A Nested Decomposition Approach to an Optical Network Design Problem

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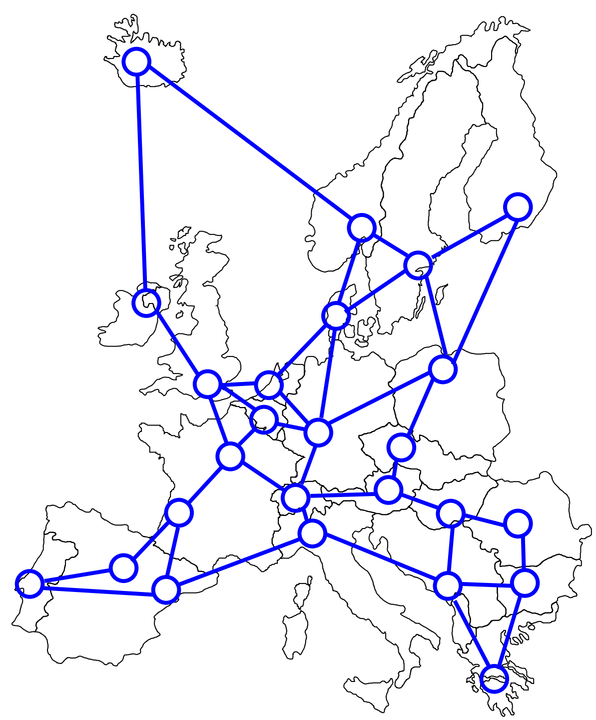
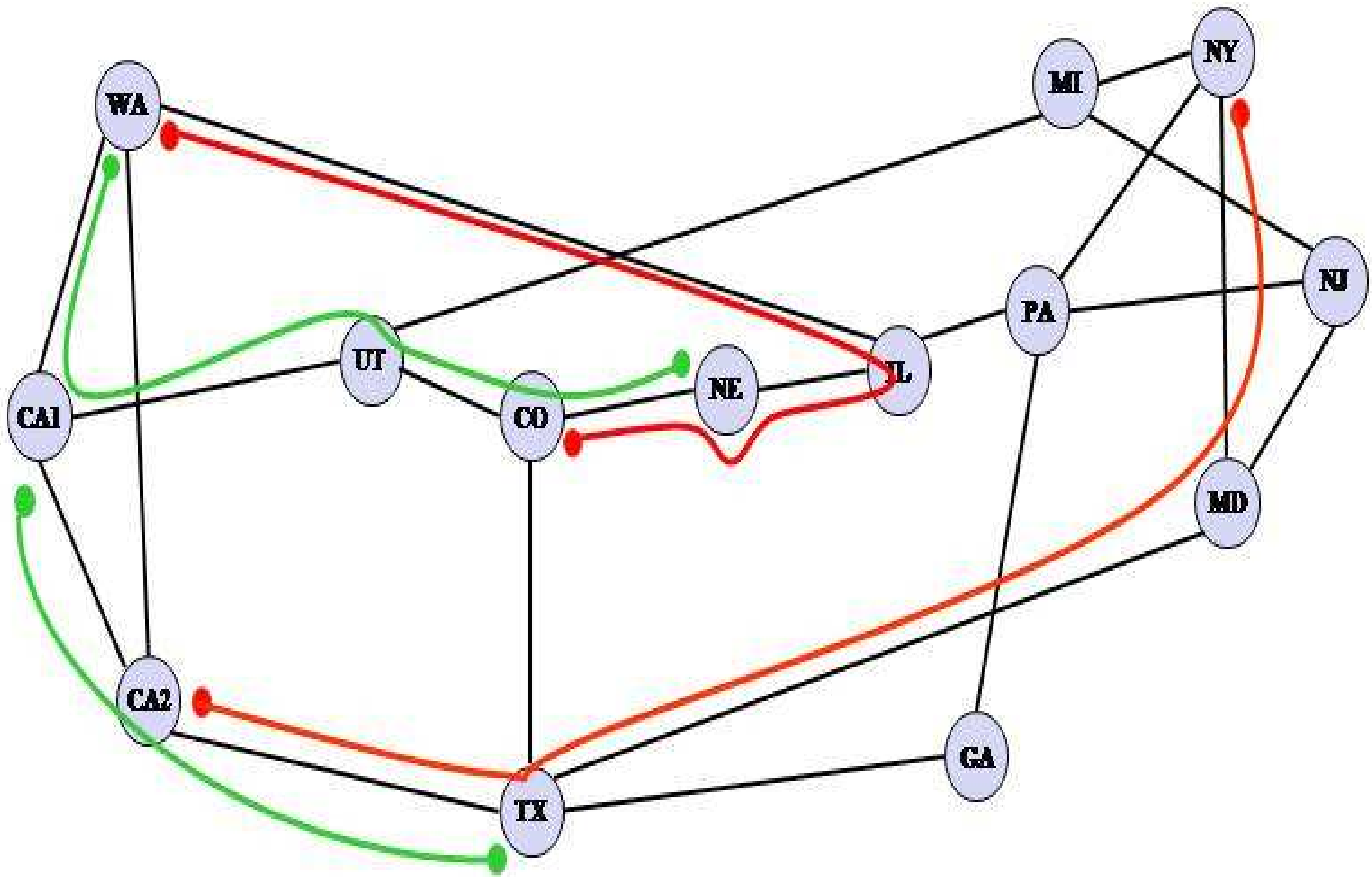


Main restriction: Single path routing



Outline

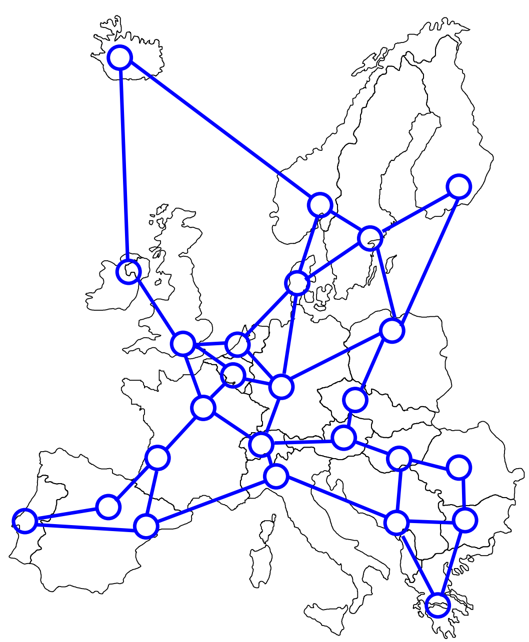
1. The problem: Grooming and Multiplexing
2. Multi-Commodity Flow Formulation
3. Nested Decomposition
4. Generating routing patterns
5. Packing them onto wavelengths
6. Solving the master
7. Adding Cuts
8. Rounding heuristic

Problem description

- A physical network $G = (V, A)$: link = optical fiber
- $W \in [17, 66]$ wavelengths of capacity $U = 192$
- $1\ 000$ to $100\ 000$ requests (s, d, t), with granularity

$$t \in \{1, 3, 12, 48\}$$

- Set K of aggregate requests with demand $D_{sdt} > 0$

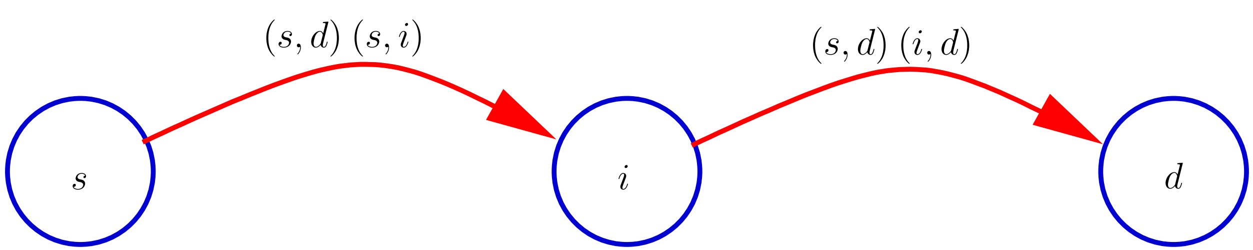


GRWA: Grooming and Multiplexing

- Grooming:
  packing several requests on the same wavelength
- Multiplexing:
  packing several wavelengths on the same light-path

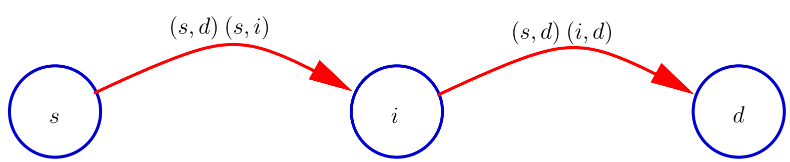
opto-electronic conversions

- adding or dropping traffic requires O/E/O
- path = $\underbrace{\text{optical hop 1}}_{light-path} + \underbrace{\text{optical hop 2}}_{light-path} + \ldots$



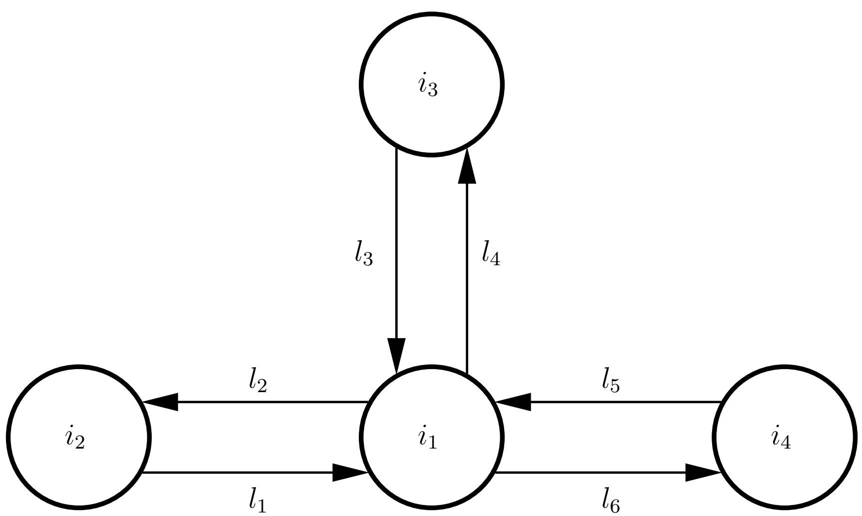
Assumptions / technical constraints

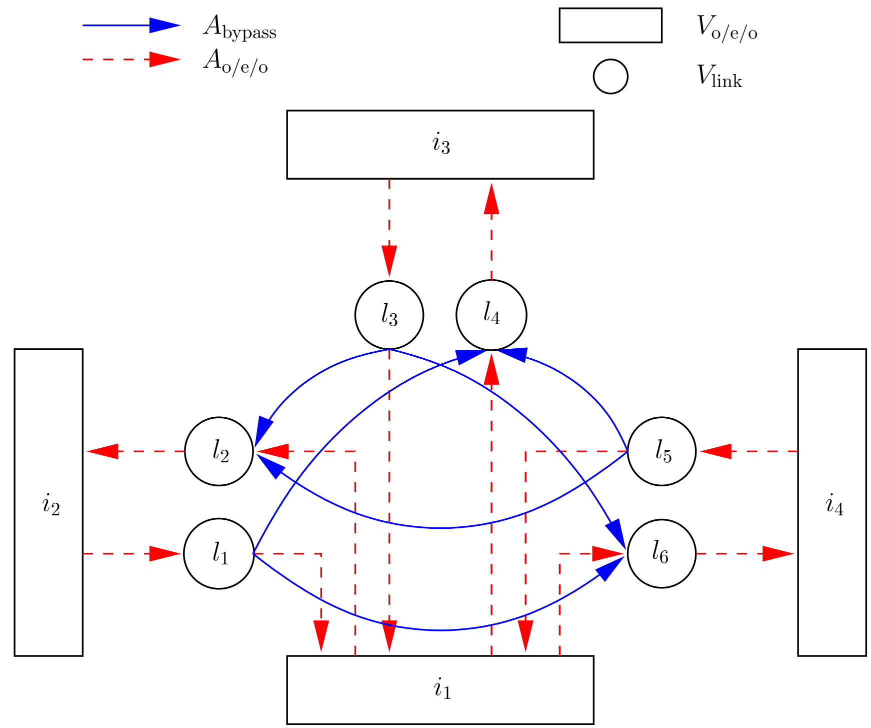
- Single path routing: each request follows a path in $\mathcal{P}_{sd}$
  - no bifurcation
  - need to follow D_{sdt} individual requests: 120 000
  - integer flows: NP-Hard problem
- wavelength continuity
  (same wavelength after O/E/O)
- Two-hop constraint: no more than 1 intermediate O/E/O conv.
  - to limit end-to-end delay



Costs = installation of O/E/O ports

electrical bypass vs O/E/O conversion





Multi-commodity flow formulation (1/3)

- expanded graph: $G' = (V_{\mathrm{o/e/o}} \cup V_{\mathrm{link}}, A_{\mathrm{o/e/o}} \cup A_{\mathrm{bypass}})$
- Unary decomposition of requests: D_{sdt} of copies of req. (s, d, t)
- $x^{u\lambda} = 1$ iff unary request u uses wavelength λ.
- $x_a^{u\lambda} = 1$ iff unary request u uses arc a in wavelength λ.
- $y_a^{\lambda} = 1$ iff wavelength λ is used on arc a.

Drawbacks

- pseudo-polynomial size due to unary decomposition
- symmetry between identical requests
- symmetry in wavelength assignment

Multi-commodity flow formulation (2/3)

wavelength continuity:

$$\sum_{\lambda} x^{u\lambda} = 1\,, \quad u \tag{1}$$

$$\sum_{a\in\omega_F^-(l)} x_a^{u\lambda} - \sum_{a\in\omega_F^+(l)} x_a^{u\lambda} = 0\,, \quad l \in V_{\text{link}},\ u,\ \lambda \tag{2}$$

flow conservation:

$$\sum_{a\in\omega_F^-(i)} x_a^{u\lambda} - \sum_{a\in\omega_F^+(i)} x_a^{u\lambda} = \begin{cases} x^{u\lambda} & \text{if } i = d_k \\ -x^{u\lambda} & \text{if } i = s_k \\ 0 & \text{otherwise} \end{cases} \quad i \in V_{\text{o/e/o}},\ u,\ \lambda \tag{3}$$

wavelength capacity:

$$\sum_{u} \sum_{a\in\omega_F^-(l)} t_k\, x_a^{u\lambda} \leq U \quad l \in V_{\text{link}},\ \lambda \tag{4}$$

$$x^{u\lambda} \in \{0,1\} \quad u,\ \lambda \tag{5}$$

$$x_a^{u\lambda} \in \{0,1\} \quad u,\ \lambda,\ a \tag{6}$$

Multi-commodity flow formulation (3/3)

$$\min \sum_{\lambda \in \Lambda} \sum_{a \in A_{\mathrm{o/e/o}}} y_a^\lambda \tag{7}$$

port installation:

$$x_a^{u\lambda} \leq y_a^\lambda \quad u,\ a,\ \lambda \tag{8}$$

single path routing:

$$\sum_{a \in \omega_F^+(l)} y_a^\lambda \leq 1 \quad l \in V_{\mathrm{link}},\ \lambda \tag{9}$$

$$\sum_{a \in \omega_F^-(l)} y_a^\lambda \leq 1 \quad l \in V_{\mathrm{link}},\ \lambda \tag{10}$$

2-hop restrictions:

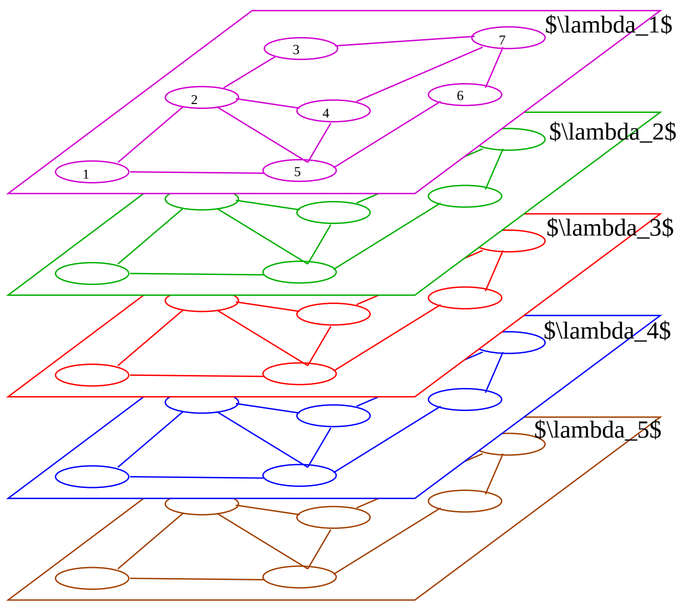
$$\sum_{i \in V_{\mathrm{o/e/o}}} \sum_{a \in \omega_F^-(i)} x_a^{u\lambda} \leq 2 \quad u,\ \lambda \tag{11}$$

$$y_a^\lambda \in \{0,1\} \quad \lambda,\ a \tag{12}$$

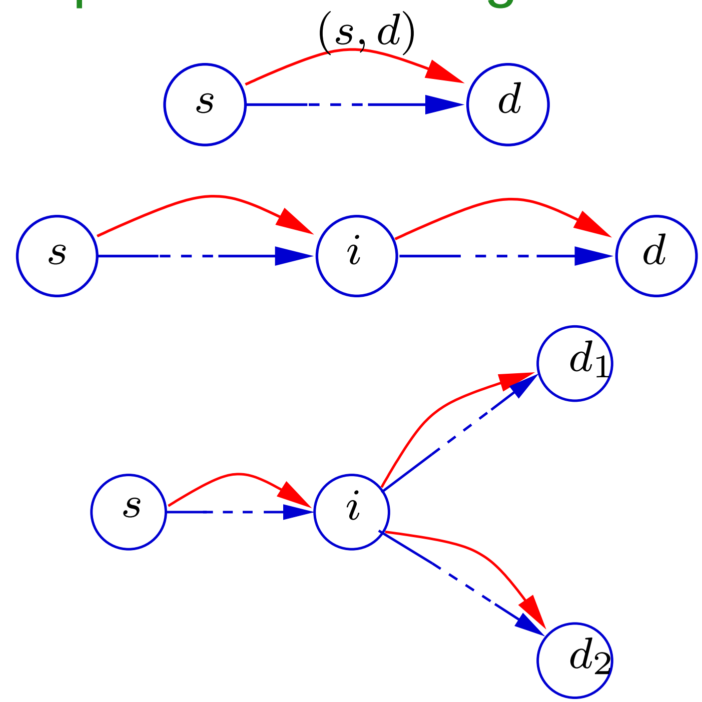
Nested decomposition

wavelength continuity + single path routing + 2-hop

Wavelength Routing Config.



Independent Routing Patterns



Column generation reformulation

configuration $c \in \mathcal{C} = \{$*wavelength routing configurations*$\}$

x^c_{sdt} = the number of requests (s, d, t) assigned to the wavelength

$cost_c$ = the number of o/e/o conversion ports for wavel. rout. conf.

$$\min \sum_{c \in \mathcal{C}} cost_c \, \gamma_c \tag{13}$$

$$\sum_{c \in \mathcal{C}} x^c_{sdt} \, \gamma_c \geq D_{sdt} \quad (s, d, t) \in K, \quad (\pi) \tag{14}$$

$$\sum_{c \in \mathcal{C}} \gamma_c \leq W \tag{15}$$

$$\gamma_c \in \mathbb{N} \quad c \in \mathcal{C} \tag{16}$$

Column generation reformulation: properties

1. polynomial number of constraints: no more unary decomp.
2. no more symmetry in wavelength assignment
3. no more symmetry between identical requests
4. even better can allow replacement between granularities:

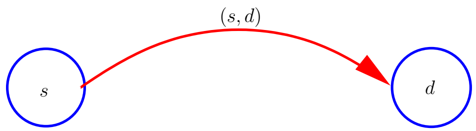
$$\sum_{c \in \mathcal{C}} x^c_{sd48}\,\gamma_c \quad \geq \quad D_{sd48} \quad s,d$$

$$\sum_{c \in \mathcal{C}} (4\,x^c_{sd48} + \ x^c_{sd12})\,\gamma_c \quad \geq \quad 4\,D_{sd48} + D_{sd12} \quad s,d$$

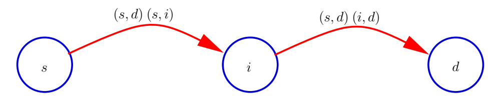
$$\sum_{c \in \mathcal{C}} (16\,x^c_{sd48} + 4\,x^c_{sd12} + \ x^c_{sd3})\,\gamma_c \quad \geq \quad 16\,D_{sd48} + 4\,D_{sd12} + D_{sd3} \quad s,d$$

implies $\pi_{sd48} \geq 4\,\pi_{sd12}\ ,\ \ \pi_{sd12} \geq 4\,\pi_{sd3}\ ,\ \ \pi_{sd3} \geq 3\,\pi_{sd1}$

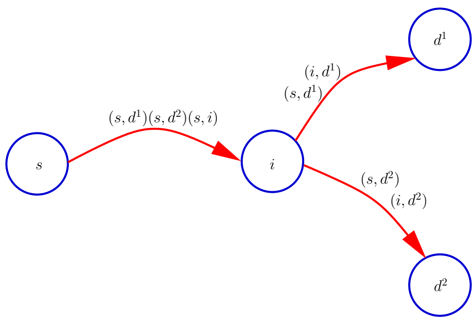
Restricted set of light path configurations



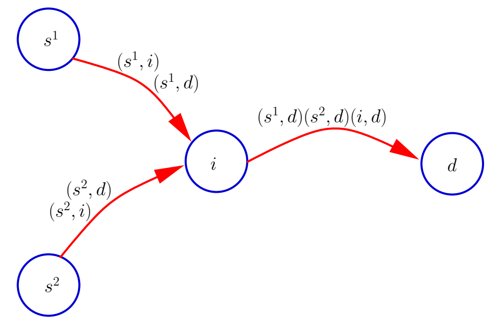
Single-hop single-path.



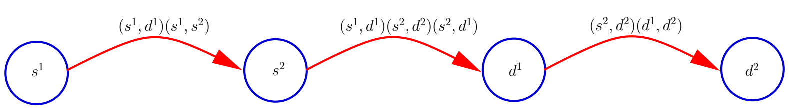
Two-hop single-path.



Two-hop splitting-path.



Two-hop merging-path.

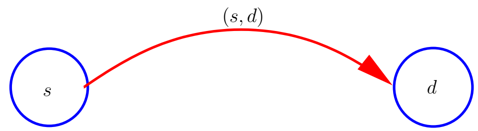


Two-hop interlaced-path

Moreover, $\left\{ \begin{array}{l} \text{each light-path admits only 3 physical paths} \\ \text{light-paths are arc disjoint} \rightarrow \text{ loaded up to } U \end{array} \right.$

Traffic loading $\rightarrow$ *independent routing patterns*

- Single-hop single-path



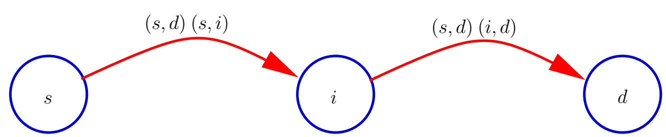
Sequent. Knapsack $\quad V^{sd}(U) \equiv \max \sum_{t \in T} \pi_{sdt}\, x_{sdt}$

$$48\, x_{sd48} + 12\, x_{sd12} + 3\, x_{sd3} + x_{sd1} \quad \leq \quad U$$

$$x_{sdt} \in I\!N \quad t \in T$$

$\pi_{sd48} \geq 4\, \pi_{sd12}\ , \quad \pi_{sd12} \geq 4\, \pi_{sd3}\ , \quad \pi_{sd3} \geq 3\, \pi_{sd1} \rightarrow$ **greedy sol**

- Two-hop single-path



$$\max_{u} \{ V^{sd}(u) + V^{si}(U - u) + V^{id}(U - u) \}$$

Packing *independent routings* onto wavelengths

$$
\begin{aligned}
\min \sum_{\text{IRP} \in \mathcal{I}} \Big(\text{cost}^{\text{IRP}} \; - \sum_{(s,d,t) \in K} \pi_{sdt}\, x_{sdt}^{\text{IRP}}\Big)\, \mu_{\text{IRP}} & \\
\sum_{\text{IRP} \in \mathcal{I}} \delta_a^{\text{IRP}} \mu_{\text{IRP}} \; &\leq \; 1 \quad a \in A \\
\sum_{\text{IRP} \in \mathcal{I}} x_{sdt}^{\text{IRP}}\, \mu_{\text{IRP}} \; &\leq \; D_{sdt} \quad (s,d) \in \mathcal{SD},\, t \in T \\
\mu_{\text{IRP}} \; &\in \; \{0,1\} \quad \text{IRP} \in \mathcal{I}
\end{aligned}
$$

- stable set problem in conflict graph of IRP
- integer cutting plane procedure
- In situ column generation of IRPs: one for each light-path conf.

Greedy heuristic first, exact solution second

Master LP solution

NSF = 14 nodes, 21 links; EON = 20 nodes, 39 links

	# Requests	W	col_{LP}	time(s)	gap (%)
NSF1	1332	18	4570	14117	18
NSF2	1949	25	201	73	28
NSF3	1667	18	227	262	8
NSF4	2292	25	2093	7043	25
EON1	60303	49	16627	78072	19
EON2	3667	37	608	323	34
EON3	120676	66	594	926	9
EON4	6639	40	7286	10506	19
average			4025	13915	20

Adding Cuts: gap $20.43\% \rightarrow 14.34\%$

- **port lower bound:** gap $20.43\% \rightarrow 19.06\%$

$$\sum_{c \in \mathcal{C}} \delta(c \ni s)\, \gamma_c \geq \left\lceil \frac{\sum_{d \in V} D_{sd}}{U} \right\rceil \quad s \in V$$

- **bound # of (s,d)-optical-paths:** gap $20.43\% \rightarrow 16.18\%$

$$\sum_{c \in \mathcal{C}} \delta(c \ni op_{sd})\, \gamma_c \geq \left\lceil \frac{D_{sd}}{U} \right\rceil \quad (s,d) \in \mathcal{SD}$$

- **bound # of (s,d,t)-optical-paths:** gap $20.43\% \rightarrow 17.83\%$

$$\sum_{c \in \mathcal{C}} \delta(c \ni op_{sd48})\ \gamma_c \geq \left\lceil \frac{D_{sd,48}}{4} \right\rceil, \quad (s,d) \in \mathcal{SD}$$

- **bound # of heavy loaded optical-paths:** gap $20.43\% \rightarrow 18.39\%$

$$\sum_{c \in \mathcal{C}} \delta(c \ni op_{sd,l>average})\, \gamma_c \leq UB, \quad (s,d) \in \mathcal{SD}$$

Rounding Heuristic

- Select largest γ_c / largest demand cover $\sum_{sdt} t\, x_{sdt}^c$
- fix $\lfloor \gamma_c \rfloor$ if $\gamma_c \geq 1$, rounded-up otherwise
- return to column generation on residual problem

	largest γ		large dem cov		truncated large dem		Tabu Search	
	time	gap	time	gap	time	gap	time	gap
NSF1	32827	18	35196	19	1256	19	317	12
NSF2	41180	16	37395	15	7752	16	1948	25
NSF3		∞	185085	8	121761	7	44896	10
NSF4		∞	36797	18	2257	21	1303	21
EON1		∞	176640	18	1556	30	1425	27
EON2	603494	16	501830	17	4788	28	35707	31
EON3		∞	2030000	9	74878	10		∞
EON4		∞		∞	24710	23	16406	29

Conclusion

- Hard Problem
  1. single path routing $\rightarrow$ thousands of requests
  2. port setup cost $\neq$ standard network design arc setup
  3. symmetry in wavelength and request assignment
  4. hop-constraints
- multi-commodity flow / meta-heuristic approaches not adapted
- Cutting plane procedure: time consuming and very fact. sol.
- Rounding heuristic on Master LP significantly better than tabu