p-Groups and Automorphism groups of curves in char. $p > 0$

M. MATIGNON[1]

[1]Laboratoire A2X Bordeaux

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Monodromy and automorphism groups.

- R is a strictly henselian DVR of inequal characteristic $(0, p)$.
  $K := \mathrm{Fr} R$; for example $K/\mathbb{Q}_p^{ur}$ finite.
  π a uniformizing parameter.
  $k := R_K/\pi R_K$.
  C/K smooth projective curve, $g(C) \geq 1$.
- C has potentially good reduction over K if there is L/K (finite) such that $C \times_K L$ has a smooth model over R_L. Then :
- There is a minimal extension L/K with this property ; it is Galois and called the **monodromy** extension.
- $\mathrm{Gal}(L/K)$ is the **monodromy group**.
- Its p-Sylow subgroup is the **wild monodromy group** .

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- The base change $C \times_K K^{alg}$ induces an homomorphism $\varphi : \mathrm{Gal}(K^{alg}/K) \to \mathrm{Aut}_k C_s$, where C_s is the special fiber of the smooth model over R_L and $L = (K^{alg})^{\ker \varphi}$.
- Let ℓ be a prime number, then, $n_\ell := v_\ell(|\mathrm{Gal}(L/K)|) \leq v_\ell(|\mathrm{Aut}_k C_s|)$.
- If $\ell \notin \{2, p\}$, then $n_\ell \leq 2g$.
- If $p > 2$, then $n_p \leq \inf_{\ell \neq 2,p} v_p(|\mathrm{GL}_{2g}(\mathbb{Z}/\ell\mathbb{Z})|) = a + [a/p] + \ldots$, where $a = [\frac{2g}{p-1}]$.
- This gives an exponential type bound in g for $|\mathrm{Aut}_k C_s|$. This justifies our interest in looking at polynomial bounds, Stichtenoth ([St 73]) and Singh ([Si 73]).

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k is an algebraically closed of char. $p > 0$.

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- $G_\infty(f) := \{\sigma \in \mathrm{Aut}_k C_f \mid \sigma(\infty) = \infty\}$.
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- ([St 73]) Let $g(C_f) \geq 2$, then $G_{\infty,1}(f)$ is a p-Sylow of $\mathrm{Aut}_k C_f$.
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Structure of $G_{\infty,1}(f)$

- Let $\rho(X) = X$, $\rho(W) = W + 1$, then $< \rho >= G_{\infty,2} \subset Z(G_{\infty,1})$
- $0 \to < \rho > \to G_{\infty,1} \to V \to 0$, $V = \{\tau_y | \tau_y(X) = X + y,\ y \in k\}$. $f(X+y) = f(X) + f(y) + (F - \mathrm{Id})(P(X,y))$, $P(X,y) \in Xk[X]$. $V \simeq (\mathbb{Z}/p\mathbb{Z})^v$ as a subgroup of k.
- Let $\tau_y(W) := W + a_y + P(X,y)$, $a_y \in \mathbb{F}_p$, then $[\tau_y, \tau_z] = \rho^{\epsilon(y,z)}$, where $\epsilon : V \times V \to \mathbb{F}_p$ is an alternating form.
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Lemma

If $f(X)=\sum_{1\leq i\leq m} t_i X^i \in k[X]$ is monic, then :

- $\Delta(f)(X,Y) := f(X+Y)-f(X)-f(Y) = R(X,Y)+(F-\mathrm{Id})(P_f(X,Y))$, where $R \in \bigoplus_{\lfloor \frac{m}{p} \rfloor \leq ip^{n(i)}<m,\ (i,p)=1} k[Y]X^{ip^{n(i)}}$ and $P_f \in Xk[X,Y]$.
- $P_f = (\mathrm{Id}+F+\ldots+F^{n-1})(\Delta(f)) \mod X^{[\frac{m-1}{p}]+1}$.
- *Let us denote by $\mathrm{Ad}_f(Y)$ the content of $R(X,Y) \in k[Y][X]$.*
- $\mathrm{Ad}_f(Y)$ *is an additive and separable polynomial.*
- $Z(\mathrm{Ad}_f(Y)) \simeq V$.

Bounds for $|G_{\infty,1}(f)|$

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- ([St 73]) $|G_{\infty,1}| = p \deg \mathrm{Ad}_f \leq p(m-1)^2$, i.e. $\frac{|G_{\infty,1}|}{g^2} \leq \frac{4p}{(p-1)^2}$.
- ([St 73]) $s=0$ i.e. $(m-1,p)=1$, then $|G_{\infty,1}|=p$.
- If $s>0$,
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  - ([St 73]) $\ell=1, m=1+p^s$, then $\frac{|G_{\infty,1}|}{g} \leq 2p^s\frac{p}{p-1}$ (with equality for $f(X)=X^{1+p^s}$).

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- ([St 73]) $|G_{\infty,1}|=p\deg \mathrm{Ad}_f\leq p(m-1)^2$, i.e. $\frac{|G_{\infty,1}|}{g^2}\leq\frac{4p}{(p-1)^2}$.
- ([St 73]) $s=0$ i.e. $(m-1,p)=1$, then $|G_{\infty,1}|=p$.
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In this section (C, G) **is a big action.** Let G_i be the lower ramifi cation groups.

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- $\mathrm{Im}\rho$ is a unipotent subgroup of $\mathrm{Gl}_t(\mathbb{F}_p)$ which is the identity iff $G_2 \subset Z(G)$. In this case $f_i(X) = c_i X + X\Sigma_i(F)(X)$ where $\Sigma_i(F) \in k\{F\}$ and $v \in V$ is a commun zero to the palyndromic polynomials $\mathrm{Ad}_{f_i} \in k\{F, F^{-1}\}$.
- For $p > 2$, we give an example such that $\mathrm{Im}\rho \neq \mathrm{Id}$
  - Let $f_1 := X(\alpha F)(X) = \alpha X^{1+p}$ with $\alpha^p + \alpha = 0$; then $\mathrm{Ad}_{f_1} = Y^{p^2} - Y$ and Let $f_2 := X^{1+2p} - X^{2+p}$, then
  - If $y \in Z(\mathrm{Ad}_{f_1}) = \mathbb{F}_{p^2}$ one has $f_2(X+y) = \frac{2(y^p - y)}{\alpha} f_1(X) + f_2(X) + \wp(P_2)$. where $y \to \frac{2(y^p-y)}{\alpha}$ is a non zero linear form over $\mathbb{F}_{p^2}$ with values in $\mathbb{F}_p$.
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- **Theorem**
  ([Le-Ma 4]) Assume G_2 is non abelian, then $G_2 = G'$.
  - Sketch proof : If $G' \neq G_2$, there is $H \lhd G$ with $G' \subset H \subset G_2$ and $[G_2 : H] = p$. $(C/H, G/H)$ is a big action ;
  - $C/H : W^p - W = f := X\Sigma(F)(X)$, $\deg(f) = 1 + p^s$.
  - $(\mathrm{Aut}C/H)_{\infty,1} := E$ is extraspecial of order p^{2s+1}.
  - G/H is abelian and normal in E.
  - ([Hu 67] Satz 13.7 p. 353) $|G/H| \leq p^{s+1}$ and so $|G/H|/g(C/H) \leq \frac{2p^{s+1}}{(p-1)p^s} = \frac{2p}{p-1}$, a contradiction.
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Riemann surfaces

- In characteristic 0, an analogue of big actions is given by the actions of a finite group G on a compact Riemann surface C with $g_C \geq 2$ such that $|G| = 84(g_C - 1)$ (we say that C is an **Hurwitz curve**) ([Co 90]).
- Let us mention Klein's quartic ($G \simeq PSL_2(\mathbb{F}_7)$) ([El 99]).
- The Fricke-Macbeath curve with genus 7 ($G \simeq PSL_2(\mathbb{F}_8)$) ([Mc] 65).
- Let C be an Hurwitz curve with genus g_C. Let $n > 1$ and C_n the maximal unramified Galois cover whose group is abelian with exponent n. The Galois group of C_n/C is $(\mathbb{Z}/n\mathbb{Z})^{2g_C}$. It follows from the unicity of C_n that the k-automorphisms of C have n^{2g} prolongations to C_n. It follows that C_n is an Hurwitz curve ([Mc] 61).

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Ray class fields

- If (C, G) is a big action then $C \to C/G$ is an étale cover of the affine line whose group is a p-group ; it follows that the Hasse-Witt invariant of C is zero ; therefore, in order to adapt the previous proof to char. $p > 0$, one needs to accept ramification. This is done with the so called ray class fields of function fields over finite fields.
- Let $K := \mathbb{F}_q(X)$ where $q = p^e$, S the set of finite rational places $(X - v)$, $v \in \mathbb{F}_q$ and $m \in \mathbb{N}$. Let K^{alg} be an algebraic closure. Let $K_S^m \subset K^{alg}$, the biggest abelian extension L of K with conductor $\leq m\infty$ and such that the places in S are completely decomposed.

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- ([La 99], [Au 00]) The constant field of K_S^m is $\mathbb{F}_q$ and $G_S(m) := \mathrm{Gal}(K_S^m/K) \simeq (1 + T\mathbb{F}_q[[T]])/ < 1 + T^m\mathbb{F}_q[[T]], 1 - vT, v \in \mathbb{F}_q >$, is a p-group.
- ([Le-Ma 4]) Let $C_m/\mathbb{F}_q$ be the smooth projective curve with function field K_S^m. The translations $X \to X + v$, $v \in \mathbb{F}_q$ stabilize S and ∞; they can be extended to $\mathbb{F}_q$-automorphisms of K_S^m. In this way, we get an action of a p-group $G(m)$ on C_m with $0 \to G_S(m) \to G(m) \to \mathbb{F}_q \to 0$
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Maximal curves (I)

Let us assume that (C, G) is a big action.

- Let i_0 with $G_2 = G_3 = ... = G_{i_0} \supsetneq G_{i_0+1}$. Then $g_{(C/G_{i_0+1})} = \frac{1}{2}(|G_2/G_{i_0+1}| - 1)(i_0 - 1)$.

- **Theorem**
  ([Le-Ma 1]) If $\frac{|G|}{g_C^2} \geq \frac{4}{(p-1)^2}$ there is $\Sigma(F) \in k\{F\}$ and $f = cX + X\Sigma(F)(X) \in k[X]$ with $C \simeq C_f$. Moreover there are two possibilities for G :
  - $\frac{|G|}{g_C^2} = \frac{4p}{(p-1)^2}$ *and $G = G_{\infty,1}(f)$ or*
  - $\frac{|G|}{g_C^2} = \frac{4}{(p-1)^2}$ *and $G \subset G_{\infty,1}(f)$ has index p.*

  - One can push the “classifi cation ” of big actions up to the condition $\frac{|G|}{g_C^2} \geq \frac{4}{(p^2-1)^2}$.

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  ([Le-Ma 4]) For all $M > 0$, the set $\frac{|G|}{g_C^2} > M$, for (C, G) a big action with G_2 abelian with exponent p, is finite.

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- **Theorem**

  ([Le-Ma 4]) For all $M > 0$*, the set* $\frac{|G|}{g_C^2} > M$*, for* (C, G) *a big action with* G_2 *abelian with exponent p, is finite.*

Maximal curves (I)

Let us assume that (C,G) is a big action.

- Let i_0 with $G_2 = G_3 = ... = G_{i_0} \supsetneq G_{i_0+1}$. Then $g_{(C/G_{i_0+1})} = \frac{1}{2}(|G_2/G_{i_0+1}| - 1)(i_0 - 1)$.

- **Theorem**
  ([Le-Ma 1]) If $\frac{|G|}{g_C^2} \geq \frac{4}{(p-1)^2}$ *there is* $\Sigma(F) \in k\{F\}$ *and* $f = cX + X\Sigma(F)(X) \in k[X]$ *with* $C \simeq C_f$.
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Monodromy polynomial ([Le-Ma 3])

- Let $C \longrightarrow \mathbb{P}^1_K$ birationally given by the equation :
  $Z_0^p = f(X_0) = \prod_{1 \leq i \leq m} (X_0 - x_i)^{n_i} \in R[X_0]$,
  $(n_i, p) = 1$ and $(\deg f, p) = 1$,
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- $f'(Y)/f(Y) = S_1(Y)/S_0(Y)$, $(S_0(Y), S_1(Y)) = 1$;
  then $\deg(S_1(Y)) = m - 1$ and $\deg(S_0(Y)) = m$.
- $f(X+Y) = f(Y)((1 + a_1(Y)X + \ldots + a_r(Y)X^r)^p - \sum_{r+1 \leq i \leq n} A_i(Y)X^i)$, où $r + 1 = [n/p]$,
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$$A_{p^\alpha}(Y) = -\binom{\frac{1}{p}}{p^\alpha - 1}^p \cdot \frac{S_1(Y)^{p^\alpha} + pT(Y)}{S_0(Y)^{p^\alpha}}.$$

- $\mathcal{L}(Y) := S_1(Y)^{p^\alpha} + pT(Y)$. This is a polynomial of degree $p^\alpha(m-1)$ which is called the *monodromy polynomial* of $f(Y)$.

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Special fiber of the easy model

We mean the R-model $\mathcal{C}_R$ defined by
$Z_0^p = f(X_0) = \prod_{1 \leq i \leq m}(X_0 - x_i)^{n_i} \in R[X_0]$ (cf. fig 1).

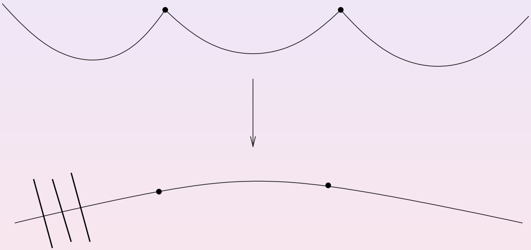

FIG.: $\mathcal{C}_R \otimes_R k \longrightarrow \mathbb{P}_k^1$ with singularities and branch locus

Potentially good reduction with $m = 1 + p^s$

Theorem

([Le-Ma 3])

- $p > 2$, $q = p^n$, $n \geq 1$, $K = \mathbb{Q}_p^{\mathrm{ur}}(p^{p/(q+1)})$ *and* $C \longrightarrow \mathbb{P}^1_K$ *is birationally defined by the equation* $Z_0^p = f(X_0) = 1 + p^{p/(q+1)} X_0^q + X_0^{q+1}$.
- *Then, C has potentially good reduction and $\mathcal{L}(Y)$ is irreducible over K.*
- *The monodromy L/K is the extension of the decomposition field of $\mathcal{L}(Y)$ obtained by adjoining the p-roots $f(y)^{1/p}$, for y describing the zeroes of $\mathcal{L}(Y)$.*
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Curves of genus 2 ([Le-Ma 3])

- Case $p = 2$ and $m = 5$ (i.e. curves with genus 2 over a 2-adic fi eld $\subset \mathbb{Q}_2^{\text{tame}}$).
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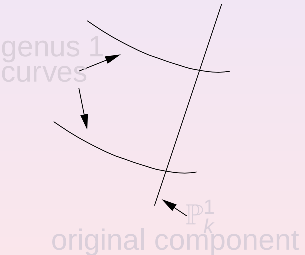

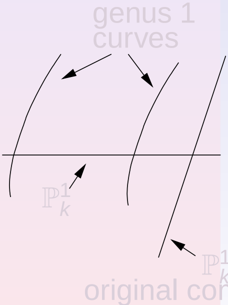

Type 1

$\mathrm{Gal}(K'/K)_w \hookrightarrow Q_8 \times Q_8$

Type 2

$\mathrm{Gal}(K'/K)_w \hookrightarrow (Q_8 \times Q_8) \rtimes \mathbb{Z}/2\mathbb{Z}$

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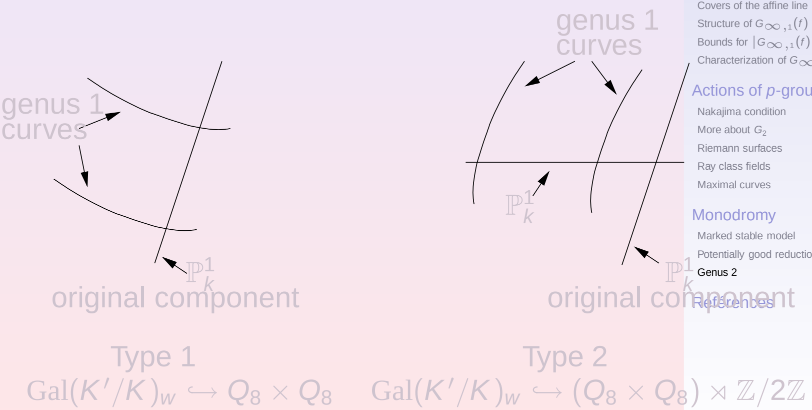

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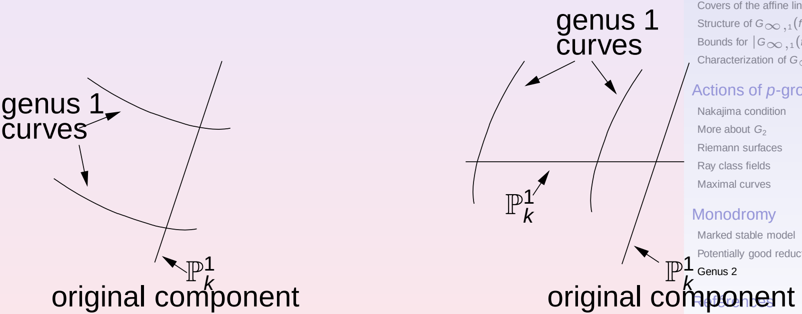

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- $C \longrightarrow \mathbb{P}^1_K$ is birationally defined by the equation $Z_0^p = f(X_0)$ with $f(X_0) = 1 + b_2X_0^2 + b_3X_0^3 + b_4X_0^4 + X_0^5 \in R[X_0]$.
- Now, we see that the monodromy can be maximal for the 3 types of degeneration.

- a) $f(X_0) = 1 + 2^{3/5}X_0^2 + X_0^3 + 2^{2/5}X_0^4 + X_0^5$ and $K = \mathbb{Q}_2^{\mathrm{ur}}(2^{1/15})$;
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- b) Let $K = \mathbb{Q}_2^{\mathrm{ur}}(a)$ with $a^9 = 2$ and $f(X_0) = 1 + a^3X_0^2 + a^6X_0^3 + X_0^5$.
- C has a marked stable model of type 2.
- The maximal wild monodromy group is $\simeq (Q_8 \times Q_8) \rtimes \mathbb{Z}/2\mathbb{Z}$, where $\mathbb{Z}/2\mathbb{Z}$ exchanges the 2 factors.

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- The maximal wild monodromy group is $\simeq (Q_8 \times Q_8) \rtimes \mathbb{Z}/2\mathbb{Z}$, where $\mathbb{Z}/2\mathbb{Z}$ exchanges the 2 factors.

- c) $K = \mathbb{Q}_2^{\mathrm{ur}}$ and . $f(X_0) = 1 + X_0^4 + X_0^5$.
- C has potentially good reduction (i.e. is of type 3)
- The maximal wild monodromy group is $\simeq Q_8 * D_8$.

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