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Poisson NL means: unsupervised non local means for Poisson noise

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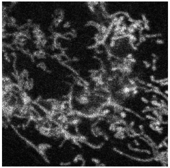
September 27, 2010

Introduction to Poisson noise

- Noise: fluctuations which corrupt a signal or an image,
- Poisson noise: due to low-light conditions when the number of collected photons is small,

  ex: optical imagery, microscopy, astronomy.



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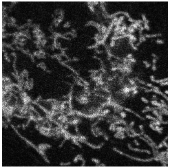
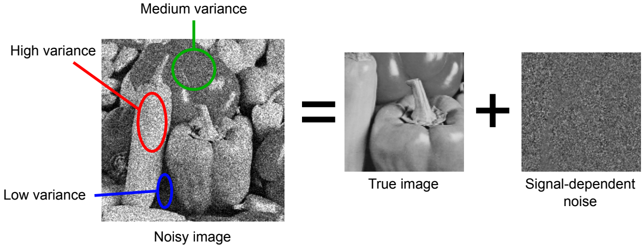
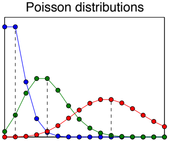


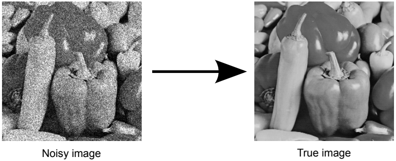
- Specificity of Poisson noise:
  - Signal-dependent,
  - True image and noise component not separable,





  - Modeled by probability distributions.

Image denoising: find an estimation of the true image from the noisy image.



Overview of denoising approaches

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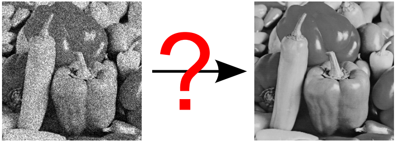
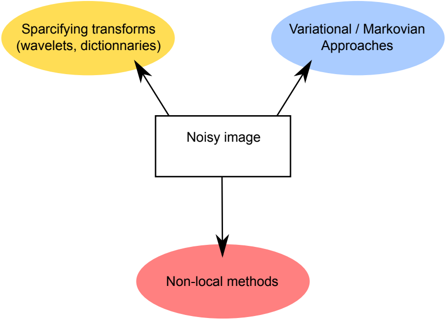


Noisy image   True image

How to denoise an image?

- Three main approaches,
- Lots of hybrid methods,
- Majority designed for Gaussian noise.



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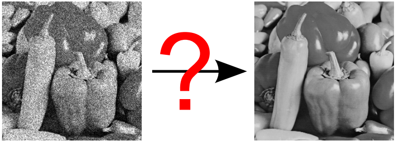

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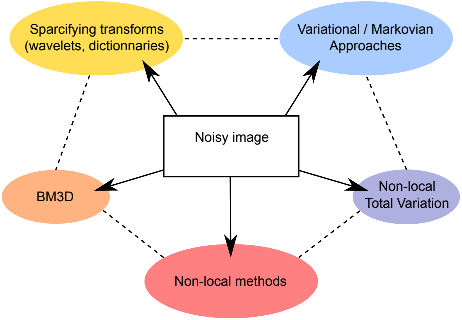

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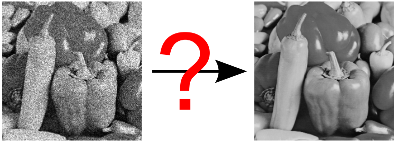

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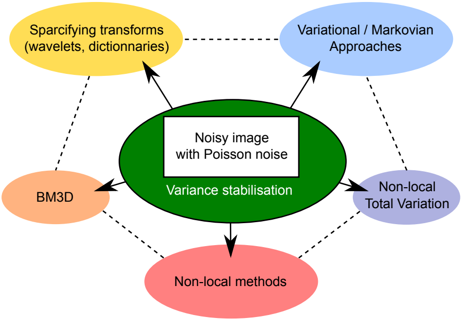
How to manage Poisson noise?

- Variance stabilisation:

  Poisson noise → Gaussian noise,
- Method adaptation:

  extend the method to Poisson noise,
- Our choice:

  adapt the NL means for Poisson noise
  with a statistical point of view.



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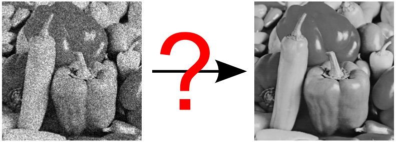

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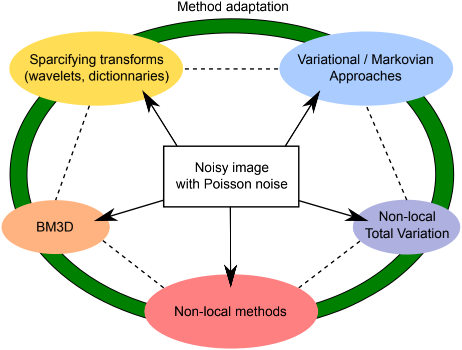

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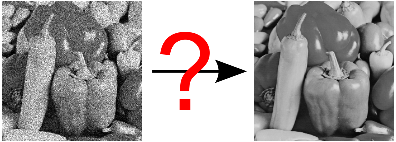

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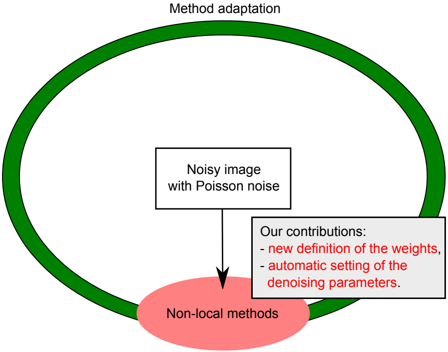

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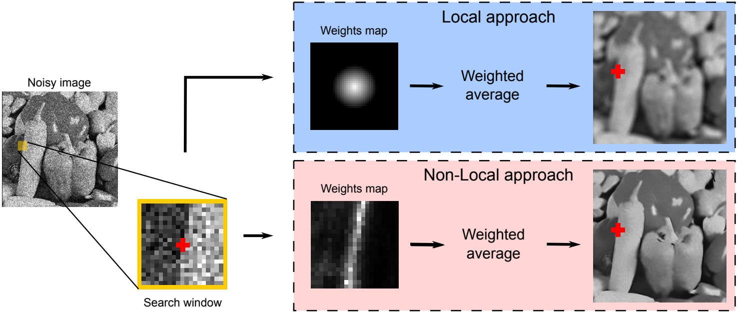
Non-local approach [Buades et al., 2005]

- Local filters: loss of resolution,
- Non-local filers: weights are defined from the image,
- Weights are based on patch similarities.



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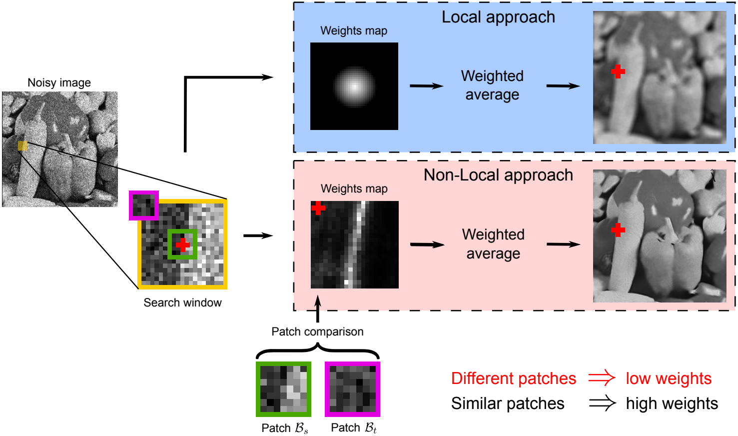

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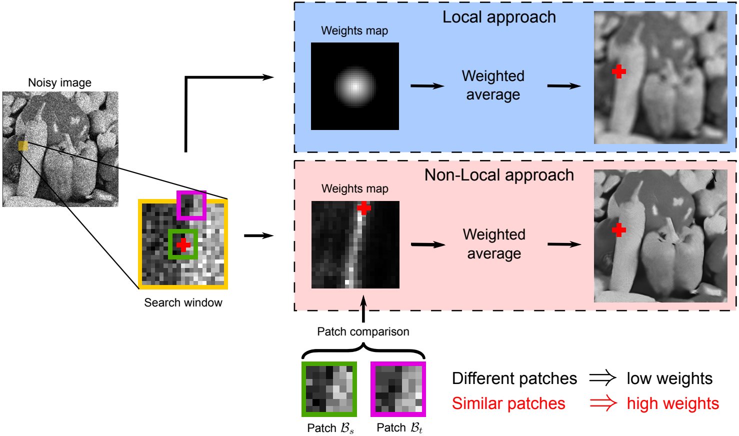
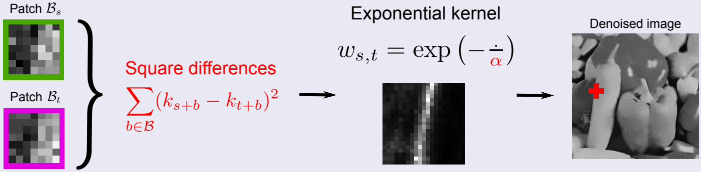


Non-local means [Buades et al., 2005]

- Define weights from the square differences between patches $\mathcal{B}_s$ and $\mathcal{B}_t$:



with $_{s+b}$ and $_{t+b}$ the b-th respective pixels in $\mathcal{B}_s$ and $\mathcal{B}_t$.

- Square differences: adapted for Gaussian noise,
- Which criterion for Poisson noise?
- How to set automatically the "optimal" parameter α?

Patch-similarities: how to replace the square difference? [Deledalle et al., 2009]

- Weights have to select pixels with same true values,
- Compare patches ⇔ test the hypotheses that patches have:

$\mathcal{H}_0$: same true values ,
$\mathcal{H}_1$: independent true values .

$$\mathrm{P}(\mathcal{H}_0 | \blacksquare_1, \blacksquare_2) = \frac{\mathrm{P}(\blacksquare_1, \blacksquare_2 | \mathcal{H}_0)}{\mathrm{P}(\blacksquare_1, \blacksquare_2 | \mathcal{H}_1)} \times \mathrm{P}(\mathcal{H}_0)$$

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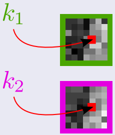
1. Similarity between noisy patches

- Based on detection theory, we propose to evaluate the generalized likelihood ratio (GLR) of both hypotheses given the noisy patches [Kay, 1998].

→ For Poisson noise, we obtain the following criterion:

$$-\log GLR(k_1, k_2) = k_1 \log k_1 + k_2 \log k_2 - (k_1 + k_2) \log \left(\frac{k_1 + k_2}{2} \right) .$$



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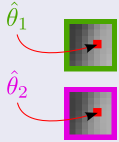
2. Similarity between pre-filtered patches

- We propose to refine weights by using the similarity between pre-filtered patches.
  Idea motivated by [Polzehl et al., 2006, Brox et al., 2007, Goossens et al., 2008, Deledalle et al., 2009]
- A statistical test for the hypothesis $\mathcal{H}_0$ can be given by the symmetrical Kullback-Leibler divergence:

$$D_{KL}(\blacksquare \| \blacksquare).$$

→ For Poisson noise, we obtain the following criterion:

$$D_{KL}(\hat{\theta}_1 \| \hat{\theta}_2) = \left(\hat{\theta}_1 - \hat{\theta}_2\right) \log \frac{\hat{\theta}_1}{\hat{\theta}_2}.$$



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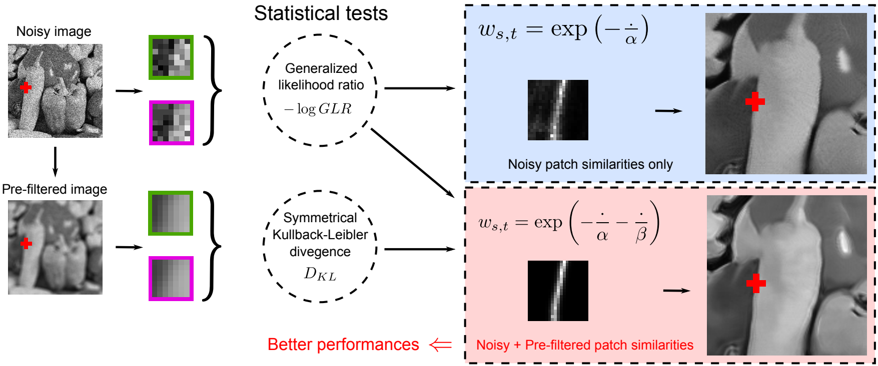

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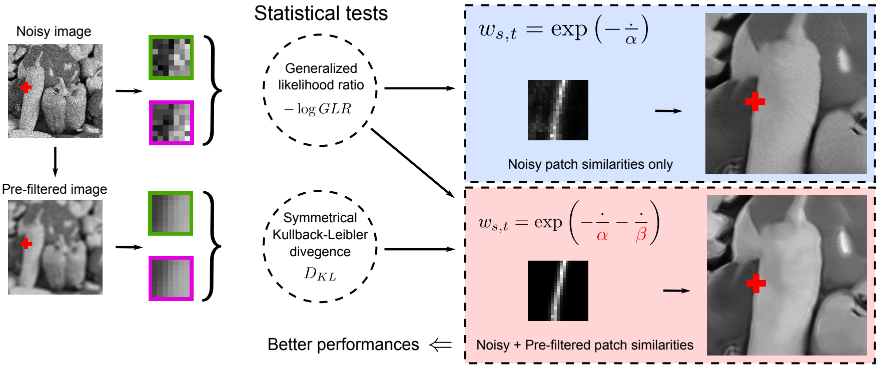
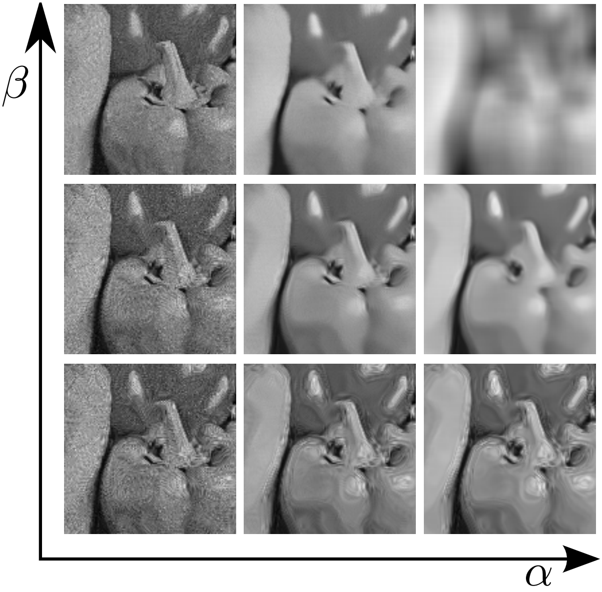
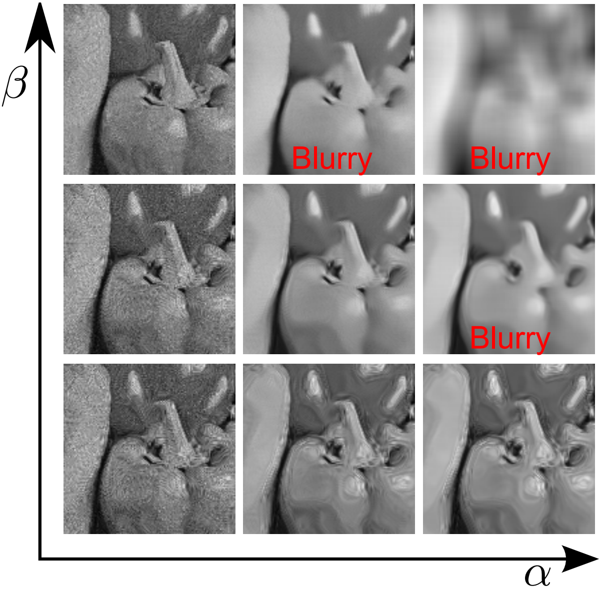


Next, how should we set the parameters α and β?

Non-local estimation under Poisson noise



How to choose the parameters?
(trade-off noisy/pre-filtered)



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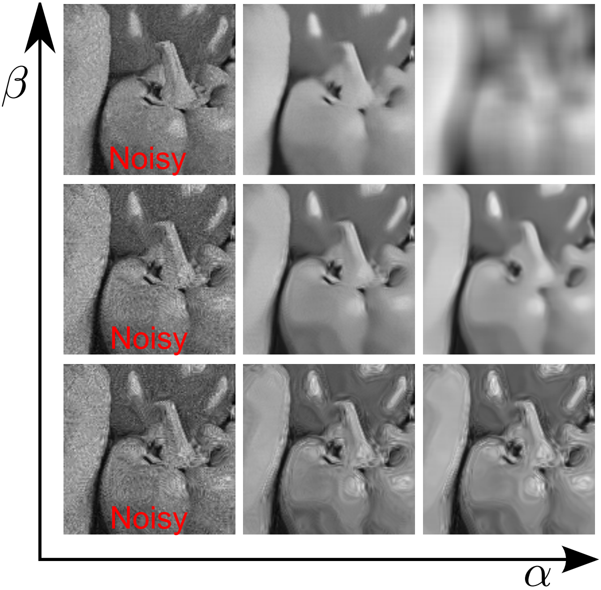
Visually?



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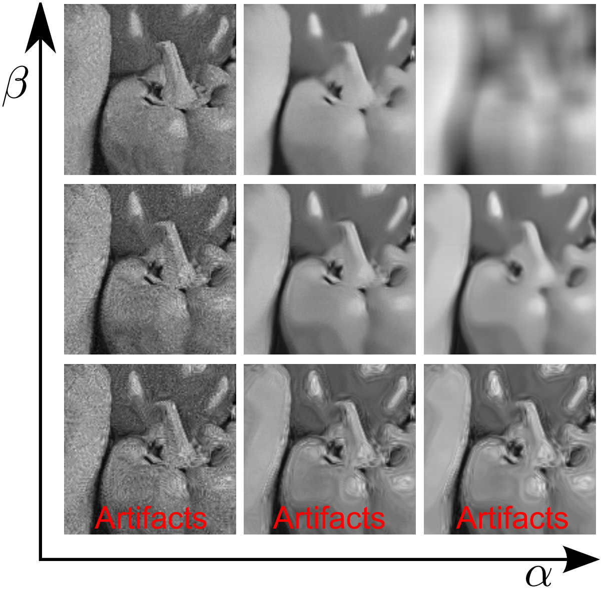

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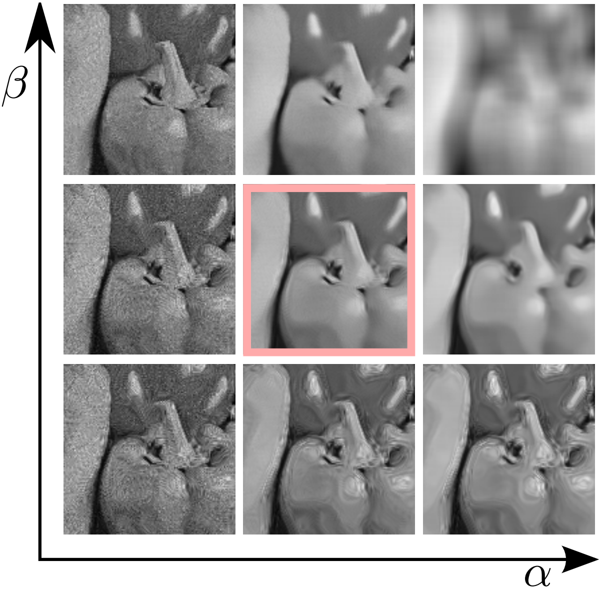

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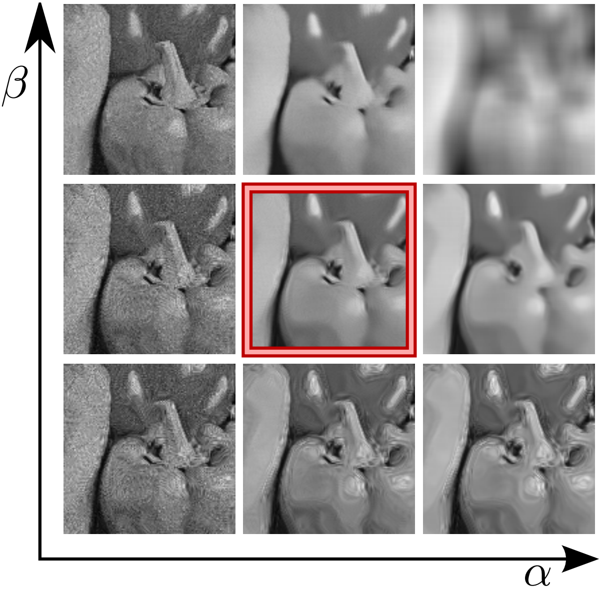
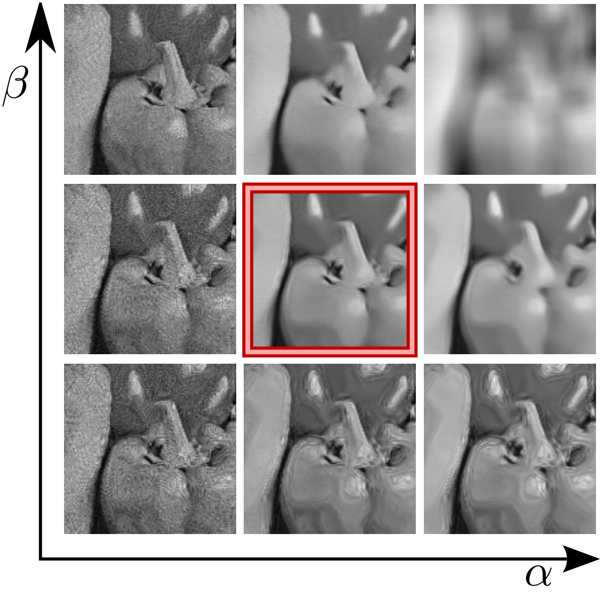


How to choose the parameters?
(trade-off noisy/pre-filtered)

Visually?

Mean square error (MSE)?

Non-local estimation under Poisson noise



How to choose the parameters?
(trade-off noisy/pre-filtered)

Visually?

Mean square error (MSE)?

How to estimate the MSE?

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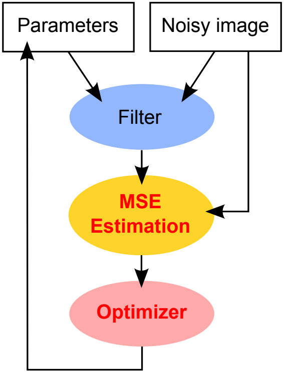


Unsupervised filtering

MSE estimators: unbiased risk estimators

Estimator	Gaussian	Poisson
General	SURE [Stein, 1981]	PURE [Chen, 1975]
Wavelet	SUREshrink [Donoho et al., 1995]	
	SURE-LET [Blu et al., 2007]	PURE-LET [Luisier et al., 2010]
NL means	SURE based NL means [Van De Ville et al., 2009]	Poisson NL means
	Local-SURE NL-means [Duval et al., 2010]	

SURE: Stein's Unbiaised Risk Estimator
PURE: Poisson Unbiaised Risk Estimator

PURE in Poisson NL means

- Based on the same ideas as *SURE based NL means*:
  - PURE is obtained in closed-form for Poisson NL means,
  - with almost same computation time.

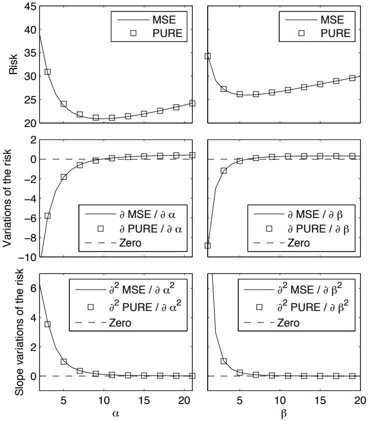
Selection of the parameters

- Optimum α and β obtained iteratively using Newton's method:

$$\begin{pmatrix} \alpha^{n+1} \\ \beta^{n+1} \end{pmatrix} = \begin{pmatrix} \alpha^{n} \\ \beta^{n} \end{pmatrix} - H^{-1}\nabla$$

  with H the Hessian and ∇ the gradiant.
- The first and second order differentials of PURE are also obtained in closed-forms.

Find the best denoising level using similarities of noisy and pre-filtered patches!



MSE and PURE and their first and second order variations with respect to the parameters α and β

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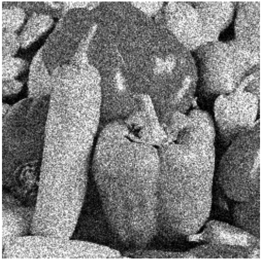
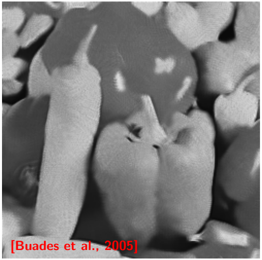
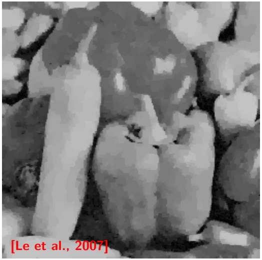
Results of Poisson NL means



(a) Noisy image (16.59)



(b) NL means (26.30)



(c) Poisson-TV (26.42)

(e) PURE-LET (26.89)

(f) Poisson NL means (**27.43**)

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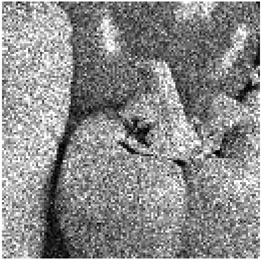

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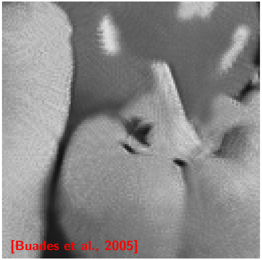

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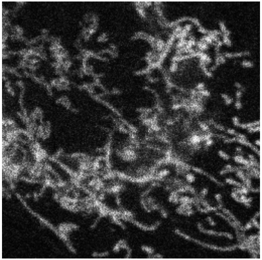
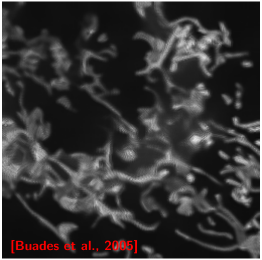
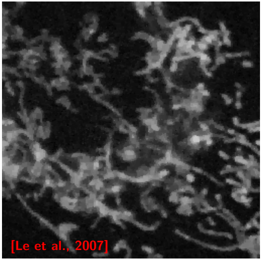
Results of Poisson NL means



(a) Noisy image

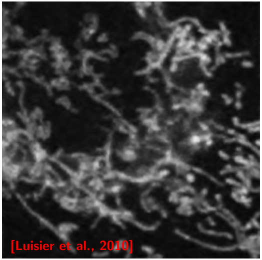


(b) NL means

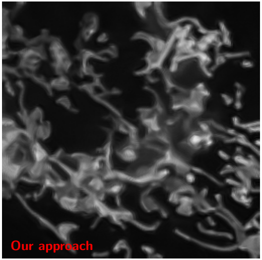


(c) Poisson-TV

Cardiac mitochondrion,
Confocal fluorescence microscopy,
Image courtesy of Y. Tourneur.

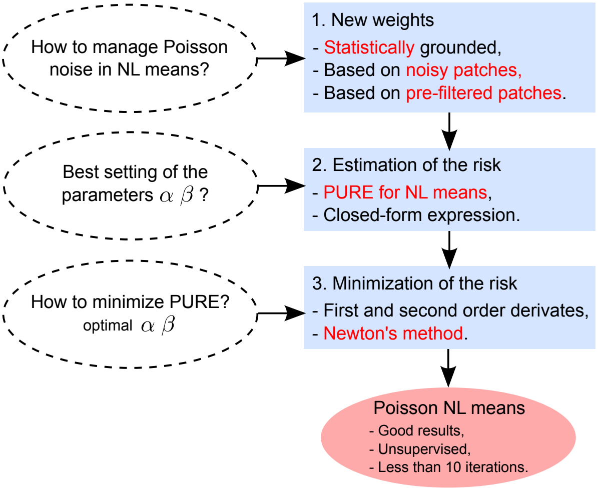


(e) PURE-LET



(f) Poisson NL means

Our contributions



Our contributions

How to manage Poisson noise in NL means? → **1. New weights**
- Statistically grounded,
- Based on noisy patches,
- Based on pre-filtered patches.

Best setting of the parameters α β ? → **2. Estimation of the risk**
- PURE for NL means,
- Closed-form expression.

How to minimize PURE? optimal α β → **3. Minimization of the risk**
- First and second order derivates,
- Newton's method.

Poisson NL means
- Good results,
- Unsupervised,
- Less than 10 iterations.

Our perspectives

1. Extend to other noise model
- Speckle noise,
- Mixture of noise,
- Vectorial images.

2. Unsupervised selection of:
- Size/shape of the search window,
- Size/shape of patches.

Questions?

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http://perso.telecom-paristech.fr/~deledall/poisson_nlmeans.php
$\rightarrow$ More details and software available.

[Alter et al., 2006] Alter, F., Matsushita, Y., and Tang, X. (2006).
An intensity similarity measure in low-light conditions.
Lecture Notes in Computer Science, 3954:267.

[Brox et al., 2007] Brox, T., Kleinschmidt, O., and Cremers, D. (2007).
Efficient Nonlocal Means for Denoising of Textural Patterns.
IEEE Transactions on Image Processing.

[Buades et al., 2005] Buades, A., Coll, B., and Morel, J. (2005).
A Non-Local Algorithm for Image Denoising.
Computer Vision and Pattern Recognition, 2005. CVPR 2005. IEEE Computer Society Conference on, 2.

[Chen, 1975] Chen, L. (1975).
Poisson approximation for dependent trials.
The Annals of Probability, 3(3):534–545.

[Deledalle et al., 2009] Deledalle, C., Denis, L., and Tupin, F. (2009).
Iterative Weighted Maximum Likelihood Denoising with Probabilistic Patch-Based Weights.
IEEE Transactions on Image Processing, 18(12):2661–2672.

[Goossens et al., 2008] Goossens, B., Luong, H., Pižurica, A., and Philips, W. (2008).
An improved non-local denoising algorithm.
In *Proc. Int. Workshop on Local and Non-Local Approximation in Image Processing (LNLA'2008), Lausanne, Switzerland*.

[Kay, 1998] Kay, S. (1998).
Fundamentals of Statistical signal processing, Volume 2: Detection theory.
Prentice Hall PTR.

[Le et al., 2007] Le, T., Chartrand, R., and Asaki, T. (2007).
A variational approach to reconstructing images corrupted by Poisson noise.
J. of Math. Imaging and Vision, 27(3):257–263.

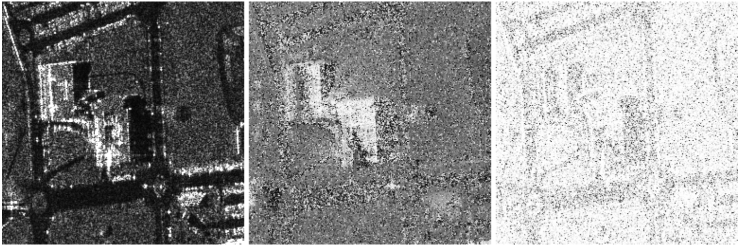
[Luisier et al., 2010] Luisier, F., Vonesch, C., Blu, T., and Unser, M. (2010).
Fast interscale wavelet denoising of Poisson-corrupted images.
Signal Processing, 90(2):415–427.

[Polzehl and Spokoiny, 2006] Polzehl, J. and Spokoiny, V. (2006).
Propagation-separation approach for local likelihood estimation.
Probability Theory and Related Fields, 135(3):335–362.

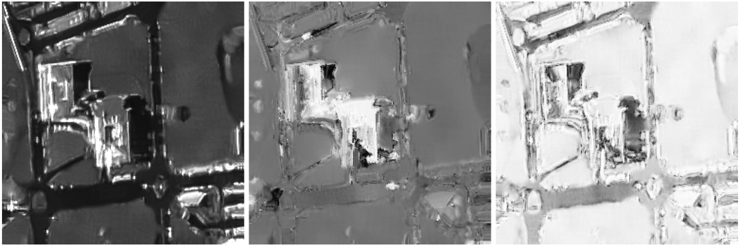
[Stein, 1981] Stein, C. (1981).
Estimation of the mean of a multivariate normal distribution.
The Annals of Statistics, pages 1135–1151.

[Van De Ville and Kocher, 2009] Van De Ville, D. and Kocher, M. (2009).
SURE-Based Non-Local Means.
IEEE Signal Processing Letters, 16(11):973–976.

Using the same model of weights for interferometric SAR data modeled by circular complexe Gaussian distributions ©DGA ©ONERA



(a) InSAR-SLC



(b) **NL-InSAR**

Numerical results - PSNRs

Peppers (256 × 256)				
Noisy	3.14	13.14	17.91	23.92
MA filter	19.20	20.93	21.11	21.16
PURE-LET [Luisier et al., 2010]	19.33	24.29	27.27	30.79
NL means [Buades et al., 2005]	18.12	23.33	26.98	30.64
Poisson NL means	**19.90**	**25.32**	**28.07**	**31.06**
α_{opt}	(209)	(13.6)	(10.05)	(9.21)
β_{opt}	(0.72)	(1.31)	(2.76)	(7.64)
$\#iterations$	(13.5)	(8.02)	(7.03)	(6.90)
Cameraman (256 × 256)				
Noisy	3.28	13.27	18.03	24.05
MA filter	18.71	20.15	20.29	20.33
PURE-LET [Luisier et al., 2010]	19.67	24.32	26.87	**30.36**
NL means [Buades et al., 2005]	18.17	23.53	26.77	29.39
Poisson NL means	**19.89**	**25.07**	**27.42**	29.47
α_{opt}	(62.1)	(9.48)	(8.81)	(7.34)
β_{opt}	(0.51)	(1.19)	(3.57)	(16.19)
$\#iterations$	(11.0)	(6.80)	(7.60)	(11.3)

PSNR values averaged over ten realisations using different methods on images damaged by Poisson noise with different levels of degradation. The averaged optimal parameters and the averaged number of iterations of the proposed Poisson NL means are given.

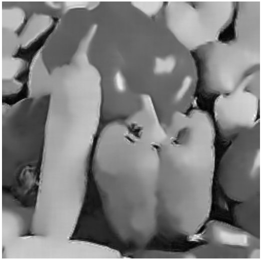
Influence of the pre-filtered images

(a) Moving avrage (24.15)

(b) Poisson-TV (26.42)

(c) PURE-LET (26.89)

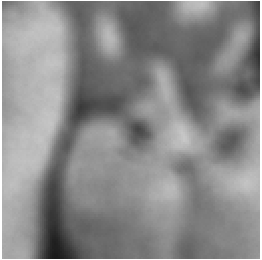
(d) PNLM+MA (27.43)



(e) PNLM+PTV (27.12)

(f) PNLM+P-LET (27.55)

Influence of the pre-filtered images



(a) Moving avrage (24.15)

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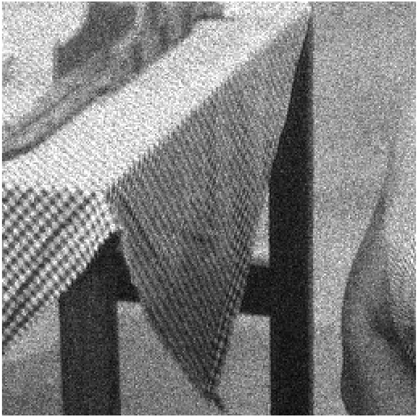
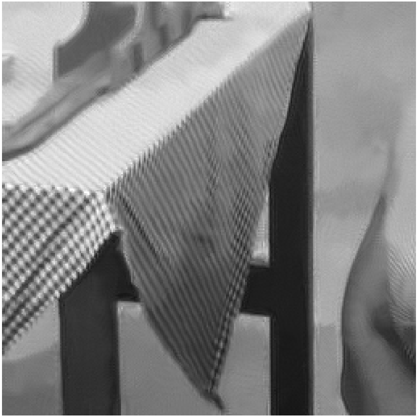
(e) PNLM+PTV (27.12)

(f) PNLM+P-LET (27.55)

Results with other images



(a) Noisy



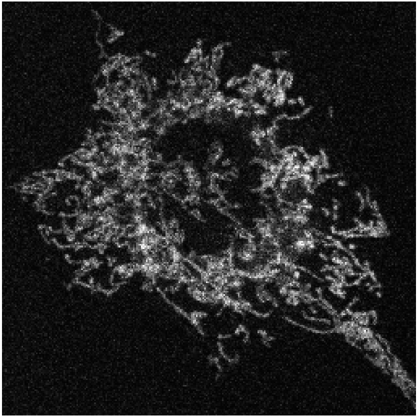
(b) Poisson NL means

Results with other images

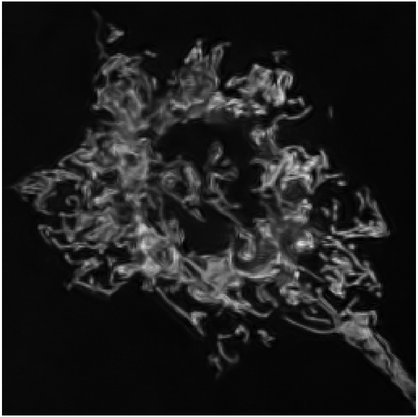
(a) Noisy

(b) Poisson NL means

Results with other images

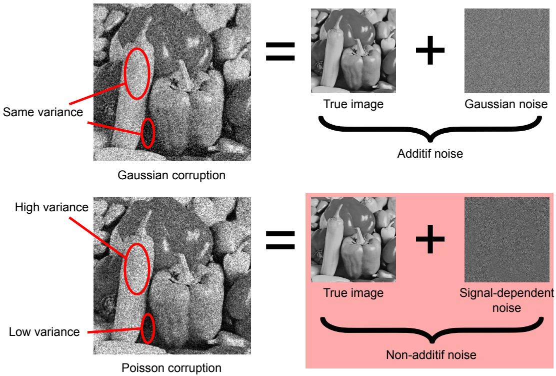


(a) Noisy



(b) Poisson NL means

What the difference between Gaussian and Poisson noise?



- Gaussian noise:
  - Constant variance,
  - True image + noise component.
- Poisson noise:
  - Signal-dependent,
  - True image and noise component not separable.

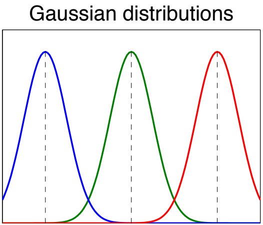
What the difference between Gaussian and Poisson noise?

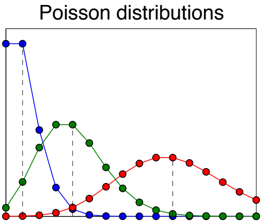
Gaussian noise

- $v \in \mathbb{R}$: measured light intensity
- $u \in \mathbb{R}$: underlying light intensity
- $v|u \sim$ Gaussian distribution

Poisson noise

- $k \in \mathbb{N}$: number of collected photons.
- $\lambda \in \mathbb{R}^+$: underlying light intensity
- $k|\lambda \sim$ Poisson distribution





Automatic setting of parameters

Risk minimisation

Choose the parameters α et β minimising the mean square error (MSE):

$$E\left[\frac{1}{N}\|\lambda - \hat{\lambda}\|^2\right] = \frac{1}{N}\sum_s \left(\lambda_s^2 + E\left[\hat{\lambda}_s^2\right] - E\left[\lambda_s \hat{\lambda}_s\right]\right)$$

- $\sum_s \lambda_s^2$ idependent of the paramaters,
- $\sum_s E\left[\hat{\lambda}_s^2\right]$ can be estimated from $\hat{\lambda}$,
- How to estimate $\sum_s E\left[\lambda_s \hat{\lambda}_s\right]$?

Poisson unbiased risk estimator (PURE) [Chen, 1975, Luisier et al., 2010]

- If k is damaged by Poisson noise and $\hat{\lambda} = h(k)$ then

$$E\left[\lambda_s \hat{\lambda}_s\right] = E\left[k_s \overline{\lambda}_s\right]$$

with $\overline{\lambda} = h(\overline{k})$ and $\overline{k}$ defined by $\overline{k}_t = \begin{cases} k_t - 1 & \text{if } t = s \\ k_t & \text{otherwise} \end{cases}$

- PURE is given by: $R(\hat{\lambda}) = \dfrac{1}{N}\sum_s \left(\lambda_s^2 + \hat{\lambda}_s^2 - 2k_s\overline{\lambda}_s\right).$

Automatic setting of parameters

Risk minimisation

Choose the parameters α et β minimising the mean square error (MSE):

$$E\left[\frac{1}{N}\|\lambda - \hat{\lambda}\|^2\right] = \frac{1}{N}\sum_s \left(\lambda_s^2 + E\left[\hat{\lambda}_s^2\right] - E\left[\lambda_s \hat{\lambda}_s\right]\right)$$

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- $\sum_s E\left[\hat{\lambda}_s^2\right]$ can be estimated from $\hat{\lambda}$,
- How to estimate $\sum_s E\left[\lambda_s \hat{\lambda}_s\right]$?

PURE - Proof

Let be k a r.v. following a Poisson distribution and $h(.)$ a function:

$$\begin{aligned} E\left[kh(k-1)\right] &= \sum_{k=1}^{\infty} kh(k-1)\frac{\lambda^k e^{-\lambda}}{k!} \\ &= \lambda \sum_{k=1}^{\infty} h(k-1)\frac{\lambda^{k-1} e^{-\lambda}}{(k-1)!} \\ &= E\left[\lambda h(k)\right] \end{aligned}$$