Rational points on the stack of roots and section conjecture (joint work with Niels Borne)

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Let k a finitely generated field of characteristic 0 and let U be a smooth, geometrically connected curve over k. One knows that any rational point $x \in U(k)$ induces a section (up to conjugation) of the fundamental exact sequence of fundamental groups of U. But, if U is affine, there are many other sections coming from objects living at infinity. Let X be the smooth compactification of U and $x \in (X - U)(k)$; the sections coming from objects over x belong to some "packet" of sections (mentioned in a letter of Grothendieck to Faltings; see also [3], [4]).

The aim of the lecture is to re-interpret the elements of a "packet" associated to x as coming from k-rational points over $x \in (X - U)(k)$ on the stack of roots X_D of the divisor at infinity D.

We will use the interpretation of sections as neutral fiber functors of a certain tannakian category, whose Galois group is the Nori tame fundamental group $\pi_1^D(X)$ and the link established in [1] and [2] between étale coverings of X_D and covering of X tamely ramified along D. This interpretation leads to a proof of the injectivity in the section conjecture: two non isomorphic k-rational points of the stack X_D give rise to non isomorphic fibre functors, or equivalently to non conjugate sections.

References

[1] N. Borne: Fibrés paraboliques et champ des racines, I.M.R.N. , 2007.

[2] N. Borne: Sur les représentations du groupe fondamental d'une varété privée d'un diviseur à croisements normaux simlpes, Indiana University Math. Journal **58**, 2009.

[3] H. Esnault and P.H. Hai: Packets in Grothendieck's section conjecture, Adv. in Math. **218**, 2008.

[4] J. Stix: On cuspidal sections of algebraic fundamental groups, preprint 2009 (arXiv: 0809.0017).