Massey products in Galois cohomology via rational points

Abstract: the Milnor conjecture identifies the cohomology ring $H^*(\mathrm{Gal}(\bar{k}/k), \mathbb{Z}/2)$ with the tensor algebra of k^* mod the ideal generated by $x \otimes (1-x)$ for $x \in k - \{0,1\}$ mod 2. In particular, $x \cup (1-x)$ vanishes, where $x \in k^*$ is identified with an element of H^1. We show that order n Massey products of $n-1$ factors of x and one factor of $1-x$ vanish by embedding $\mathbb{P}^1 - \{0,1,\infty\}$ into its Picard variety and constructing $\mathrm{Gal}(\bar{k}/k)$ equivariant maps from π_1^{et} applied to this embedding to unipotent matrix groups. This also identifies Massey products of the form $\langle 1-x, x, \ldots, x, 1-x \rangle$ with $f \cup (1-x)$, where f is a certain cohomology class which arises in the description of the action of $\mathrm{Gal}(\bar{k}/k)$ on $\pi_1^{\mathrm{et}}(\mathbb{P}^1 - \{0,1,\infty\})$.