Some applications of the random projection method

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This talk is based on some joint research with:

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What are we going to talk about?

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- easy to compute
- reasonable
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"...simple methods typically yield performance almost as good as more sophisticated methods to the extent that the difference in performance may be swamped by other sources of uncertainty..."

HAND, D.J., 2006. Classifier technology and the illusion of progress. *Statist. Sci.*, **21**(1) 1-14.

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How many one-dimensional marginals

are required to determine a probability measure

on a separable Hilbert space?

Only a one-dimensionial projection suffices

if it is randomly chosen

(under some assumptions on the moments)

Notation:

- $\boldsymbol{H}$ will denote a separable Hilbert space

  $\| - \|$ and $\langle \cdot, \cdot \rangle$ its norm and scalar product

- Given P a probability on $\boldsymbol{H}$ and $v \in \boldsymbol{H}$

  P_v is the marginal of P on the subspace generated by v

- Given P, Q two probabilities

$$\boldsymbol{E}(P, Q) := \{v \in \boldsymbol{H} : P_v = Q_v\}$$

The result. Separable Hilbert spaces.

Assume that:

1. *P is determined by its moments*
2. *$Q \neq P$*

Then $\mu[\boldsymbol{E}(P, Q)] = 0$ (remember: $\boldsymbol{E}(P, Q) = \{v : P_v = Q_v\}$)

Here μ is any continuous distribution

For instance:

μ absolutely continuous w.r.t. the Lebesgue measure

μ Gaussian, with non-degenerate 1-dimensional marginals

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Extension to Banach spaces in Cuevas & Fraiman (2009)

The result. How to apply it.

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2. $Q \neq P$

Then $\mu[\boldsymbol{E}(P,Q)] = 0$ (remember: $\boldsymbol{E}(P,Q) = \{v : P_v = Q_v\}$)

If you want to test $H_0 : P = Q$

only select v at random at test $H_0^v : P_v = Q_v$

because, with probability one, H_0 and H_0^v are equivalent

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   Two possibilites.

   2.1 Simulate v from $N_d(0, Id)$.

   Multiply v by the appropriate matrix and add a function m

   2.2 Apply a property of μ

   Assume that (theoretically) $v \in L_2[0,1]$

   that we have measured the data at points $t_1 < \ldots < t_d$

   that μ is the distribution of the standard Brownian motion

   take $\delta_i, i = 1, \ldots, d$ i.i.d. N(0,1)

   define

$$
\begin{aligned}
v(t_0) &= 0, \text{ where } t_0 = 0, \\
v(t_i) &= v(t_{i-1}) + (t_i - t_{i-1})^{1/2}\delta_i, \; i = 1, \ldots, d
\end{aligned}
$$

Two-way factorial ANOVA for functional data

We have two factors with R and S levels respectively

Thus, for every $r = 1, \ldots, R$ and $s = 1, \ldots, S$ we have
$\mathbf{X}_i^{r,s}, i = 1, ..., n_{r,s} \in \boldsymbol{N}$ random functions in $L_2[0, 1]$

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$\mathbf{X}_i^{r,s}, i = 1, ..., n_{r,s} \in \boldsymbol{N}$ random functions in $L_2[0,1]$

$$\mathbf{X}_i^{r,s}(t) = m(t) + f^r(t) + g^s(t) + h^{r,s}(t) + \epsilon_i^{r,s}(t), \; t \in [0,1],$$

1. $m \in L_2[0,1]$ is non random. Describes the overall shape of the process
2. $f^r, g^s, h^{r,s} \in L_2[0,1]$ are non random. Account for the main effects of the factors and for the interaction between them; and

$$\sum_r f^r(t) = \sum_s g^s(t) = \sum_r h^{r,s_0}(t) = \sum_s h^{r_0,s}(t) = 0, \forall t, r_0, s_0$$

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3. $\epsilon_i^{r,s} \in L_2[0,1]$, are random, independent and $E[\epsilon_i^{r,s}] = 0$ for each r, s, $\epsilon_i^{r,s}$ are i.d.

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We want to test the null hypotheses:

$H_0^A:$	$f^1 = \ldots = f^R = 0$	the first factor has no effect
$H_0^B:$	$g^1 = \ldots = g^S = 0$	the second factor has no effect
$H_0^I:$	$h^{1,1} = \ldots = h^{R,S} = 0$	there is no interaction between factors

The theorem

Theorem (Cuesta-Albertos and Febrero-Bande, 2009)

Let us assume the previous model. If μ is Gaussian, then

1. *If H_0^A fails, then* $\mu\left\{v \in L_2[0,1] : \langle v, f^1\rangle = \ldots = \langle v, f^R\rangle\right\} = 0$
2. *If H_0^B fails, then* $\mu\left\{v \in L_2[0,1] : \langle v, g^1\rangle = \ldots = \langle v, g^S\rangle\right\} = 0$
3. *If H_0^I fails, then* $\mu\left\{v \in L_2[0,1] : \langle v, h^{1,1}\rangle = \ldots = \langle v, h^{R,S}\rangle\right\} = 0$

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3. *If H_0^I fails, then* $\mu\left\{v \in L_2[0,1] : \langle v, h^{1,1}\rangle = \ldots = \langle v, h^{R,S}\rangle\right\} = 0$

PROOF.- Let $r \in \{1, ..., R\}$, and let P^r be such that $P^r[f^r] = 1$

Obviously, P^r is determined by its moments

Thus, we can apply the result on random projections to every pair of probability distributions P^{r_1} and P^{r_2}

The proofs of 1 and 2 are identical.

Two-way factorial ANOVA. The procedure

To test H_0^A:

Select a vector $v \in L_2[0,1]$ (with the distribution of a Brownian motion)

Compute the (real) projections of the sample

$$\langle v, \mathbf{X}_i^{r,s} \rangle, \; i = 1, \ldots, n_{r,s}, r = 1, \ldots, R, s = 1, \ldots, S$$

Apply an ANOVA procedure to test the null hypothesis

$$H_0^{A,v} : \langle v, f^1 \rangle = \ldots = \langle v, f^R \rangle = 0$$

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Nothing!

We only need a (one-dimensional) procedure valid for heteroscedastic data

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Well, at least if we have a (one-dimensional) procedure allowing ...

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We are losing information, this should bring some loss of power

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Where is the price we have paid for this?

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A solution: Choose $v_1, ..., v_k$ at random.

Apply the ANOVA to the hypotheses $H_0^{A,v_1}, \ldots, H_0^{A,v_k}$

And base the decision on the k tests

Technical problem: How to handle multiple tests?

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Take $p_0 = \min(p_1, \ldots, p_k)$

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Correct via Bonferroni $\longrightarrow$ too conservative

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Correct via Bootstrap $\rightarrow$ too time consuming

Cuesta-Albertos et al, 2007

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a test at level α in our problem: Theo. 1.3, Benjamini&Yekutyeli, 2001

sort the p-values to obtain $p_{(1)} \leq \ldots \leq p_{(k)}$

reject the null hypothesis under consideration if

$$\left\{ i \in \{1, \ldots, k\} : \; p_{(i)} \leq \frac{i}{k} \frac{\alpha}{\phantom{\sum_{j=1}^{k} 1/j}} \right\} \neq \emptyset$$

if the tests are positively dependent

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always!!!

functional ANOVA. Orthosis data.

How do individuals cope with a perturbation while stepping-in-place?

Seven volunteers wore a spring-loaded orthosis of adjustable stiffness

Experimental conditions: Control condition (without orthosis)

Orthosis condition (with the orthosis only)

Spring1, Spring2: a spring-loaded orthosis onto the knee joint

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For each of the seven subjects,

10 stepping-cycles of 20 seconds were analyzed under each condition

Moment at the knee was computed at 64 time points equally spaced and scaled so that a time interval corresponds to an individual gait cycle

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We consider subjects and treatments as factors. 10 observations per cell

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RP	Subj.	Treat.	Inter.	Spr1&2 vs Co&Or	Cont vs Orth	Spr1 vs Spr2
5	0	0	0	0	1.86e-05	.0908
15	0	0	0	0	2.67e-05	.0231
30	0	0	0	0	3.22e-05	.0279
A&S				0	.001	.020

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using Bonferroni's correction:

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Comparison with MANOVA

Multidimensional data can be considered as functional

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Data on the production of plastic film (Krzanowski, 1988):

- three characteristics: tear, gloss, opacity
- two factors: rate, additive
- with two levels each: low, high

Five measurements under each set of production conditions

$\Rightarrow$ 3-dimensional, 2-way MANOVA. 2 levels in each factor. $n_{i,j} = 5$

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We use the usual ANOVA test (Krzanowski uses Normal MANOVA)

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	Pillai test *p*-value	Random projection tests *p*-value		
		$k = 5$	$k = 15$	$k = 30$
rate	.003	.018	.007	.001
additive	.025	.005	.009	.008
interact.	.302	.263	.174	.192

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	Pillai test	Random projection tests					
		p-value			Rate of rejections		
	p-value	$k = 5$	$k = 15$	$k = 30$	$k = 5$	$k = 15$	$k = 30$
rate	.003	.018	.007	.001	.882	.998	1
additive	.025	.005	.009	.008	.772	.974	1
interact.	.302	.263	.174	.192	0	0	0

functional ANCOVA.

We have two factors with R and S levels respectively and a covariable

Thus, for every $r = 1, \ldots, R$ and $s = 1, \ldots, S$ we have
$\mathbf{X}_i^{r,s}, i = 1, ..., n_{r,s} \in \boldsymbol{N}$ random functions in $L_2[0,1]$

$$\mathbf{X}_i^{r,s}(t) = m(t) + f^r(t) + g^s(t) + h^{r,s}(t) + \epsilon_i^{r,s}(t) + \gamma Y_i^{r,s}(t), \;\; t \in [0,1]$$

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We handle the covariable exactly in the same way as the factors:

Select a vector $v \in L_2[0,1]$ (with the distribution of a Brownian motion)

Compute the (real) projections of the sample

$$\langle v, \mathbf{X}_i^{r,s} \rangle, \;\; i = 1, \ldots, n_{r,s}, r = 1, \ldots, R, s = 1, \ldots, S$$

Apply an ANCOVA procedure to test the null hypothesis

$$H_0^{C,v} : \langle v, Y \rangle = 0$$

Notice that $\gamma_i^{r,s}$ has no influence iff $Y \equiv 0$

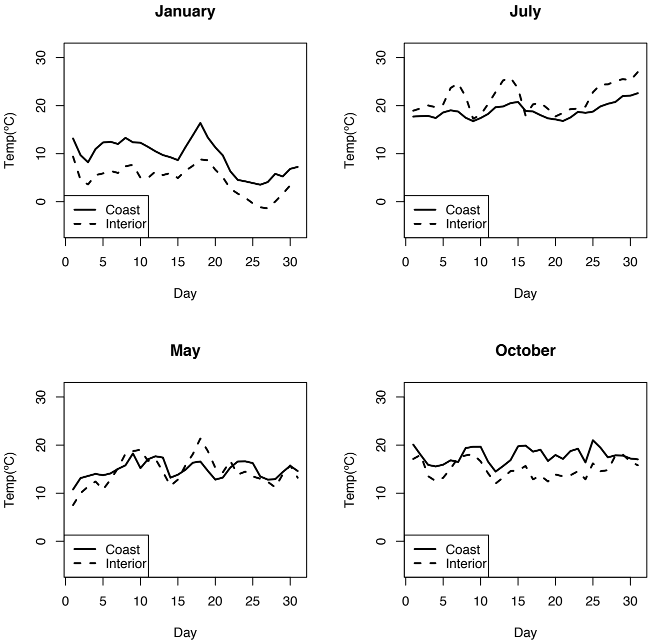
Spanish temperature data. Description of the data

Data: daily mean temp, certain locations and months. An annual cycle

* Months (4 levels):	October-06, January-07, May-07 and July-07
* Locations (2 levels):	
Coast:	A Coruña, Avilés, Bilbao, San Sebastián, Santander, Vigo
Inland:	Burgos, León, Madrid, Salamanca, Segovia, Soria, Valladolid, Vitoria and Zamora.
* Covariable (γ):	Monthly Total Amount of Rainfall
	γ is a real known r.v. which multiplies the unknown, non random function Y measuring the influence of γ each day in the month

downloaded from http://clima.meteored.com

Spanish temperature data. Means by cells



Spanish temperature data. Random projected ANCOVA

$\#$ of projections = 30

Correction method: Bonferroni and Bootstrap (B=500)

	Location	Month	Interaction	Rainfall
Bonf: p-value	$4.9 \cdot 10^{-6}$	$2.4 \cdot 10^{-33}$	$6.8 \cdot 10^{-9}$	.029
Boot: p-value	0	0	0	.037

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Proportions of rejections of the null hypotheses (level $\alpha = 0.05$)

Bonferroni	1	1	1	.804
Bootstrap	1	1	1	.808

T H A N K Y O U !!!

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