Numerical Simulation of Cavitation Bubbles by Compressible Two-Phase Fluids

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Outline

- Motivation
- Modelling of Two-Phase Fluid Flows
- Numerical Discretization
- Numerical Results
- Outlook

Cavitation in Liquids

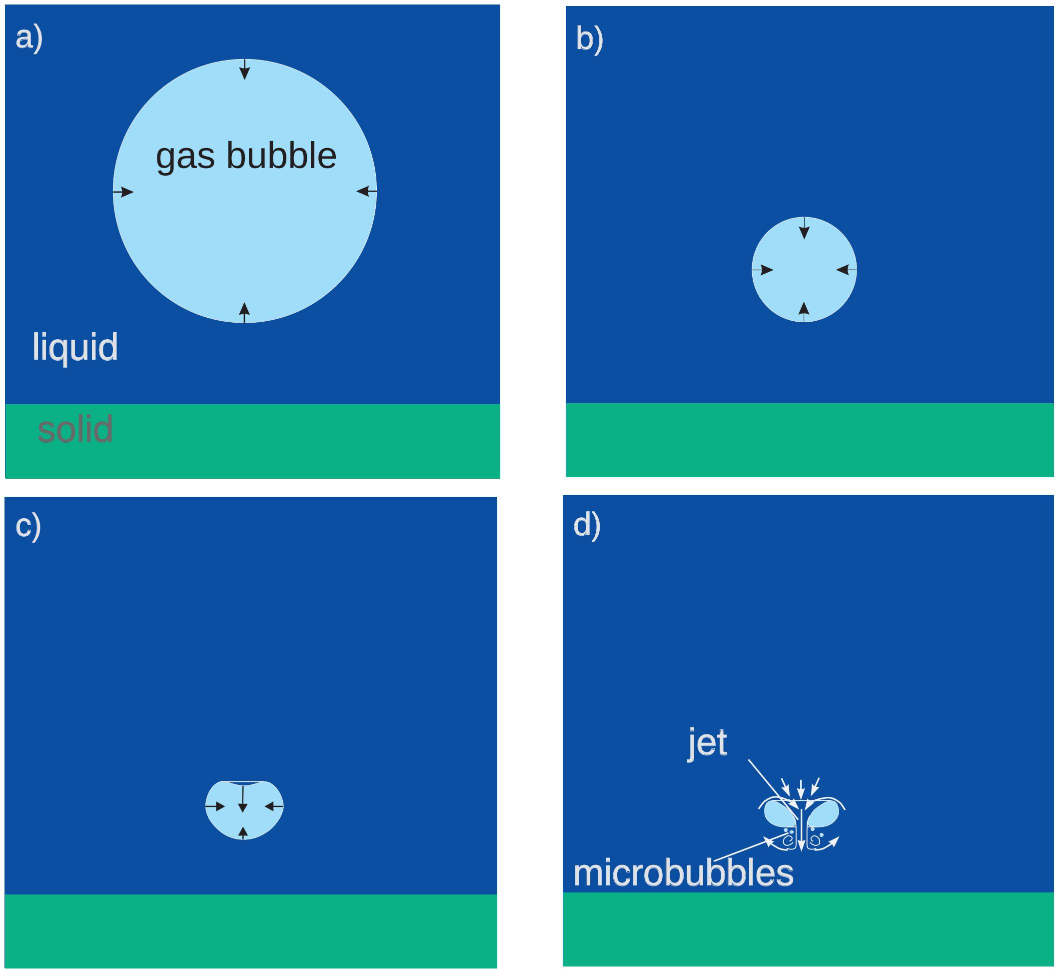
- Phenomena
  - Growing of previously inviscible small gas bubbles due to pressure drop
  - Forming of vapor bubbles if pressure drops below vapor pressure, e.g. very high acceleration

- Occurence
  - at ship propellers (famous explanation of cavitation damaging by Lord Rayleigh)
  - in oil pumps and liquid injectors
  - in medical applications, e.g. lithotripsy

- Basic experiments
  - Laser induced single bubbles far from and near solid boundaries of different stiffness (Courtesy of Lauterborn; Brujan, Nahen, Schmidt, Vogel — JFM2001)

- Observed Processes
  - Bubble collapse accompanied by strong shock waves in the bubble and the liquid as well
  - Interactions of shock/rarefaction waves with phase boundaries
  - Bubble oscillation, splitting and coalescing
  - Formation of liquid jets

Cavitation near a Solid



Goals of the Project

- Experiments with laser induced bubbles

  Group of Lauterborn/Kurz (Göttingen)

  - high speed photographic observations
  - flow field measurements (PIV)
  - shock wave visualization and measurement

- Numerical investigation

  Group of Ballmann/M. (Aachen), Helluy (Strasbourg)

  - pressure and density contours
  - velocity vectors

$\rightsquigarrow$ Expected Results

  - better understanding of complicated flow dynamics
  - mechanism of material damage

Modelling of Two-Phase Flows

I. Requirements

II. Phase Boundary Motion

III. Two-Phase Flow Model

Requirements

- Basic analysis
  - spherical bubble collapse using 2 equations of state taking both phases as compressible (Hanke)
  - Results:
    * Extreme changes of state due to compression and shocks in gas and liquid
    * Inhomogeneous fields even in small bubbles due to strong wave processes
    * Shocks can be formed in the gas as well as in the liquid

- Consequences for modelling
  - real gas and compressible liquid approach with two equations of state
  - extended by non-equilibrium thermodynamics including phase transition

Phase Boundary Motion

I. **Lagrangian approach** (interface-tracking)

- particle methods, front tracking methods, marker methods
- **Disadvantages**
  - ∗ extremely difficult and expensive in multi-dimensions
  - ∗ the extreme jump of the acoustic impedance $[\rho\, c]$
  - ∗ topological changes due to deformation, splitting, coalescing of p.b.

II. **Eulerian approach** (interface-capturing)

- **evolution equation for interface position**
  - ∗ colour function, levet set function, fraction of phase
- **Problem**: Pressure oscillation at interface
  - ∗ quasi-conservative formulation Abgrall (96), Saurel/Abgrall (99), Abgrall/Karni (01))
  - ∗ ghost fluid method (Fedkiv et al. (99))
  - ∗ two-flux method (Abgrall/Karni (01))

Two-Phase Flow Model

Compressible Navier-Stokes Equations:

$$\partial_t \rho + \text{div}\,(\rho\,\mathbf{v}) = 0$$

$$\partial_t\,(\rho\,\mathbf{v}) + \text{div}\,(\rho\,\mathbf{v}\,\mathbf{v}^T + p\,\mathbf{I}) = \text{div}\,(\tau_v) + \gamma\,\kappa\,\delta(d)\,\mathbf{n}$$

$$\partial_t\,(\rho\,E) + \text{div}\,(\rho\,\mathbf{v}\,(E + p/\rho)) = \text{div}\,(\tau_v\,\mathbf{v} - \mathbf{q}) + \gamma\,\kappa\,\delta(d)\,\mathbf{v}^T\,\mathbf{n}$$

Vaporization Model:

$$\partial_t\,\varphi + \mathbf{v}^T\,\text{grad}\varphi = \mu$$

$$0 \leq \varphi \leq 1 \qquad \text{(gas fraction)}$$

Equation of State:

$$p = p(\rho, e, \varphi)$$

Mixture of Fluids

- pressure law behaves as a stiffened gas

$$p = (\gamma(\varphi) - 1)\,\rho\,e - \gamma(\varphi)\,\pi(\varphi), \quad e = c_v\,T + \pi(\varphi)/\rho$$

- linear interpolation

$$\begin{cases} \beta_1 = \dfrac{1}{\gamma - 1} \\ \beta_2 = \dfrac{\gamma\pi}{\gamma - 1} \end{cases} \quad \Leftrightarrow \quad \begin{cases} \gamma = 1 + \dfrac{1}{\beta_1} \\ \pi = \dfrac{\beta_2}{1 + \beta_1} \end{cases}$$

$$\begin{aligned} \beta_1(\varphi) &= \varphi\beta_1(1) + (1-\varphi)\beta_1(0), \\ \beta_2(\varphi) &= \varphi\beta_2(1) + (1-\varphi)\beta_2(0). \end{aligned}$$

Discretization of Fluid Equations

- Finite volume discretization of conserved quantities $(\rho, \rho\,\boldsymbol{v}, \rho\,E)$
  - explicit time discretization
  - 2nd order reconstruction of primitive variables $(\rho, \boldsymbol{v}, p, \boldsymbol{\beta})$
  - exact Riemann solver
- Upwind discretization of non-conservative transport equation for gas fraction (Abgrall/Saurel)
  - **Idea**: Preserve constant pressure and velocity fields
- Multiscale-based grid adaptation
- Multilevel time stepping

Conclusion

- Discretization of Stiffened Gas Model
  - higher order reconstruction
  - multiscale-based grid adaptation
  - multilevel time stepping
  - $\rightsquigarrow$ locally high resolution of physical effects
  - $\rightsquigarrow$ efficient computations
- Application: Bubble Collapse near to a Rigid Wall
  - Interaction of shock/rarefaction waves with phase boundary
  - Formation of liquid jets
  - Bubble splitting and moving
- Future
  - enhanced physical model
    - $\ast$ viscosity and heat conduction
    - $\ast$ surface tension
    - $\ast$ phase transition
  - sharp interface model using level set method