C. R. Acad. Sci. Paris, t. 330, Série I, p. 957–962, 2000
Équations aux dérivées partielles/*Partial Differential Equations*

Existence of solutions for a class of partial neutral differential equations

Mostafa ADIMY [a], Khalil EZZINBI [b], Mostafa LAKLACH [a]

[a] **Département de mathématiques appliquées, URS 2055 CNRS, Université de Pau, avenue de l'Université, 64000 Pau, France**
E-mail: Mostafa.Adimy@univ-pau.fr

[b] **Département de Mathématiques, faculté des sciences Semlalia, B.P. 2390, Marrakech, Maroc**

(Reçu le 31 janvier 2000, accepté après révision le 27 avril 2000)

Abstract. In this work, we obtain the local and the global existence for a class of partial neutral differential equations with a non dense domain. We assume that the nonlinear part is continuous and the linear part satisfy a compactness property. © 2000 Académie des sciences/Éditions scientifiques et médicales Elsevier SAS

Existence de solutions pour une classe d'équations aux dérivées partielles de type neutre

Résumé. *Dans ce travail, nous obtenons des résultats d'existence locale et globale pour une classe d'équations aux dérivées partielles de type neutre avec un domaine non dense. Nous supposons que la partie non linéaire est continue et que la partie linéaire satisfait une propriété de compacité.* © 2000 Académie des sciences/Éditions scientifiques et médicales Elsevier SAS

Version française abrégée

1. Introduction

Dans la description mathématique d'un circuit électrique formé de plusieurs oscillateurs identiques, connectés entre eux par une résistance et formant une boucle fermée, Hale [6,7], Wu [11], et Wu et Xia [12, 13] ont abouti à un système d'équations de type neutre (au sens de Hale [5]) avec des termes de diffusions discrètes. Dans ce système, chaque équation correspond à un oscillateur. Dans la pratique le nombre N d'oscillateurs est très grand. Par passage à la limite, $N \to \infty$, ils ont obtenu une équation aux dérivées partielles dite de type neutre, de la forme :

$$\frac{\partial}{\partial t}\mathcal{D}v_t = K\frac{\partial^2}{\partial x^2}\mathcal{D}v_t + f(v_t), \quad t \geqslant 0. \tag{1}$$

Note présentée par Pierre-Louis LIONS.

S0764-4442(00)00307-4/FLA

Cette équation (1) appartient à la classe d'équations aux dérivées partielles suivante :

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{D}u_t = A\mathcal{D}u_t + F(t,u_t), \quad t \geqslant 0, \tag{2}$$

où $\mathcal{D} : \mathrm{C}([-r,0];X) \to X$ est un opérateur linéaire continu, X est un espace de Banach et A est un opérateur non borné de domaine $D(A)$ contenu dans X.

Dans ce travail, nous supposons que : (i) l'opérateur A est à domaine non dense et satisfait la condition de Hille–Yosida (H1) dans X, (ii) l'opérateur $\mathcal{D}$ satisfait la condition (H2), (iii) l'opérateur A satisfait la propriété de compacité (H3) sur $\overline{D(A)}$.

2. Résultats

THÉORÈME 1. – *Soit U un ouvert de l'espace de Banach $C_0 := \{\varphi \in \mathrm{C}_X : \mathcal{D}\varphi \in \overline{D(A)}\}$. Supposons que $F : [0,T] \times U \to X$ est continue. Alors, pour chaque $\varphi \in U$, il existe $t_1 := t_1(\varphi) \in (0,T]$ et une solution intégrale $u \in \mathrm{C}([-r,t_1];X)$ de l'équation* (2)*.*

THÉORÈME 2. – *Supposons que $F : [0,+\infty) \times \mathrm{C}_X \to X$ est continue et transforme tout borné de $[0,+\infty) \times \mathrm{C}_X$ en un borné de X. Alors, pour chaque $\varphi \in C_0$, il existe un intervalle maximal d'existence $[0,t_\varphi)$, $t_\varphi > 0$, et une solution intégrale $u := u(\cdot,\varphi)$ de l'équation* (2)*, définie sur $[0,t_\varphi)$ et satisfaisant l'une des deux conditions $t_\varphi = +\infty$ ou bien $\limsup_{t\to t_\varphi^-} \|u_t\| = +\infty$.*

THÉORÈME 3. – *Supposons que les hypothèses du théorème* 2 *sont satisfaites, et qu'il existe deux fonctions localement intégrables l_1 et l_2 telles que $|F(t,\varphi)| \leqslant l_1(t)\|\varphi\| + l_2(t)$, pour $\varphi \in C_0$ et $t \geqslant 0$. Alors, pour chaque $\varphi \in C_0$, l'équation* (2) *admet une solution intégrale globale sur $[-r,+\infty)$.*

---

1. Introduction

Suppose that $r > 0$ is a given real number, $(X,|\cdot|)$ is a Banach space and $\mathcal{L}(X)$ is the space of bounded linear operators from X into X. We denote by $\mathrm{C}_X := \mathrm{C}([-r,0];X)$ the space of continuous functions from $[-r,0]$ to X. For $u \in \mathrm{C}([-r,b];X)$, $b > 0$ and $t \in [0,b]$, let u_t denote the element of C_X defined by $u_t(\theta) = u(t+\theta)$, $-r \leqslant \theta \leqslant 0$. By an abstract semilinear neutral functional differential equation on X we mean an evolution system of the type:

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}\mathcal{D}u_t = A\mathcal{D}u_t + F(t,u_t), & t \geqslant 0, \\ u(t) = \varphi(t), & t \in [-r,0], \end{cases} \tag{1}$$

where $A : D(A) \subseteq X \to X$ is a linear operator, $\mathcal{D} : \mathrm{C}_X \to X$ is a continuous linear operator, F is a nonlinear function from $[0,T] \times \mathrm{C}_X$, $T > 0$, into X and $\varphi \in \mathrm{C}_X$ is given.

Xia and Wu [12,13], Hale [6,7] and Wu [11] considered a class of partial neutral functional differential equations of the form:

$$\frac{\partial}{\partial t}\mathcal{D}v_t = K\frac{\partial^2}{\partial x^2}\mathcal{D}v_t + f(v_t), \quad t \geqslant 0, \tag{2}$$

with $\mathrm{C}([-r,0];\mathrm{H}^1(\mathrm{S}^1))$ as the space of initial data, where S^1 is the unit circle. This system is a model for a continuous circular array of resistively coupled transmission lines with mixed initial boundary conditions. They considered the Laplace operator $A = K\,\partial^2/\partial x^2$ with domain $\mathrm{H}^2(\mathrm{S}^1)$, which yields an infinitesimal generator of a C_0-semigroup of bounded linear operators on $X = \mathrm{H}^1(\mathrm{S}^1)$. In [6,7] Hale presented the basic theory of existence and uniqueness, and, properties of the solution operator associated to Equation (2).

It has been shown in [1,2] and [3] that the density condition is not necessary (in a certain sense) to deal with partial neutral functional differential equations (*see* examples in [4]). In all our previous works, we assumed that the nonlinear term satisfies a Lipschitz condition. In this paper, we show that the additional compactness of the semigroup allows us to remove this Lipschitz condition.

Throughout this paper, we will assume that:

(H1) A is a Hille–Yosida operator on X, i.e., there exist $M_0 \geqslant 0$ and $\omega_0 \in \mathbb{R}$ such that $(\omega_0, +\infty) \subseteq \rho(A)$ and $\sup\left\{(\lambda-\omega_0)^n \|(\lambda I - A)^{-n}\|_{\mathcal{L}(X)} :\ n \in \mathbb{N},\ \lambda > \omega_0\right\} \leqslant M_0$;

(H2) the operator $\mathcal{D} : \mathrm{C}_X \to X$ is defined by $\mathcal{D}\varphi = \varphi(0) - \mathcal{D}_0\varphi$, for $\varphi \in \mathrm{C}_X$, where $\mathcal{D}_0$ is a bounded linear operator from C_X into X given by: $\mathcal{D}_0\varphi = \int_{-r}^0 [\mathrm{d}\eta(\theta)]\varphi(\theta),\ \varphi \in \mathrm{C}_X$, with η is of bounded variation and nonatomic at 0. That is, there is a continuous nondecreasing function $\delta : [0, r] \to [0, +\infty)$ such that: $\delta(0) = 0$ and $\left|\int_{-s}^0 [\mathrm{d}\eta(\theta)]\varphi(\theta)\right| \leqslant \delta(s)\|\varphi\|,\ \varphi \in \mathrm{C}_X,\ s \in [0, r]$.

2. Results

DEFINITION 1. – We say that a function $u \in \mathrm{C}([-r, b]; X)$, $0 < b \leqslant T$, is an integral solution of Equation (1) if: (i) $\int_0^t \mathcal{D}u_s \,\mathrm{d}s \in D(A)$, for $t \in [0, b]$; (ii) $\mathcal{D}u_t = \mathcal{D}\varphi + A\int_0^t \mathcal{D}u_s \,\mathrm{d}s + \int_0^t F(s, u_s)\,\mathrm{d}s$, for $t \in [0, b]$; (iii) $u(t) = \varphi(t)$, for $t \in [-r, 0]$.

Let A_0 be the part of the operator A in $\overline{D(A)}$. A_0 is defined by:

$$\begin{cases} D(A_0) = \left\{x \in D(A)\ :\ Ax \in \overline{D(A)}\right\}; \\ A_0 x = Ax \text{ for } x \in D(A_0). \end{cases}$$

It is well known that the operator A_0 generates a strongly continuous semigroup $\{T_0(t)\}_{t\geqslant 0}$ on $\overline{D(A)}$. We deduce, from [1–3] and [9], that integral solutions of Equation (1) are given, for $\varphi \in \mathrm{C}_X$ such that $\mathcal{D}\varphi \in \overline{D(A)}$, by the following system:

$$\begin{cases} \mathcal{D}u_t = T_0(t)\mathcal{D}\varphi + \lim\limits_{\lambda\to\infty}\displaystyle\int_0^t T_0(t-s)B_\lambda F(s, u_s)\,\mathrm{d}s, & t \in [0, b], \\ u(t) = \varphi(t), & t \in [-r, 0], \end{cases}$$

where $B_\lambda = \lambda(\lambda I - A)^{-1}$, for $\lambda > \omega_0$.

We will assume also that

(H3) the semigroup $\{T_0(t)\}_{t\geqslant 0}$ is compact on $\left(\overline{D(A)}, |\cdot|\right)$. It means that all operators $T_0(t)$, $t > 0$, are compact on $\overline{D(A)}$.

The local existence result for Equation (1) is the following.

THEOREM 1. – *Assume that* (H1)–(H3) *hold. Let U be an open subset of the Banach space $C_0 := \left\{\varphi \in \mathrm{C}_X : \mathcal{D}\varphi \in \overline{D(A)}\right\}$. If $F : [0, T] \times U \to X$ is continuous, then for each $\varphi \in U$ there exist $t_1 := t_1(\varphi) \in (0, T]$ and an integral solution $u \in \mathrm{C}([-r, t_1]; X)$ of Equation* (1).

Proof. – The proof of this result is based on the Sadovskii's fixed-point theorem [14]. Let $\varphi \in U$ and $0 < t_1 \leqslant T$. We choose $0 < \alpha \leqslant T$ small enough such that $\{\psi \in C_0\ :\ \|\psi - \varphi\| \leqslant \alpha\} \subset U$.
Consider the following set:

$$Z_{\varphi,t_1} := \left\{u \in \mathrm{C}\big([-r, t_1]; X\big) :\ u(t) = \varphi(t),\ \text{for } t \in [-r, 0] \text{ and } \|u_t - \varphi\| \leqslant \alpha,\ \text{for } t \in [0, t_1]\right\},$$

where $\mathrm{C}([-r, t_1]; X)$ is endowed with the uniform convergence topology. It is clear that Z_{φ,t_1} is a nonempty, closed, bounded and convex subset of $\mathrm{C}([-r, t_1]; X)$. Consider the nonlinear mapping H :

$Z_{\varphi,t_1} \to \mathrm{C}([-r,t_1];\, X)$, defined by:

$$H(u)(t) = \begin{cases} \mathcal{D}_0 u_t + T_0(t)\mathcal{D}\varphi + \lim\limits_{\lambda\to\infty} \int_0^t T_0(t-s) B_\lambda F(s,u_s)\,\mathrm{d}s, & t\in[0,t_1],\\ \varphi(t), & t\in[-r,0]. \end{cases}$$

We will show that there exists $t_1 := t_1(\varphi) \in (0,T]$ such that $H(Z_{\varphi,t_1}) \subseteq Z_{\varphi,t_1}$. Let $u \in Z_{\varphi,t_1}$. We have the following translation property:

$$(H(u))_t(\theta) = \begin{cases} \mathcal{D}_0 u_{t+\theta} + T_0(t+\theta)\mathcal{D}\varphi + \lim\limits_{\lambda\to\infty} \int_0^{t+\theta} T_0(t+\theta-s) B_\lambda F(s,u_s)\,\mathrm{d}s, & \text{for } t+\theta\in[0,t_1],\\ \varphi(t+\theta), & \text{for } t+\theta\in[-r,0]. \end{cases}$$

Choose $\beta_1 > 0$ such that $|\varphi(t+\theta)-\varphi(\theta)| \leqslant (\alpha/5)\min\{1, 1/\mathrm{Var}_{[-r,0]}(\eta)\}$, for $t\in[0,\beta_1]$ and $\theta\in[-r,0]$ such that $t+\theta\in[-r,0]$. This implies in particular that $\left|(H(u))_t(\theta)-\varphi(\theta)\right| \leqslant \alpha$, for $t\in[0,\beta_1]$ and $\theta\in[-r,0]$ such that $t+\theta\in[-r,0]$.
Choose $\beta_2>0$ such that $|T_0(t)\mathcal{D}\varphi - \mathcal{D}\varphi| \leqslant \alpha/5$, for $t\in[0,\beta_2]$, and $s\in(0,r]$ such that $\delta(s)\leqslant 1/5$. If $0\leqslant t+\theta\leqslant s$, then

$$\begin{aligned}\big(H(u)\big)_t(\theta)-\varphi(\theta) &= \int_{-r}^{-s} \big[\mathrm{d}\eta(\tau)\big]\big(\varphi(t+\theta+\tau)-\varphi(\tau)\big) + \int_{-s}^{0} \big[\mathrm{d}\eta(\tau)\big]\big(u_{t+\theta}(\tau)-\varphi(\tau)\big)\\ &\quad + \varphi(0)-\varphi(\theta) + T_0(t+\theta)\mathcal{D}\varphi - \mathcal{D}\varphi + \lim_{\lambda\to\infty}\int_0^{t+\theta} T_0(t+\theta-s)B_\lambda F(s,u_s)\,\mathrm{d}s.\end{aligned}$$

Hence,

$$\begin{aligned}\left|\big(H(u)\big)_t(\theta)-\varphi(\theta)\right| &\leqslant \mathrm{Var}_{[-r,0]}(\eta) \sup_{\tau\in[-r,-s]} \left|\varphi(t+\theta+\tau)-\varphi(\tau)\right| + \delta(s)\|u_{t+\theta}-\varphi\| + \left|\varphi(0)-\varphi(\theta)\right|\\ &\quad + \left|T_0(t+\theta)\mathcal{D}\varphi-\mathcal{D}\varphi\right| + \lim_{\lambda\to\infty}\int_0^{t+\theta} \left\|T_0(s)\right\|_{\mathcal{L}(\overline{D(A)})}\|B_\lambda\|\left|F(t+\theta-s,u_{t+\theta-s})\right|\mathrm{d}s.\end{aligned}$$

We use the principle of uniform boundedness to find $N>0$ such that $\|T_0(t)\|_{\mathcal{L}(\overline{D(A)})} \leqslant N$, for $0\leqslant t\leqslant t_1$.

We deduce from (H1) that $\|B_\lambda\| \leqslant \lambda M_0/(\lambda-\omega_0) \underset{\lambda\to+\infty}{\longrightarrow} M_0$. As F is continuous, we can choose $\alpha>0$ small enough such that there exists $K>0$ so that $|F(t,\psi)|\leqslant K$, for $t\in[0,\alpha]$ and $\|\psi-\varphi\|\leqslant\alpha$. Then, if $t_1\leqslant\alpha$ we obtain

$$\lim_{\lambda\to\infty}\int_0^{t+\theta} \left\|T_0(s)\right\|_{\mathcal{L}(\overline{D(A)})}\|B_\lambda\|\left|F(t+\theta-s,u_{t+\theta-s})\right| \leqslant (t+\theta)NM_0K.$$

Finally, we choose

$$t_1 = \min\left\{\beta_1,\, \beta_2,\, s,\, \alpha,\, \frac{\alpha}{5NM_0K}\right\}. \tag{3}$$

Then, for $0\leqslant t+\theta\leqslant t_1$, we obtain, $|(H(u))_t(\theta)-\varphi(\theta)|\leqslant\alpha$. So, we have proved that there exists $t_1 := t_1(\varphi)\in(0,T]$ such that $H(Z_{\varphi,t_1})\subseteq Z_{\varphi,t_1}$.

We construct now maps H_1 and H_2 such that $H=H_1+H_2$ on Z_{φ,t_1}, H_1 is compact and H_2 is a strict contraction. In such case the Sadovskii's fixed-point theorem can be applied to the mapping H. Let $H_1 : Z_{\varphi,t_1}\to \mathrm{C}([-r,t_1];\, X)$ be the operator defined by:

$$H_1(u)(t) = \begin{cases} T_0(t)\mathcal{D}\varphi + \lim\limits_{\lambda\to\infty}\int_0^t T_0(t-s)B_\lambda F(s,u_s)\,\mathrm{d}s, & t\in[0,t_1],\\ \mathcal{D}\varphi, & t\in[-r,0], \end{cases}$$

and $H_2 : Z_{\varphi,t_1} \to \mathrm{C}([-r,t_1]\,;X)$ be the operator defined by:

$$H_2(u)(t) = \begin{cases} \mathcal{D}_0 u_t, & t \in [0,t_1], \\ \varphi(t) - \mathcal{D}\varphi, & t \in [-r,0]. \end{cases}$$

It is not difficult to see that $H = H_1 + H_2$ on Z_{φ,t_1}.

(i) We will show that $H_1(Z_{\varphi,t_1})$ is an equicontinuous family of functions.

Let $u \in Z_{\varphi,t_1}$, $\varepsilon > 0$ small enough and $0 \leqslant \tau < t \leqslant t_1$, when t_1 is given by (3). If $\tau > 0$, we can take $\varepsilon < \tau$. Then

$$\begin{aligned} \big|H_1(u)(t) - H_1(u)(\tau)\big| \leqslant{} & \big|T_0(t)\mathcal{D}\varphi - T_0(\tau)\mathcal{D}\varphi\big| + \lim_{\lambda\to\infty}\left|\int_\tau^t T_0(t-s)B_\lambda F(s,u_s)\,\mathrm{d}s\right| \\ & + \lim_{\lambda\to\infty}\left|\int_{\tau-\varepsilon}^{\tau} \big(T_0(t-s) - T_0(\tau-s)\big)B_\lambda F(s,u_s)\,\mathrm{d}s\right| \\ & + \lim_{\lambda\to\infty}\left|\int_0^{\tau-\varepsilon} \big(T_0(t-s) - T_0(\tau-s)\big)B_\lambda F(s,u_s)\,\mathrm{d}s\right|. \end{aligned}$$

Hence,

$$\begin{aligned} \big|H_1(u)(t) - H_1(u)(\tau)\big| \leqslant{} & \big|T_0(t)\mathcal{D}\varphi - T_0(\tau)\mathcal{D}\varphi\big| + NM_0K|t-\tau| + 2NM_0K\varepsilon \\ & + \tau \sup_{s\in[0,\tau-\varepsilon]} \big\|T_0(t-s) - T_0(\tau-s)\big\|_{\mathcal{L}(\overline{D(A)})} M_0K. \end{aligned}$$

If we take $|t-\tau|$ small enough, it follows from the uniform continuity of $T_0(\cdot) : [\varepsilon,t_1] \to \mathcal{L}(\overline{D(A)})$ the claimed equicontinuity.

Let $\tau = 0$ and $\varepsilon > 0$, we choose $\alpha > 0$ such that $|H_1(u)(t) - \mathcal{D}\varphi| \leqslant \varepsilon$, for $0 < t < \alpha$. This proves the equicontinuity.

We will prove now that, for each $0 < t \leqslant t_1$, the set $\{H_1(u)(t) : u \in Z_{\varphi,t_1}\}$ is precompact in X.

Let $0 < t \leqslant t_1$ and $\varepsilon \in (0,t)$. Consider the set

$$\Omega_\varepsilon(t) := \left\{T_0(t)\mathcal{D}\varphi + \lim_{\lambda\to\infty}\int_0^{t-\varepsilon} T_0(t-s)B_\lambda F(s,u_s)\,\mathrm{d}s \,:\, u \in Z_{\varphi,t_1}\right\}.$$

We have

$$\Omega_\varepsilon(t) = \left\{T_0(t)\mathcal{D}\varphi + T_0(\varepsilon)\left(\lim_{\lambda\to\infty}\int_0^{t-\varepsilon} T_0(t-\varepsilon-s)B_\lambda F(s,u_s)\,\mathrm{d}s\right) \,:\, u \in Z_{\varphi,t_1}\right\}.$$

Since $T_0(t)$ is compact for $t > 0$, then $\Omega_\varepsilon(t)$ is a precompact set of X. Then, by the approximation theorem for compact operators, we conclude that $\{H_1(u)(t) : u \in Z_{\varphi,t_1}\}$ is a precompact set of X. Hence, by Arzela–Ascoli theorem we deduce that $H_1(Z_{\varphi,t_1})$ is precompact in $\mathrm{C}([-r,t_1]\,;X)$.

(ii) Let u, $v \in Z_{\varphi,t_1}$ and $0 < t \leqslant t_1$, when t_1 is given by (3). Then, $u(t+\theta) = v(t+\theta) = \varphi(t+\theta)$, for $\theta \in [-r,-s]$. This means that

$$\big|H_2(u)(t) - H_2(v)(t)\big| \leqslant \int_{-s}^0 \big|\big[\mathrm{d}\eta(\theta)\big]\big(u(t+\theta) - v(t+\theta)\big)\big|.$$

Consequently,

$$\sup_{t\in[0,t_1]} \big|H_2(u)(t) - H_2(v)(t)\big| \leqslant \delta(s) \sup_{t\in[0,t_1]} \big|u(t) - v(t)\big|,$$

with $\delta(s) \leqslant 1/5$. Then, H_2 is a strict contraction on Z_{φ,t_1}. Apply the Sadovskii's fixed-point theorem to obtain the existence of a fixed point of H on Z_{φ,t_1}. We conclude that for each $\varphi \in U$ there exist $t_1 := t_1(\varphi) \in (0,T]$ and an integral solution $u \in \mathrm{C}([-r,t_1];X)$ of Equation (1). □

In the sequel, we will need the following lemma.

LEMMA 1 ([11]). – *Assume that* (H2) *hold and let* $g \in \mathrm{C}([0,T);X)$, $T > 0$, *and* $\psi \in \mathrm{C}_X$. *Suppose that there exists a function* $v \in \mathrm{C}([-r,T);X)$ *such that*:

$$\begin{cases} \mathcal{D}v_t = g(t), & t \in [0,T), \\ v(t) = \psi(t), & t \in [-r,0]. \end{cases}$$

Then, there exist positive constants a, b, *and* c *such that*

$$\|v_t\| \leqslant \Big(a\|\psi\| + b \sup_{\tau\in[0,t]} |g(\tau)|\Big)\mathrm{e}^{ct}, \quad t \in [0,T].$$

It follows from classical arguments the following results.

THEOREM 2. – *Assume that* (H1)–(H3) *hold. If* $F : [0,+\infty) \times \mathrm{C}_X \to X$ *is continuous and maps bounded subsets of* $[0,+\infty) \times \mathrm{C}_X$ *into bounded subsets of* X, *then for each* $\varphi \in C_0$, *Equation* (1) *has an integral solution* $u := u(\cdot,\varphi)$ *on a maximal interval of existence* $[0,t_\varphi)$, $t_\varphi > 0$ *and either* $t_\varphi = +\infty$ *or* $\limsup_{t\to t_\varphi^-} \|u_t\| = +\infty$.

THEOREM 3. – *Under the same assumptions as in Theorem* 2, *if there exist locally integrable functions* l_1 *and* l_2 *such that* $|F(t,\varphi)| \leqslant l_1(t)\|\varphi\| + l_2(t)$ *for* $\varphi \in C_0$ *and* $t \geqslant 0$, *then Equation* (1) *has global integral solutions.*

References

[1] Adimy M., Ezzinbi K., A class of linear partial neutral functional differential equations with non-dense domain, J. Differ. Eq. 147 (1998) 285–332.
[2] Adimy M., Ezzinbi K., Strict solutions of nonlinear hyperbolic neutral differential equations, Appl. Math. Let. 12 (1999) 107–112.
[3] Adimy M., Ezzinbi K., Existence and linearized stability for partial neutral functional differential equations, Eq. and Dynam. Sys. (2000)(to appear).
[4] Da Prato G., Sinestrari E., Differential operators with non-dense domains, Ann. Scu. Norm. Sup. Pisa Cl. Sci. 14 (1987) 285–344.
[5] Hale J.K., Theory of Functional Differential Equations, Springer-Verlag, New York, 1977.
[6] Hale J.K., Partial neutral functional differential equations, Rev. Roumaine Math. Pure Appl. 39 (1994) 339–344.
[7] Hale J.K., Coupled oscillators on a circle, Resenhas IME-USP 1 (4) (1994) 441–457.
[8] Hale J.K., Lunel S., Introduction to Functional Differential Equations, Springer-Verlag, New York, 1993.
[9] Thieme H.R., Semiflows generated by Lipschitz perturbations of non-densely defined operators, Differ. Int. Eq. 3 (1990) 1035–1066.
[10] Travis C.C., Webb G.F., Existence and stability for partial functional differential equations, Trans. Amer. Math. Soc. 200 (1974) 395–418.
[11] Wu J., Theory and Applications of Partial Functional Differential Equations, Springer-Verlag, 1996.
[12] Wu J., Xia H., Self-sustained oscillations in a ring array of coupled lossless transmission lines, J. Differ. Eq. 124 (1996) 247–278.
[13] Wu J., Xia H., Rotating waves in neutral partial functional differential equations, Preprint.
[14] Zeidler E., Nonlinear Functional Analysis and its Applications I: Fixed-Point Theorems, Springer-Verlag, 1986.