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Patch similarity under non Gaussian noise

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September 13, 2011

Motivation

Increasing use of patches to model images

- Texture synthesis,
- Inpainting,
- Image editing,
- Denoising,
- Super-resolution,
- Image registration,
- Stereo vision,
- Object tracking.

Image model based on the natural redundancy of patches

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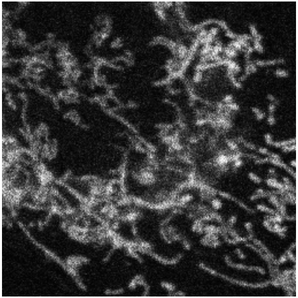
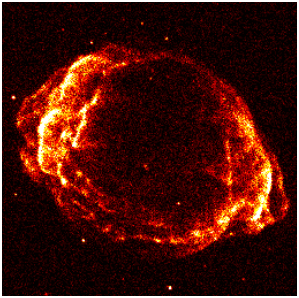
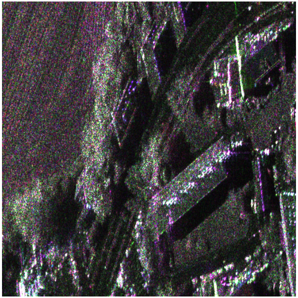


(a) Microscopy



(b) Astronomy



(c) SAR polarimetry

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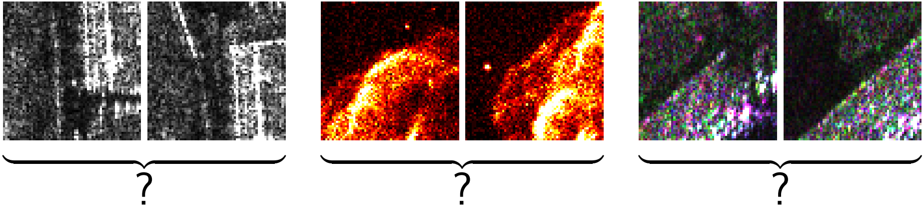
How to compare noisy patches?



How to take into account the noise model?

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1. Limits of the Euclidean distance
2. Variance stabilization approach
3. How to adapt properly to the noise distribution?
4. Evaluation of similarity criteria

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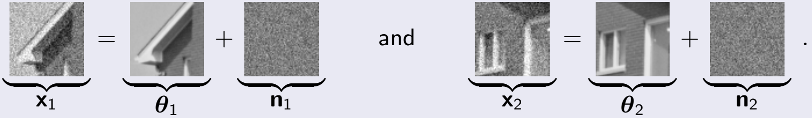
Noise models

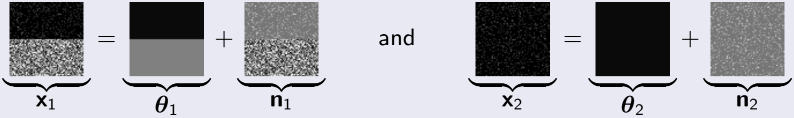
Gaussian noise assumption

- A pair $(\mathbf{x}_1, \mathbf{x}_2)$ of noisy patches can be decomposed as:



Beyond Gaussian noise

- Noise can be non-Gaussian, e.g., Poisson or Gamma distributed,
- Non-additive decomposition for Poisson noise:



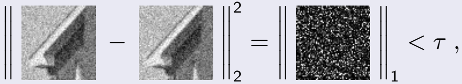
- The noise level is signal-dependent.

Euclidean distance under noisy conditions

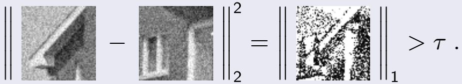
Why the Euclidean distance under Gaussian noise?

1. Euclidean distance estimates the dissimilarity between noise-free patches:

2. When $\boldsymbol{\theta}_1 = \boldsymbol{\theta}_2 = \boldsymbol{\theta}_{12}$, the residue is statistically small and independent on $\boldsymbol{\theta}_{12}$:



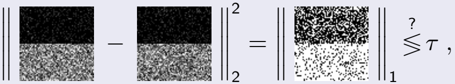
3. When $\boldsymbol{\theta}_1 \neq \boldsymbol{\theta}_2$, the residue is statistically higher:



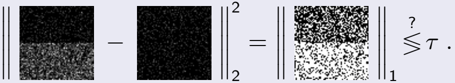
Limit of the Euclidean distance with Poisson noise

1. Euclidean distance does not estimate the dissimilarity between noise-free patches:

2. When $\boldsymbol{\theta}_1 = \boldsymbol{\theta}_2 = \boldsymbol{\theta}_{12}$, the residue cannot be controlled and is dependent on $\boldsymbol{\theta}_{12}$:



3. Then, when $\boldsymbol{\theta}_1 \neq \boldsymbol{\theta}_2$, there is no guarantee that the residue is statistically higher:



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1. Limits of the Euclidean distance
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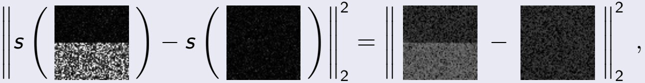
Variance stabilization approach

- Use an application *s* which stabilizes the variance for a specific noise model,
- Evaluate the Euclidean distance between the transformed patches:



(1)

Variance stabilization approach

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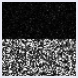
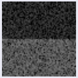
$\Big\| s \Big($  $\Big) - s \Big($  $\Big) \Big\|_2^2 = \Big\|$  $-$  $\Big\|_2^2 ,$ (1)

Example

- Gamma noise (multiplicative) and the homomorphic approach:

$$\mathbf{s}(X) = \log X \Rightarrow \mathrm{Var}[\mathbf{s}(X)] = \mathrm{Var}[\log X] = \Psi(1, L) \qquad (2)$$

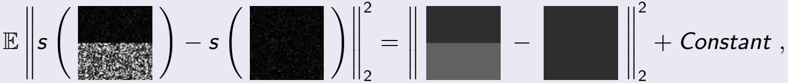
  where L is the shape parameter of the gamma distribution.

- Poisson noise and the Anscombe transform:

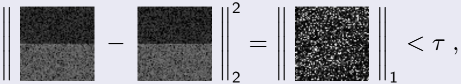
$$\mathbf{s}(X) = 2\sqrt{X + \frac{3}{8}} \Rightarrow (\theta \gtrsim 4 \Rightarrow \mathrm{Var}[\mathbf{s}(X)] = 1) \ . \qquad (3)$$

Why does it *seem* to work?

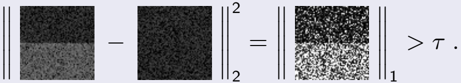
1. Euclidean distance estimates the dissimilarity between transformed noise-free patches:



2. When $\boldsymbol{\theta}_1 = \boldsymbol{\theta}_2 = \boldsymbol{\theta}_{12}$, the residue is statistically small:



3. Then, when $\boldsymbol{\theta}_1 \neq \boldsymbol{\theta}_2$, the residue is statistically higher:

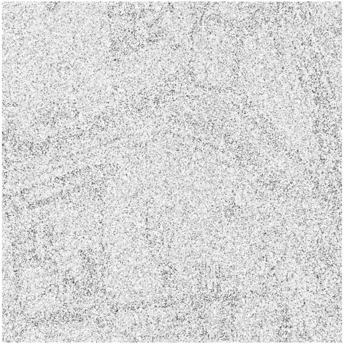


Variance stabilization approach

Limits

- Only heuristic,
- No optimality results,
- Does not take into account the statistics of the transformed data,
- Does not exist for all noise distribution models.

(a) Image with impulse noise



(b) SAR cross correlation

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What are we looking for?

Definitions and properties

- A similarity criterion can be based on the hypothesis test (i.e., a parameter test):

$$\mathcal{H}_0 : \boldsymbol{\theta}_1 = \boldsymbol{\theta}_2 \equiv \boldsymbol{\theta}_{12} \qquad \text{(null hypothesis)},$$
$$\mathcal{H}_1 : \boldsymbol{\theta}_1 \neq \boldsymbol{\theta}_2 \qquad \text{(alternative hypothesis)}.$$

- Its performance can be measured as:

$$P_{FA} = \mathbb{P}(\text{answer dissimilar}; \boldsymbol{\theta}_{12}, \mathcal{H}_0) \qquad \text{(false-alarm rate)},$$
$$P_D = \mathbb{P}(\text{answer dissimilar}; \boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \mathcal{H}_1) \qquad \text{(detection rate)}.$$

- The likelihood ratio (LR) test minimizes P_D for any P_{FA}:

$$L(\mathbf{x}_1, \mathbf{x}_2) = \frac{p(\mathbf{x}_1, \mathbf{x}_2; \boldsymbol{\theta}_{12}, \mathcal{H}_0)}{p(\mathbf{x}_1, \mathbf{x}_2; \boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \mathcal{H}_1)}. \qquad \leftarrow \text{given by the noise distribution model}$$

$\rightarrow$ Problem: $\boldsymbol{\theta}_{12}$, $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ are unknown.

Generalized likelihood ratio (GLR)

- Estimate $\boldsymbol{\theta}_{12}$, $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ with the maximum likelihood estimate (MLE),
- Define the (negative log) generalized likelihood ratio test:

$$\begin{aligned} -\log GLR(\mathbf{x}_1, \mathbf{x}_2) &= -\log \frac{\sup_{\mathbf{t}} p(\mathbf{x}_1, \mathbf{x}_2; \boldsymbol{\theta}_{12} = \mathbf{t}, \mathcal{H}_0)}{\sup_{\mathbf{t}_1, \mathbf{t}_2} p(\mathbf{x}_1, \mathbf{x}_2; \boldsymbol{\theta}_1 = \mathbf{t}_1, \boldsymbol{\theta}_2 = \mathbf{t}_2, \mathcal{H}_1)} \\ &= -\log \frac{p(\mathbf{x}_1; \boldsymbol{\theta}_1 = \hat{\mathbf{t}}_{12}) p(\mathbf{x}_2; \boldsymbol{\theta}_2 = \hat{\mathbf{t}}_{12})}{p(\mathbf{x}_1; \boldsymbol{\theta}_1 = \hat{\mathbf{t}}_1) p(\mathbf{x}_2; \boldsymbol{\theta}_2 = \hat{\mathbf{t}}_2)} \end{aligned}$$

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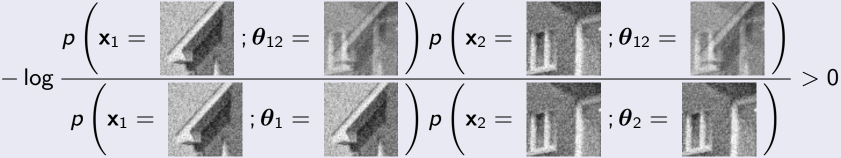
&= -\log \frac{p(\mathbf{x}_1; \boldsymbol{\theta}_1 = \hat{\mathbf{t}}_{12}) p(\mathbf{x}_2; \boldsymbol{\theta}_2 = \hat{\mathbf{t}}_{12})}{p(\mathbf{x}_1; \boldsymbol{\theta}_1 = \hat{\mathbf{t}}_1) p(\mathbf{x}_2; \boldsymbol{\theta}_2 = \hat{\mathbf{t}}_2)}
\end{aligned}
$$

Maximal self similarity

- Assume $\mathbf{x}_1 \neq \mathbf{x}_2$, then:



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\end{aligned}
$$

Equal self similarity

- Assume $\mathbf{x}_1 = \mathbf{x}_2$, then:

Numerator: $p\left(\mathbf{x}_1 =\right.$  $; \boldsymbol{\theta}_{12} =$  $\left.\right) p\left(\mathbf{x}_2 =\right.$  $; \boldsymbol{\theta}_{12} =$  $\left.\right)$

Denominator: $p\left(\mathbf{x}_1 =\right.$  $; \boldsymbol{\theta}_1 =$  $\left.\right) p\left(\mathbf{x}_2 =\right.$  $; \boldsymbol{\theta}_2 =$  $\left.\right)$

$$-\log \frac{\text{Numerator}}{\text{Denominator}} = 0$$

Patch similarity criteria – Generalized likelihood ratio

name	pdf	− log GLR	Stabilization	Euclidean
Gaussian	$\frac{e^{-\frac{(x-\theta)^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma}$	$(x_1 - x_2)^2$		
Poisson	$\frac{\theta^x e^{-\theta}}{x!}$	$\log\left(\frac{2^{x_1+x_2} x_1^{x_1} x_2^{x_2}}{(x_1+x_2)^{x_1+x_2}}\right)$	$\left(\sqrt{x_1+3/8} - \sqrt{x_2+3/8}\right)^2$	
Gamma	$\frac{L^L x^{L-1} e^{-\frac{Lx}{\theta}}}{\Gamma(L)\theta^L}$	$\log\left(\sqrt{\frac{x_1}{x_2}} + \sqrt{\frac{x_2}{x_1}}\right) - \log 2$	$\left(\log \frac{x_1}{x_2}\right)^2$	

The three criteria for three noise models

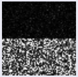
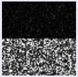
Patch similarity criteria – Generalized likelihood ratio

name	pdf	$-\log$ GLR	Stabilization	Euclidean
Gaussian	$\dfrac{e^{-\frac{(x-\theta)^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma}$	$(x_1-x_2)^2$		
Poisson	$\dfrac{\theta^x e^{-\theta}}{x!}$	$\log\left(\dfrac{2^{x_1+x_2} x_1^{x_1} x_2^{x_2}}{(x_1+x_2)^{x_1+x_2}}\right)$	$\left(\sqrt{x_1+3/8}-\sqrt{x_2+3/8}\right)^2$	
Gamma	$\dfrac{L^L x^{L-1} e^{-\frac{Lx}{\theta}}}{\Gamma(L)\theta^L}$	$\log\left(\sqrt{\dfrac{x_1}{x_2}}+\sqrt{\dfrac{x_2}{x_1}}\right)-\log 2$	$\left(\log \dfrac{x_1}{x_2}\right)^2$	

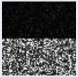
The three criteria for three noise models

Does it work? Illustration with Gamma noise

1. When $\boldsymbol{\theta}_1 = \boldsymbol{\theta}_2 = \boldsymbol{\theta}_{12}$, the residue is statistically small:

$-\log GLR \Big($  ,  $\Big) = \Big\|$  $\Big\|_1 < \tau$

2. Then, when $\boldsymbol{\theta}_1 \neq \boldsymbol{\theta}_2$, the residue is statistically higher:

$-\log GLR \Big($  ,  $\Big) = \Big\|$  $\Big\|_1 > \tau$

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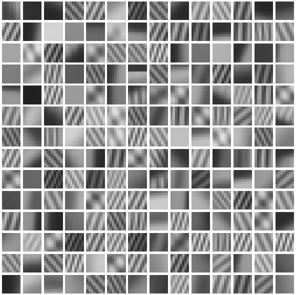
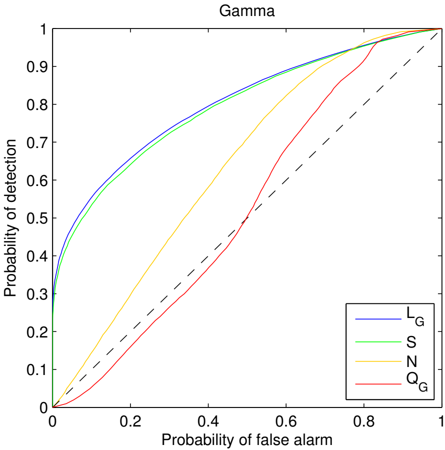
Detection

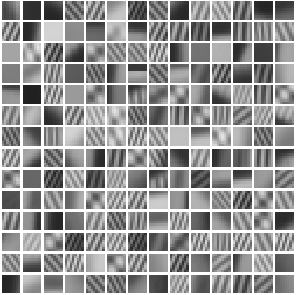


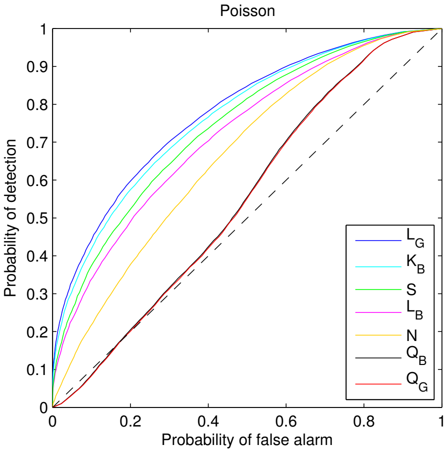
- Generalized likelihood ratio
- Variance stabilization
- Euclidean distance
- Maximum joint likelihood



[Alter et al., 2006]

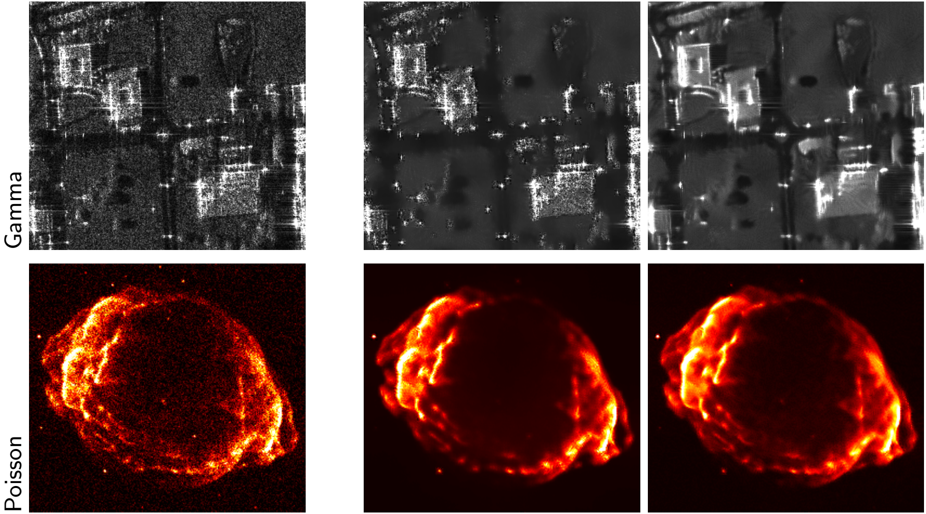
Detection





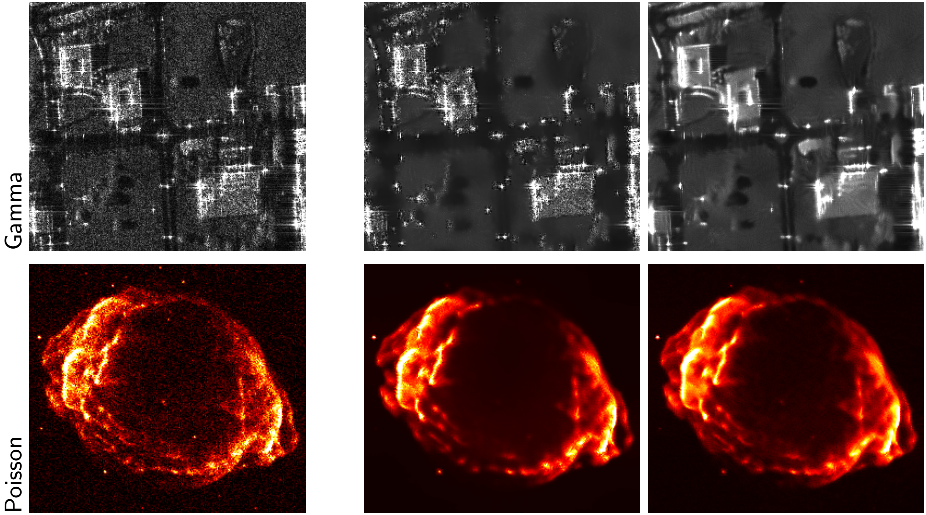
- Generalized likelihood ratio
- Variance stabilization
- Euclidean distance
- Maximum joint likelihood [Alter et al., 2006]
- Mutual information kernel [Seeger, 2002]
- Bayesian likelihood ratio [Minka, 1998, Minka, 2000]
- Bayesian joint likelihood [Yianilos, 1995, Matsushita and Lin, 2007]

Evaluation on denoising – Non-local filtering with GLR



(a) Noisy image (b) Euclidean distance (c) GLR

NB: Variance stabilization provides visual quality very close to GLR.



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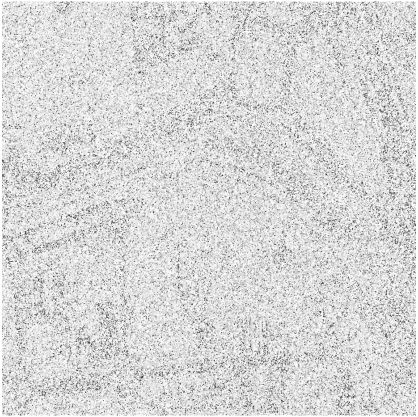
Evaluation on denoising – Iterative non-local filtering with GLR

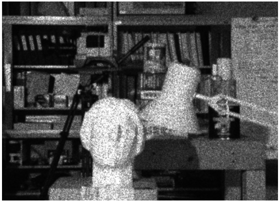


(a) SAR cross correlation

(b) Non-local filtering with GLR

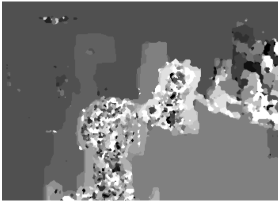
GLR can be used when the variance stabilization approach cannot be applied.

Stereo-vision



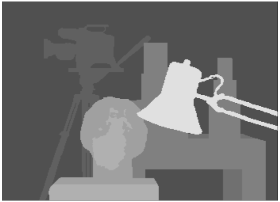
(a) Noisy image

Gamma



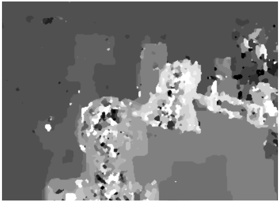
(b) Euclidean distance

(c) GLR

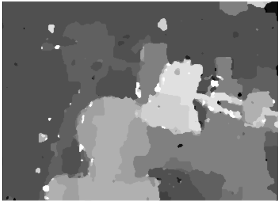


(d) Ground truth

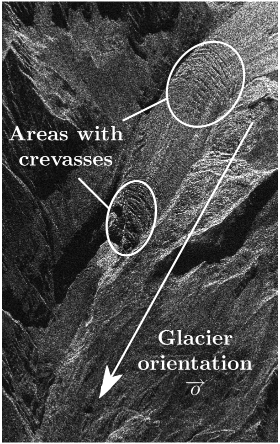
Poisson



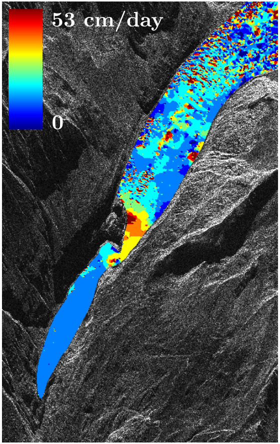
(e) Euclidean distance



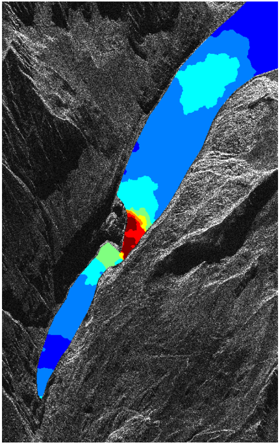
(f) GLR



(g) Noisy image



(h) Euclidean distance



(i) GLR

Glacier of Argentière. With GLR, the estimated speeds matches with the ground truth: average over the surface of 12.27 cm/day and a maximum of 41.12 cm/day in the areas with crevasses.

Conclusion

- GLR behaves well to compare patches under non-Gaussian noise conditions,
- It can be used when variance stabilization cannot be applied,
- Under high levels of gamma and Poisson noise, it outperforms six other criteria:
  $\rightarrow$ Best probability of detection for any probability of false-alarm.
- We have shown the interest of GLR in:
  - Patch-based denoising,
  - Patch-based stereo vision, and
  - Patch-based motion tracking.

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  - Patch-based denoising,
  - Patch-based stereo vision, and
  - Patch-based motion tracking.

Future work

- Under high SNR the variance stabilization approach defeats GLR:
  - Why such a change of relative behavior?
  - Euclidean distance estimates the dissimilarity between noise-free patches,
  - While GLR evaluates the equality of noise-free patches.
- Extend this approach to derive criteria with contrast invariance
  (e.g., for stereo-vision or flickering).

Questions?

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http://perso.telecom-paristech.fr/~deledall/

[Alter et al., 2006] Alter, F., Matsushita, Y., and Tang, X. (2006).
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Evaluation on denoising – Numerical performance

		Noisy	$\mathcal{Q}_G$	$\mathcal{L}_G$	$\mathcal{S}$	$\mathcal{G}$
		Gamma				
Medium noise levels	barbara	14.34	22.61	25.66	**25.67**	23.83
	boat	13.78	23.40	**25.50**	**25.50**	24.06
	bridge	14.58	20.17	**22.36**	**22.36**	21.01
	cameraman	13.96	23.88	**25.04**	25.01	14.93
	couple	14.37	23.19	**25.08**	25.06	23.68
	fingerprint	13.00	18.37	21.88	**21.89**	20.27
	hill	14.80	21.46	**24.24**	**24.24**	22.47
	house	13.35	22.52	26.33	**26.34**	24.36
	lena	14.09	24.61	27.71	**27.72**	25.61
	man	14.88	23.49	26.00	**26.01**	24.50
	mandril	14.02	21.61	**23.20**	**23.20**	22.22
	peppers	14.02	22.95	**25.54**	25.51	23.41

PSNR values: the higher the better

- Generalized likelihood ratio
- Variance stabilization
- Euclidean distance
- Maximum joint likelihood [Alter et al., 2006]

Evaluation on denoising – Numerical performance

		Noisy	$\mathcal{Q}_G$	$\mathcal{L}_G$	$\mathcal{S}$	$\mathcal{G}$
		Gamma				
Strong noise levels	barbara	5.86	20.25	**20.97**	20.90	20.33
	boat	5.32	20.90	**21.47**	21.42	20.97
	bridge	6.09	18.44	**19.21**	19.16	18.49
	cameraman	5.54	18.56	**20.88**	20.87	7.48
	couple	5.98	20.93	**21.54**	21.51	20.99
	fingerprint	4.60	15.34	**16.30**	16.22	15.57
	hill	6.35	20.18	**20.68**	20.61	20.20
	house	4.84	20.54	**21.20**	21.13	20.64
	lena	5.64	22.14	**22.89**	22.83	22.23
	man	6.47	21.56	**22.16**	22.10	21.64
	mandril	5.52	20.22	**20.44**	20.41	20.27
	peppers	5.56	18.59	**20.44**	20.43	18.65

PSNR values: the higher the better

- Generalized likelihood ratio
- Variance stabilization
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Evaluation on denoising – Numerical performance

		Noisy	$\mathcal{Q}_B$	$\mathcal{Q}_G$	$\mathcal{L}_B$	$\mathcal{L}_G$	$\mathcal{K}_B$	$\mathcal{S}$	$\mathcal{G}$
		Poisson							
Medium noise levels	barbara	14.43	23.59	23.57	25.43	25.40	25.41	**25.44**	24.79
	boat	13.99	24.00	23.98	25.28	25.26	25.27	**25.29**	24.74
	bridge	14.58	21.06	21.04	22.30	22.29	22.30	**22.31**	21.84
	cameraman	14.33	23.63	23.57	25.01	25.02	25.02	**25.03**	24.22
	couple	14.31	23.54	23.52	**24.88**	24.85	24.86	**24.88**	24.29
	fingerprint	13.62	20.59	20.58	22.03	21.99	22.00	**22.04**	21.60
	hill	14.62	22.49	22.48	**23.98**	23.96	23.97	**23.98**	23.36
	house	13.73	24.36	24.34	**26.58**	26.57	26.57	**26.58**	25.76
	lena	14.20	25.57	25.55	**27.40**	27.37	27.38	**27.40**	26.58
	man	14.64	24.08	24.06	25.66	25.65	25.66	**25.67**	25.09
	mandril	14.03	22.18	22.17	23.03	23.01	23.02	**23.04**	22.68
	peppers	14.20	23.38	23.35	**25.45**	25.41	25.43	**25.45**	24.41

PSNR values: the higher the better

- Generalized likelihood ratio
- Variance stabilization
- Euclidean distance
- Maximum joint likelihood [Alter et al., 2006]
- Mutual information kernel [Seeger, 2002]
- Bayesian likelihood ratio [Minka, 1998, Minka, 2000]
- Bayesian joint likelihood [Yianilos, 1995, Matsushita and Lin, 2007]

Evaluation on denoising – Numerical performance

		Noisy	$\mathcal{Q}_B$	$\mathcal{Q}_G$	$\mathcal{L}_B$	$\mathcal{L}_G$	$\mathcal{K}_B$	$\mathcal{S}$	$\mathcal{G}$
		Poisson							
Strong noise levels	barbara	5.68	20.25	20.25	20.52	**20.68**	20.65	20.59	20.42
	boat	5.23	20.90	20.90	21.11	**21.21**	21.19	21.15	21.04
	bridge	5.83	18.36	18.36	18.65	**18.81**	18.78	18.72	18.53
	cameraman	5.59	18.61	18.61	19.17	**19.56**	19.49	19.37	19.01
	couple	5.55	20.91	20.91	21.11	**21.20**	21.18	21.15	21.04
	fingerprint	4.87	15.48	15.48	16.18	**16.41**	16.38	16.30	15.96
	hill	5.88	20.13	20.13	20.41	**20.54**	20.52	20.47	20.31
	house	4.94	20.48	20.49	20.81	**20.97**	20.94	20.88	20.67
	lena	5.44	22.14	22.15	22.44	**22.59**	22.56	22.49	22.30
	man	5.89	21.55	21.55	21.77	**21.89**	21.87	21.82	21.69
	mandril	5.31	20.23	20.23	20.34	**20.38**	20.37	20.36	20.30
	peppers	5.46	18.55	18.56	19.09	**19.46**	19.38	19.25	18.88

PSNR values: the higher the better

- Generalized likelihood ratio
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	Max. self sim.	Eq. self sim.	Id. of indiscernible	Invariance	Asym. CFAR	Asym. UMPI
$\mathcal{Q}_B$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
$\mathcal{Q}_G$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
$\mathcal{L}_B$	$\times$	$\times$	$\times$	$\surd$	$\times$	$\times$
$\mathcal{L}_G$	$\surd$	$\surd$	$\surd^{(\dagger)}$	$\surd$	$\surd$	$\surd$
$\mathcal{K}_B$	$\surd$	$\surd$	$\surd^{(\ddagger)}$	$\surd$	$\times$	$\times$
$\mathcal{G}$	$\surd$	$\surd$	$\surd$	$\times$	$\times$	$\times$
$\mathcal{S}$	$\surd^{(\star)}$	$\surd^{(\star)}$	$\surd^{(\star)}$	$\surd^{(\star)}$	$\surd^{(\star)}$	$\times$

Properties of the different studied criteria. Legend: ($\surd$) the criterion holds, ($\times$) the criterion does not hold. Holds only if the observations are statistically identifiable (†) through their MLE or (‡) through their likelihood (such assumptions are frequently true). (*) Holds only for an exact variance stabilizing transform $s(\cdot)$ (such an assumption is usually wrong). The proofs of all these properties are available in the Online Resource 1.

name	pdf	$\mathcal{Q}_B$	$\mathcal{Q}_G$	$\mathcal{L}_B$	$\mathcal{L}_G$	$\mathcal{K}_B$	$\mathcal{S}$	$\mathcal{G}$
Gaussian	$\frac{e^{-\frac{(x-\theta)^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma}$	$e^{-(x_1-x_2)^2}$						
Gamma	$\frac{L^L x^{L-1} e^{-\frac{Lx}{\theta}}}{\Gamma(L)\theta^L}$	$\frac{1}{x_1 x_2}\left(\frac{x_1 x_2}{(x_1+x_2)^2}\right)^L$		$\frac{x_1 x_2}{(x_1+x_2)^2}$			$e^{-\left(\log \frac{x_1}{x_2}\right)^2}$	
Poisson	$\frac{\theta^x e^{-\theta}}{x!}$	$\frac{\Gamma'(x_1+x_2)}{2^{x_1+x_2} x_1! x_2!}$	$\frac{(x_1+x_2)^{x_1+x_2}}{(2e)^{x_1+x_2} x_1! x_2!}$	$\frac{\Gamma'(x_1+x_2)}{2^{x_1+x_2}\Gamma'(x_1)\Gamma'(x_2)}$	$\frac{(x_1+x_2)^{x_1+x_2}}{2^{x_1+x_2} x_1^{x_1} x_2^{x_2}}$	$\frac{\Gamma'(x_1+x_2)}{\sqrt{\Gamma'(2x_1)\Gamma'(2x_2)}}$	$e^{-\left(\sqrt{x_1+a}-\sqrt{x_2+a}\right)^2}$	

Instances of the seven criteria for Gaussian, gamma and Poisson noise (parameters σ and L are fixed and known). All Bayesian criteria are obtained with Jeffreys' *priors* (resp. $1/\sigma$, $\sqrt{L}/\theta$, $\sqrt{1/\theta}$). All constant terms which do not affect the detection performance are omitted. For clarity reason, we define $\Gamma'(x) = \Gamma(x + 0.5)$ and the Anscombe constant $a = 3/8$.