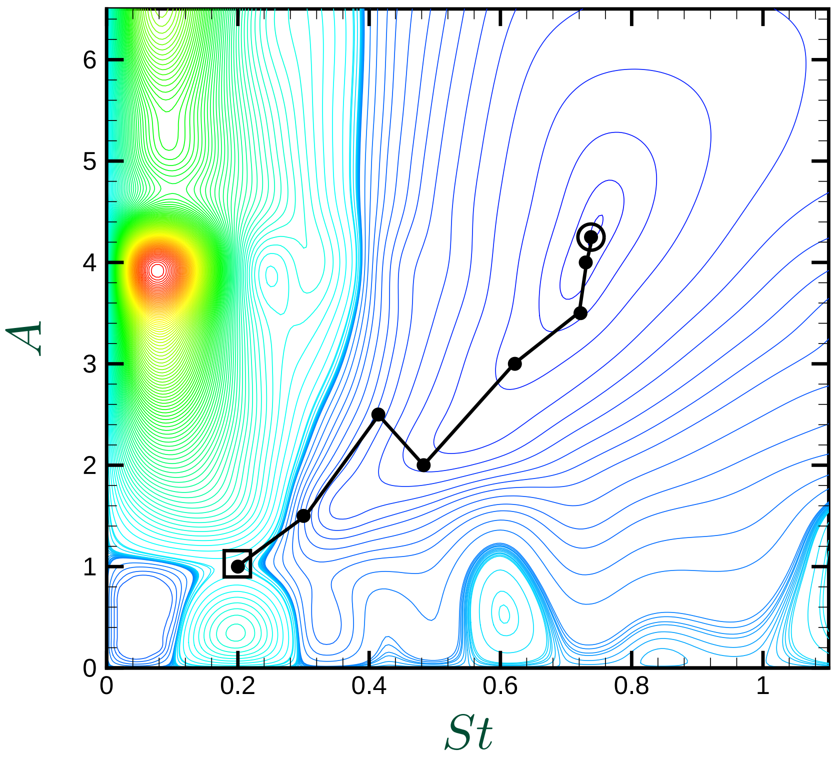
- Embed the POD technique into the concept of trust-region methods : Trust-Region Proper Orthogonal Decomposition (Fahl, 2000)

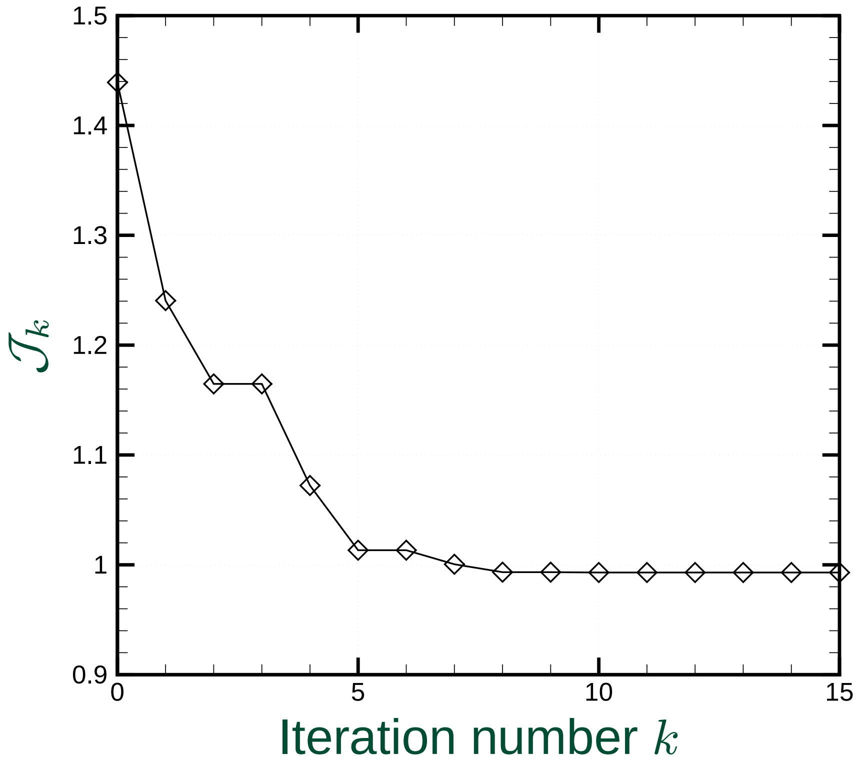
Conn, A.R., Gould, N.I.M. et Toint, P.L. (2000) : Trust-region methods. *SIAM, Philadelphia.*

Initialization : $\boldsymbol{c}_0$, Navier-Stokes resolution, $\mathcal{J}_0$. $k = 0$.



Initial control parameters : $A = 1.0$ et $St = 0.2$



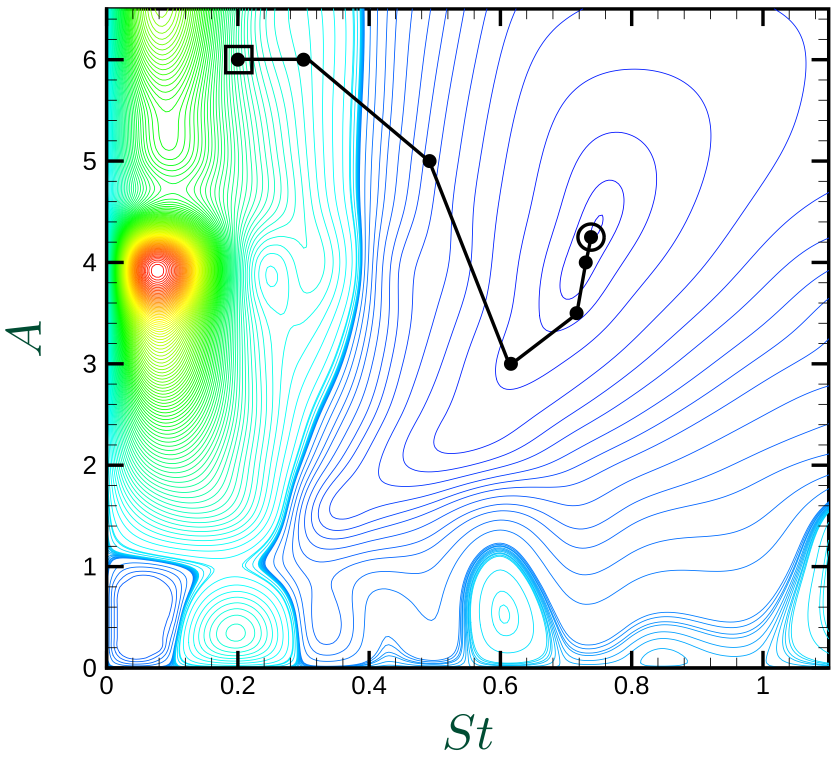


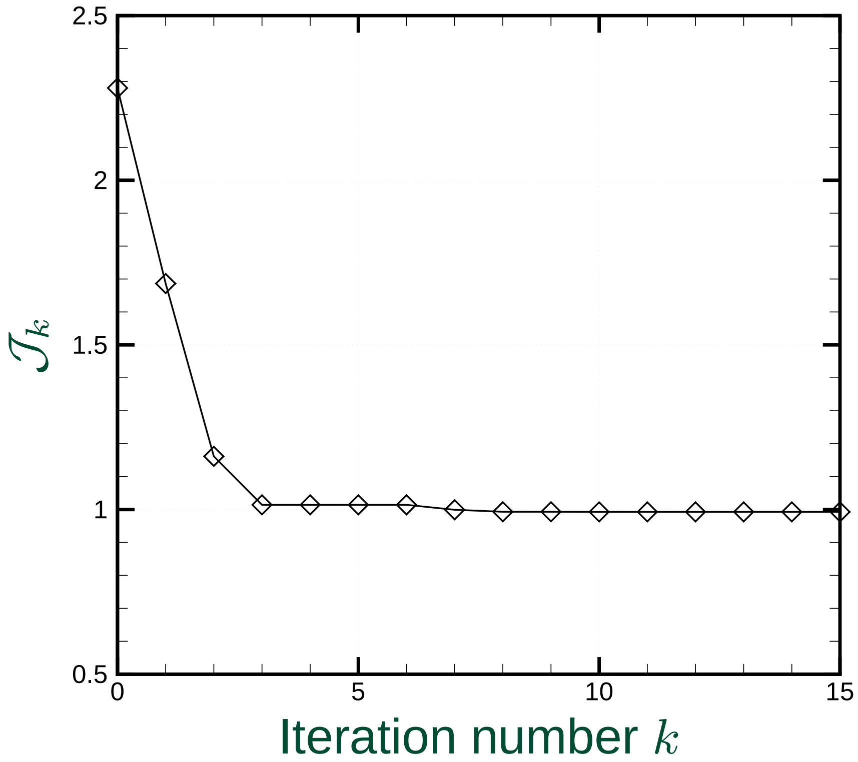
Optimal control parameters : $A = 4.25$ et $St = 0.74$

Mean drag coefficient : $\mathcal{J} = 0.993$

8 resolutions of the Navier-Stokes equations

Initial control parameters : $A = 6.0$ et $St = 0.2$



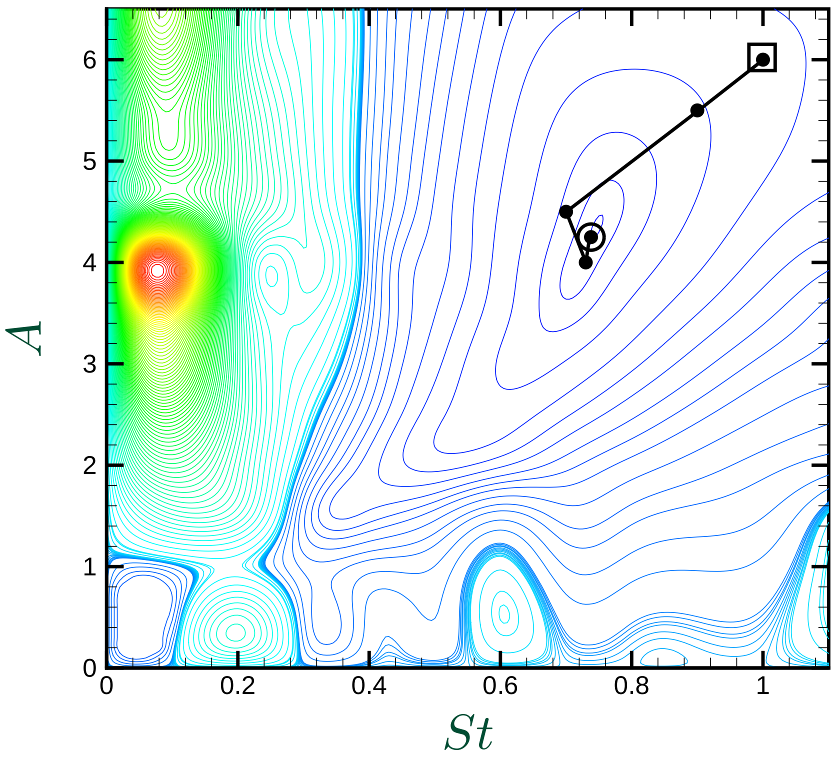


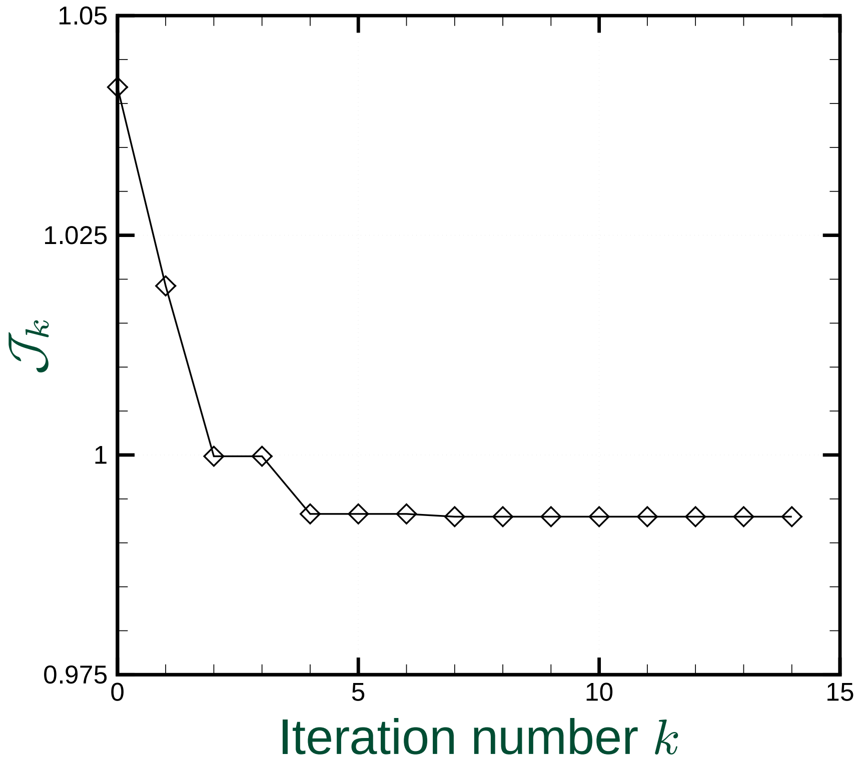
Optimal control parameters : $A = 4.25$ and $St = 0.74$

Mean drag coefficient : $\mathcal{J} = 0.993$

6 resolutions of the Navier-Stokes equations

Initial control parameters : $A = 6.0$ et $St = 1.0$



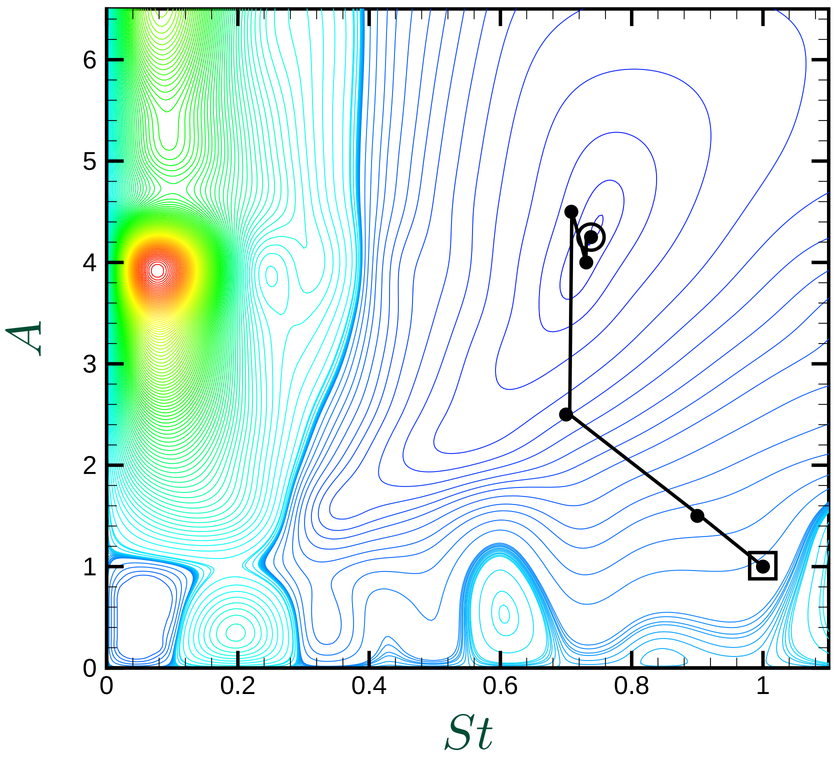


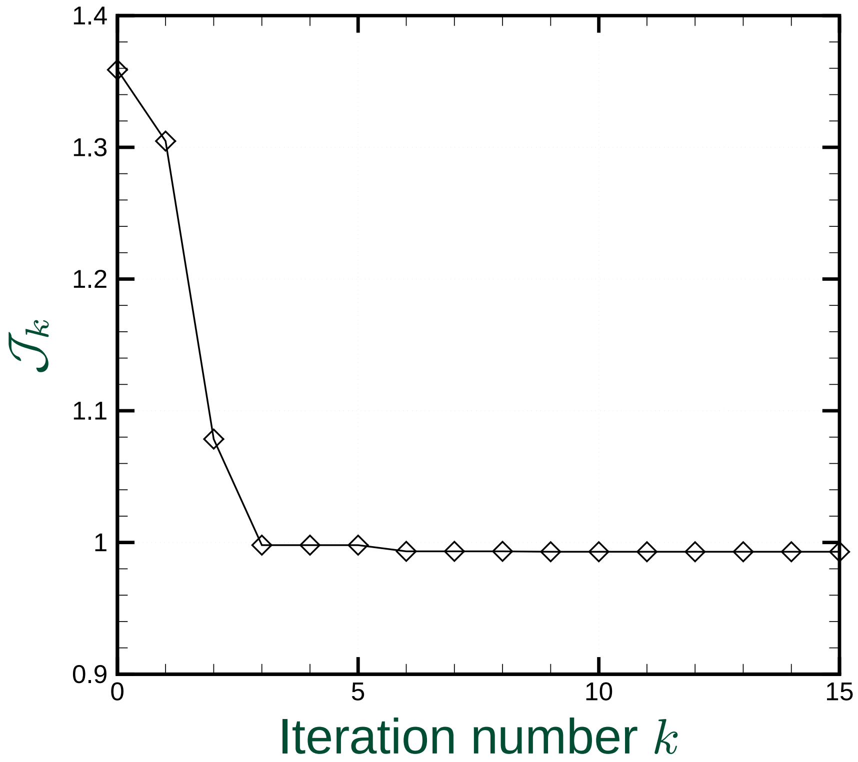
Optimal control parameters : $A = 4.25$ and $St = 0.74$

Mean drag coefficient : $\mathcal{J} = 0.993$

4 resolutions of the Navier-Stokes equations

Initial control parameters : $A = 1.0$ et $St = 1.0$



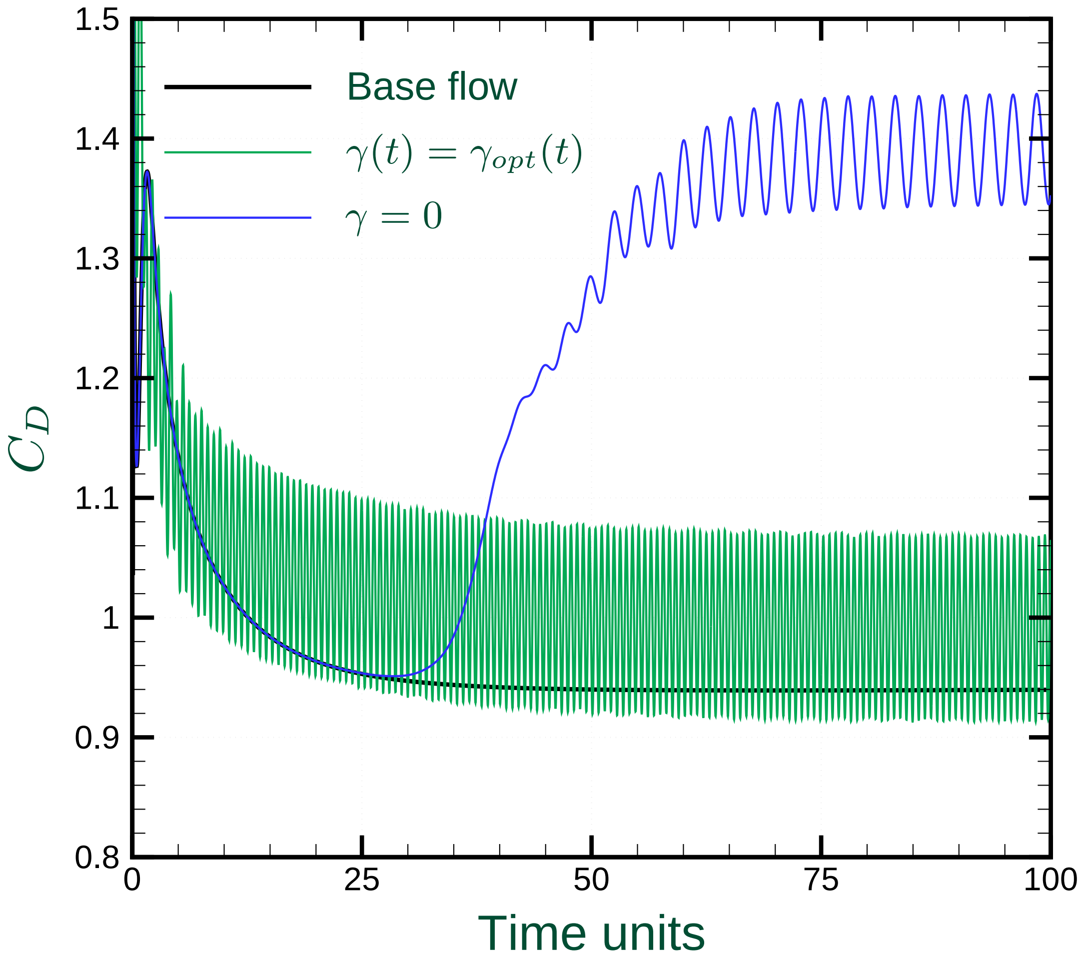


Optimal control parameters : $A = 4.25$ and $St = 0.74$

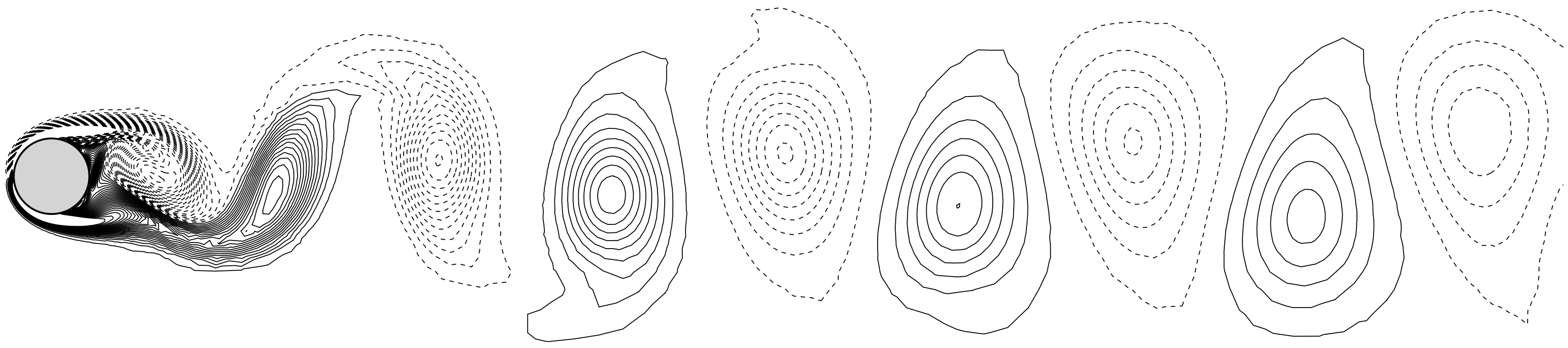
Mean drag coefficient : $\mathcal{J} = 0.993$

5 resolutions of the Navier-Stokes equations

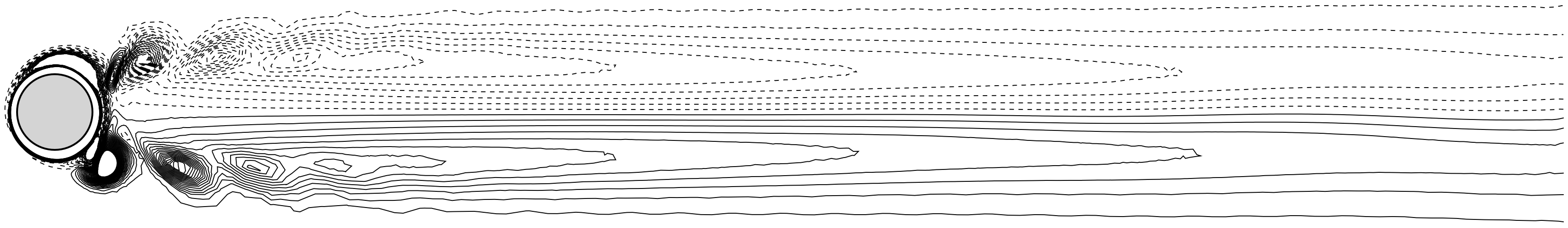
- Optimal control law : $\gamma_{opt}(t) = A\sin(2\pi St\, t)$ avec $A = 4.25$ et $St = 0.74$



- Relative drag reduction of 30% ($\mathcal{J}_0 = 1,4 \;\Rightarrow\; \mathcal{J}_{opt} = 0,99$)



Uncontrolled flow, $\gamma = 0$.



Controlled flow, $\gamma = \gamma_{opt}$.

Fig. : *Iso-values of vorticity* ω_z.

Controlled flow : near wake strongly unsteady, far wake (after 5 diameters) steady and symmetric $\rightarrow$ steady unstable base flow

Numerical costs

- Optimal control of NSE by He *et al.* (2000) :

  $\Rightarrow 30\%$ drag reduction for $A = 3$ and $S_t = 0.75$.

- Optimal control POD ROM by Bergmann *et al.* (2005) with no reactualization of the POD ROM :

  $\Rightarrow 25\%$ drag reduction for $A = 2.2$ and $S_t = 0.53$.

  - Reduction costs compared to NSE :
    - CPU time : 100
    - Memory storage : 600

  but no mathematical proof concerning the Navier-Stokes optimality.

- TRPOD (this study) :

  $\Rightarrow$ More than 30% of drag reduction for $A = 4.25$ and $S_t = 0.738$.

  - Reduction costs compared to NSE :
    - CPU time : 4
    - Memory storage : 400

  but global convergence.

$\hookrightarrow$ "Optimal" control of 3D flows becomes possible !

Conclusions and perspectives

- Conclusions on TRPOD
  - Important relative drag reduction : more than 30% of relative drag reduction
  - Global convergence : mathematical assurance that the solution is identical to the one of the high-fidelity model
  - TRPOD compared to NSE $\Rightarrow$ important reduction of numerical costs :
    $\hookrightarrow$ Reduction factor of the CPU time : 4
    $\hookrightarrow$ Reduction factor of the memory storage : 400

"OPTIMAL" CONTROL OF 3D FLOWS POSSIBLE BY POD ROM

- Perspectives
  - Test mode interpolations for controlled flows (Morzynski and Tadmor's talks, GAMM 2006)
  - Test other reduced basis method than classical POD
    - Centroidal Voronoi Tessellations (Gunzburguer, 2004) : "intelligent" sampling in the control parameter space
    - Model-based POD (Willcox, 2005) : modify the definitions of the POD modes
  - Optimal control of the channel flow at $Re_\tau = 180$