A Fluid-Mixture Type Algorithm for Compressible Multicomponent Flow: Eulerian versus Lagrangian-like Formulation

Keh-Ming Shyue
Department of Mathematics
National Taiwan University

Abstract

This talk consists of two parts. In the first part, we describe a simple Eulerian interface-capturing approach for the efficient numerical resolution of a hybrid barotropic and non-barotropic two-fluid flow problem in more than one space dimension. We use the compressible Euler equations as a model system with the thermodynamic property of each of the barotropic and non-barotropic fluid components characterized by the Tait and Noble-Abel equations of state, respectively. The algorithm is based on a volume fraction formulation of the equations together with an extended equation of state that is devised to give an approximate treatment for the mixture of more than one fluid component within a grid cell. A standard high-resolution wave propagation method is employed to solve the proposed two-fluid model with the dimensional-splitting technique incorporated in the method for multidimensional problems. Several numerical results are presented in one, two, and three space dimensions that show the feasibility of the algorithm as applied to a reasonable class of practical problems without the occurrence of any spurious oscillation in the pressure near the smeared material interfaces.

In the second part, we generalize the above multicomponent flow model to a so-called unified coordinate system proposed by Hui and coworkers, see references shown below. In this formulation, a new coordinate system (τ, ξ, η, ζ) in three space dimensions is introduced via the transformation from the Eulerian system (t, x, y, z):

$$\begin{cases} dt = d\tau \\ dx = U d\tau + A d\xi + L d\eta + P d\zeta \\ dy = V d\tau + B d\xi + M d\eta + Q d\zeta \\ dz = W d\tau + C d\xi + N d\eta + R d\zeta, \end{cases}$$

where $\vec{q} = (U, V, W)$ is the grid velocity, and A, B, C, L, M, N, P, Q, and R are the geometric variables which are related to $\vec{q}$ by the compatibility conditions. In this instance, it includes the Eulerian as a special case when $\vec{q}$ is stationary and the Lagrangian when $\vec{q}$ is the underlying fluid velocity. Here, the arbitrary functions U, V, and W give three-degree of freedoms in the problem formulation. One major objective of this talk is to explore this extra degree of freedom so as to obtain an improved resolution of the material interfaces in a multicomponent flow computation. This is a joint work with W. H. Hui and J. J. Hu.

References

- W. H. Hui and P. Y. Li and Z. W. Li, A unified coordinate system for solving the two-dimensional Euler equations, J. Comput. Phys., 153:596-637, 1999.
- W. H. Hui, G. P. Zhao, J. H. Hu, and Y. Zheng. Automatic grid-generation method for two-dimensional flow using the unified coordinates, submitted for publication, 2005.