Column Generation based Tactical Planning Method for Inventory Routing

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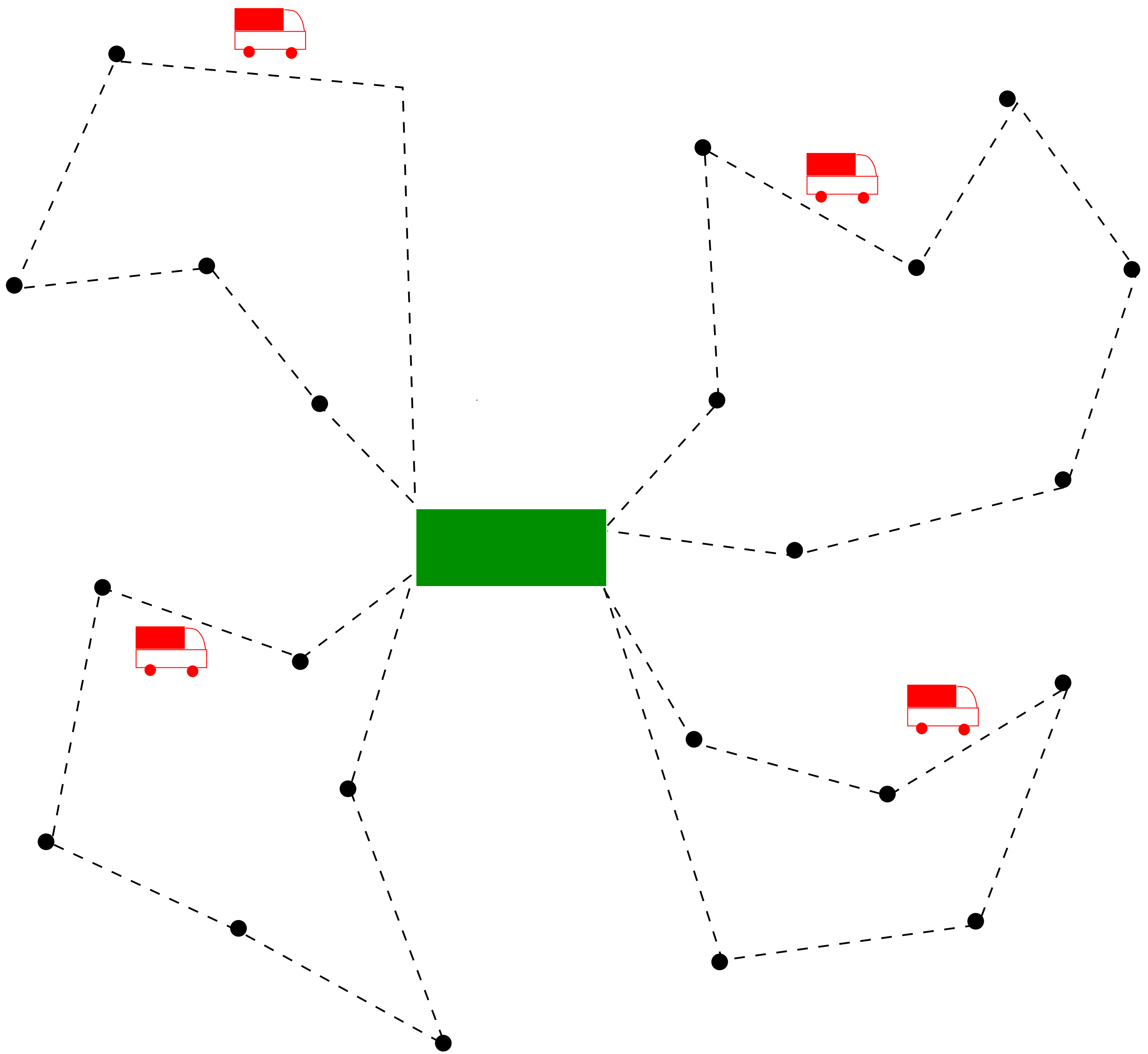
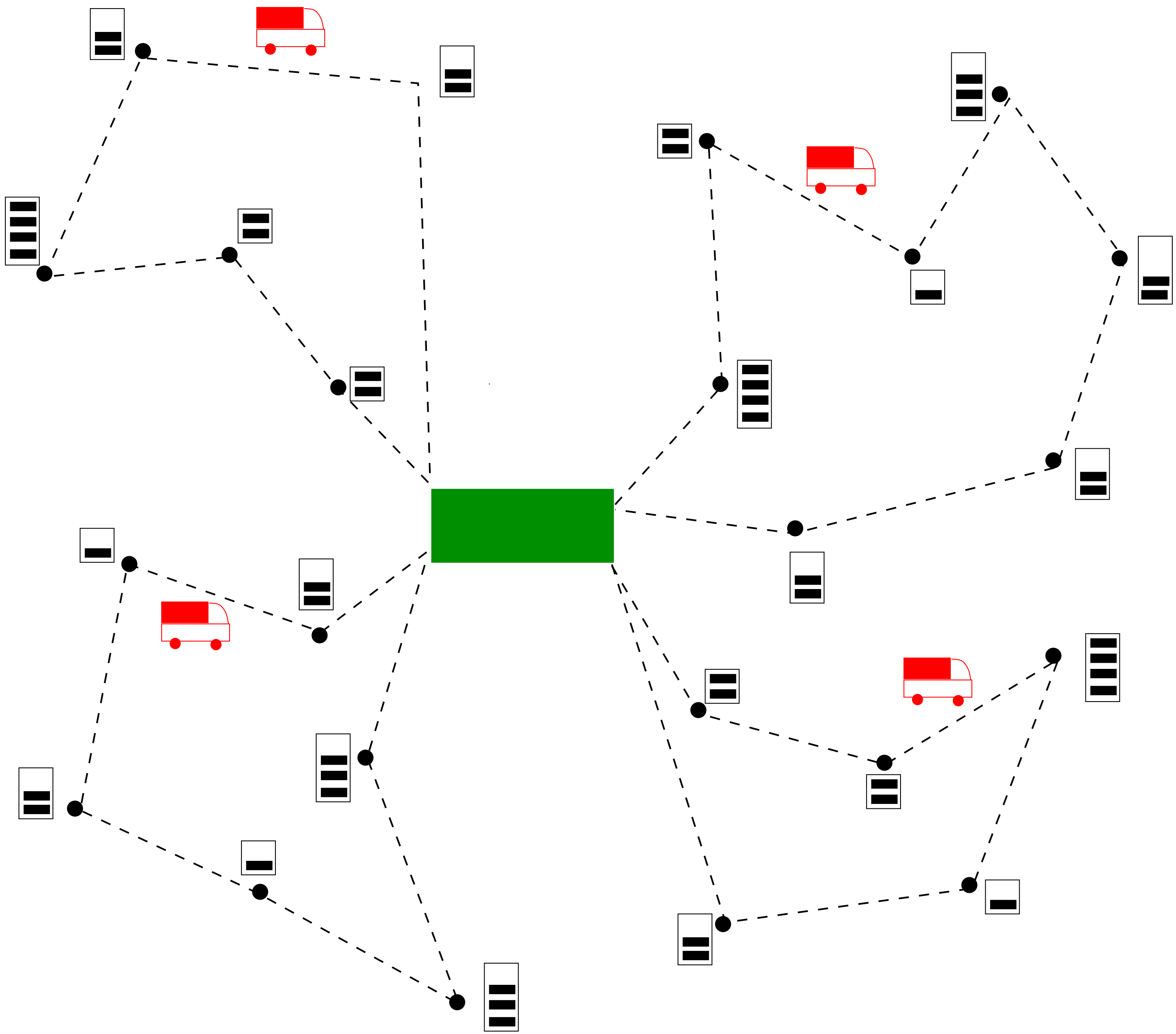
Vehicle routing

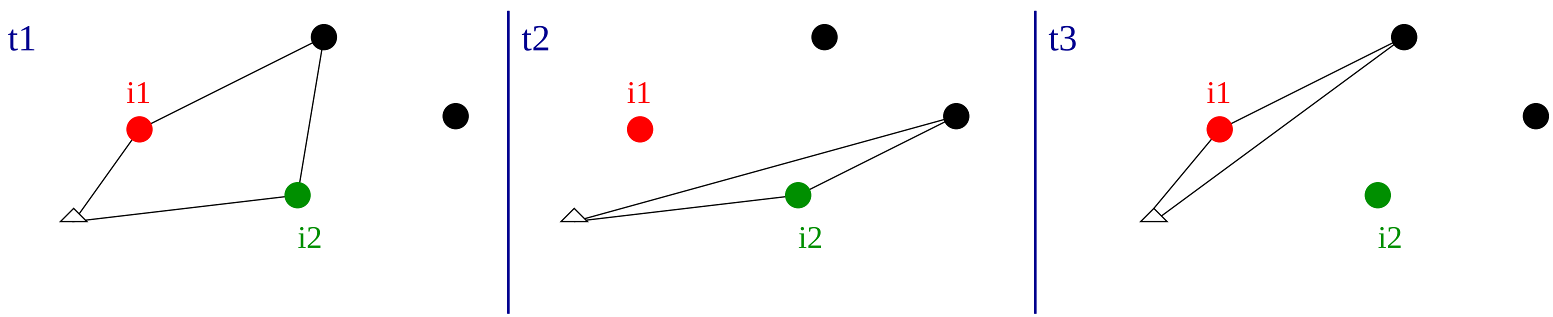


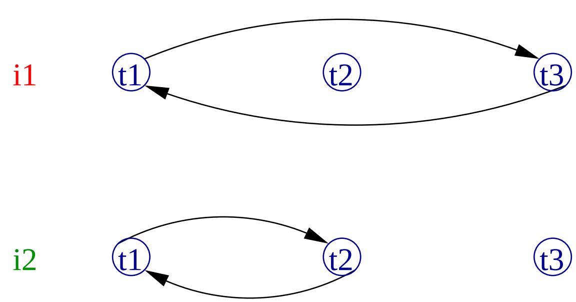
Inventory Routing



Inventory Routing

- → 3 decisions:
  1. **when** to visit a site?
  2. **how much to collect** from the site?
  3. **which tasks are "assigned" to a vehicle** (cluster/routes) ?
- → underlying models: Multi-Period VRP + Inventory manag.





Literature

many variants: # of products - Time horizon - det./stoch demands - stock manag. policy - routing vs clustering

various applications: ammonia shipping, petrol stations, supermarkets

low size of instances: 15 customers, or 1 period, or 2 customers per vehicles

Non exact solution approaches:

- **restrictive assumptions:** "fixed partition policy" (sets of customers that are serviced together)

  (Bramel and Simchi-Levi 1995)

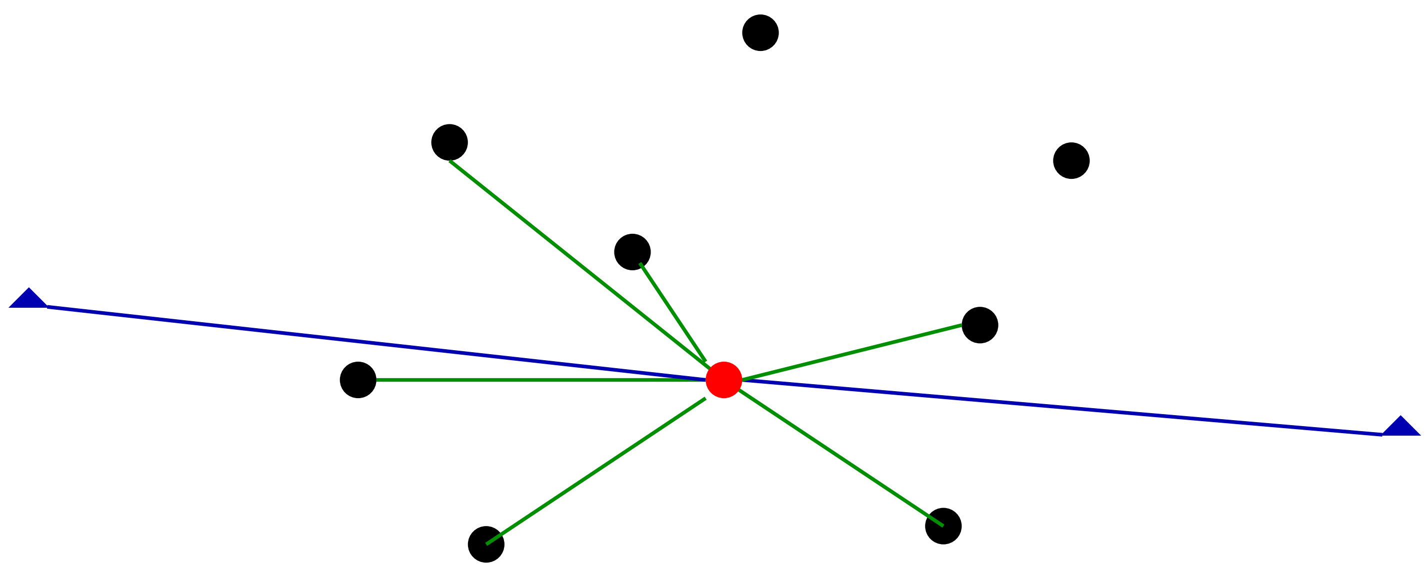
- **hierarchical approach:** planning first, routing second

  (Campbell and Savelsbergh 2004)

- Mostly **heuristics**. Some studies use **branch-and-price**.

Collecting recycle waste from deposit points

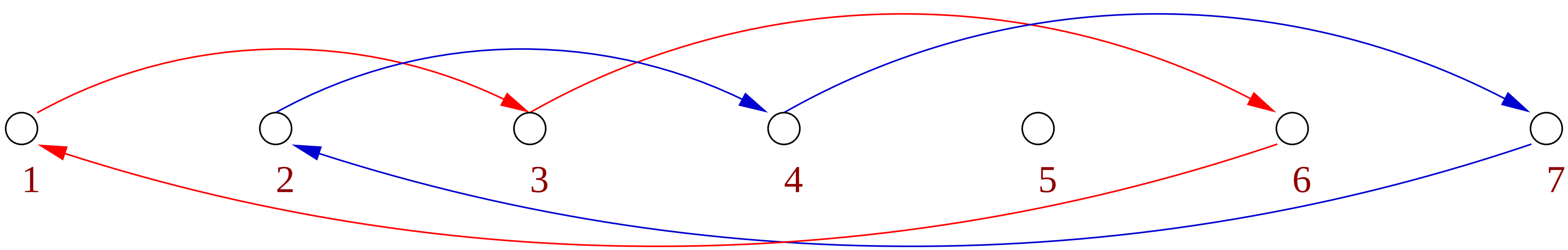
- Inventory management model:
  - order up-to level policy: empty the bin on each visit
  - deterministic filling rates: inventory management = $t_{\max}$ between visits; costs = transportation.
  - $p-$periodic routes: ∞ horizon, periodic planning constraint periodicities $p \in P = \{1, 2, 3, 4, 5, 6\} \Rightarrow T = LCM(P) = 60$.
- Routing model: form compact clusters first (tactical planning), route second (left for operational planning – drivers'decision)



Challenges

- problem size:
  - 260 sites
  - 10 vehicles
  - Frequency of visits: $t_{\max} \in \{1, \ldots, 14\}$
  - 1 vehicle = 10 sites visited on average

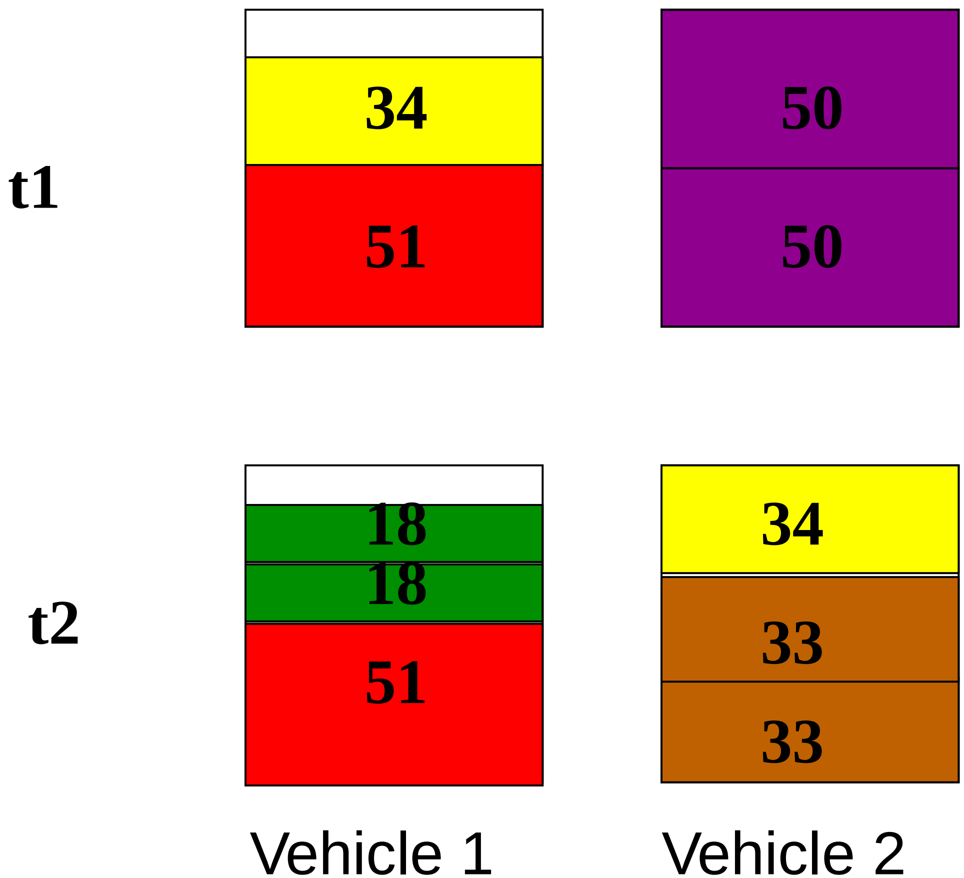
- symmetry drawback: in selecting periodic solutions



$\longrightarrow$ we shall model an average behavior

Saving vehicles compared to Bin-Packing sol

filling rates $= (51, 50, 34, 33, 18), \quad W = 100 \quad \Rightarrow \quad V(bpp) = 3$



Solution approach & results

- Truncated branch-and-price-and-cut
- Column generation based primal heuristics
- state space relaxation

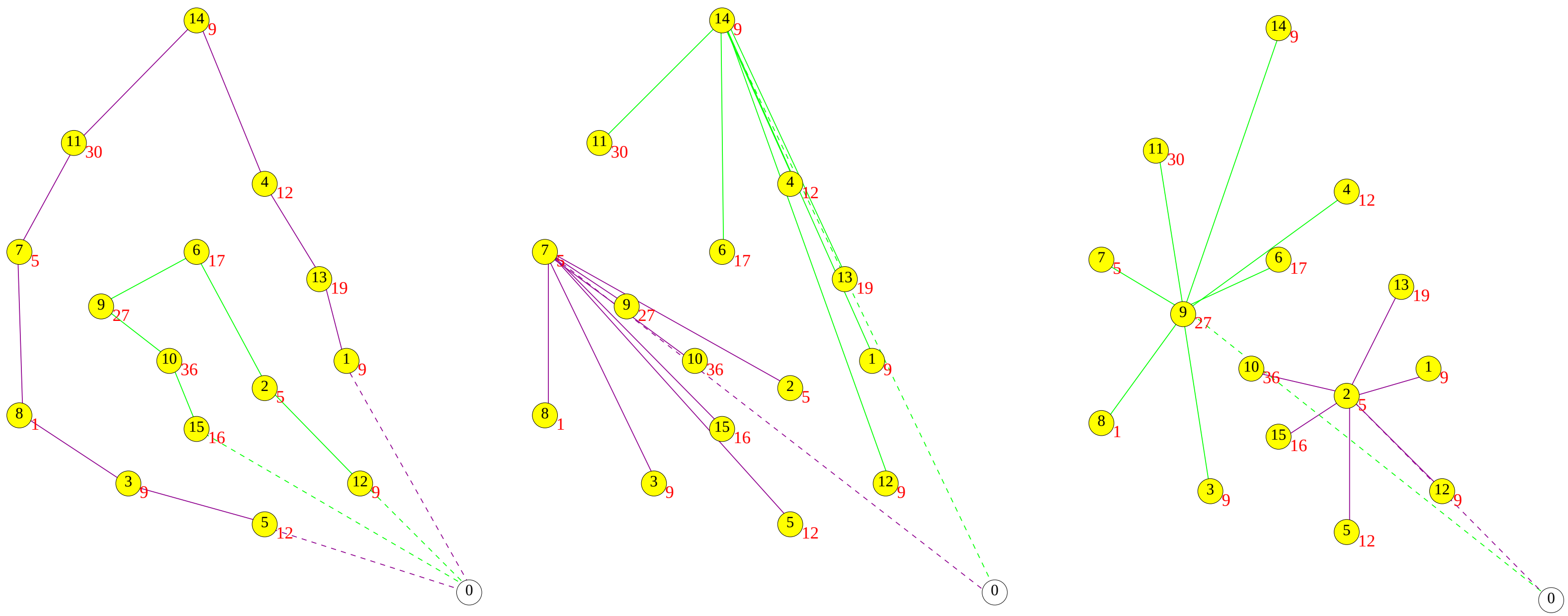
	av # of customer visits per week	# of vehicles	av travel av distance
our sol	98	9	711 km
indust. solution	59	10	782 km

+ regional partition of routes

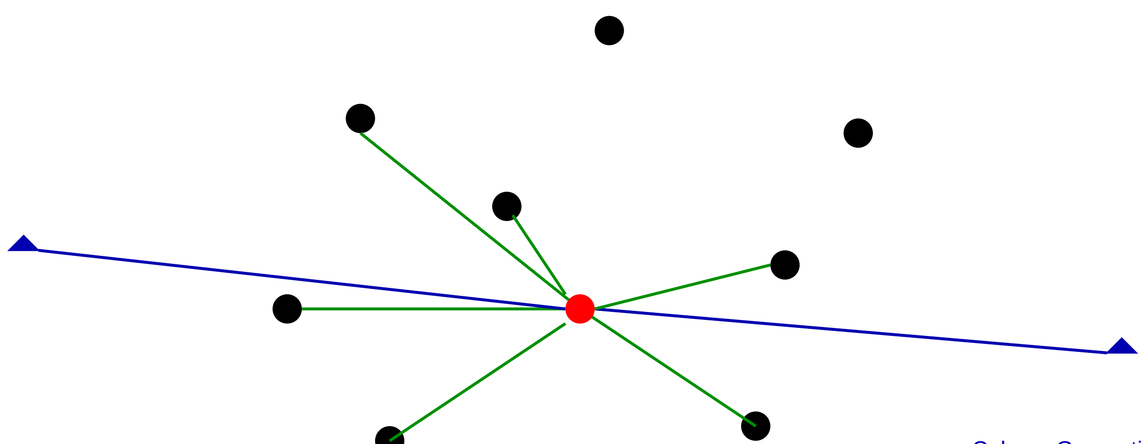
Outline

1. Clustering Model
2. “Compact” Formulation
3. Decomposition into Vehicle Tasks
4. DW reformulations
   - Discrete time
   - Aggregate time
5. Dual bounds
   - Use the aggregate formulation
   - Adding cuts
   - Doing partial branching
6. Primal Bounds: heuristics
   - Use of the Discrete formulation
   - Col Gen based heuristics: Restricted Master, Greedy, Rounding, Local Search

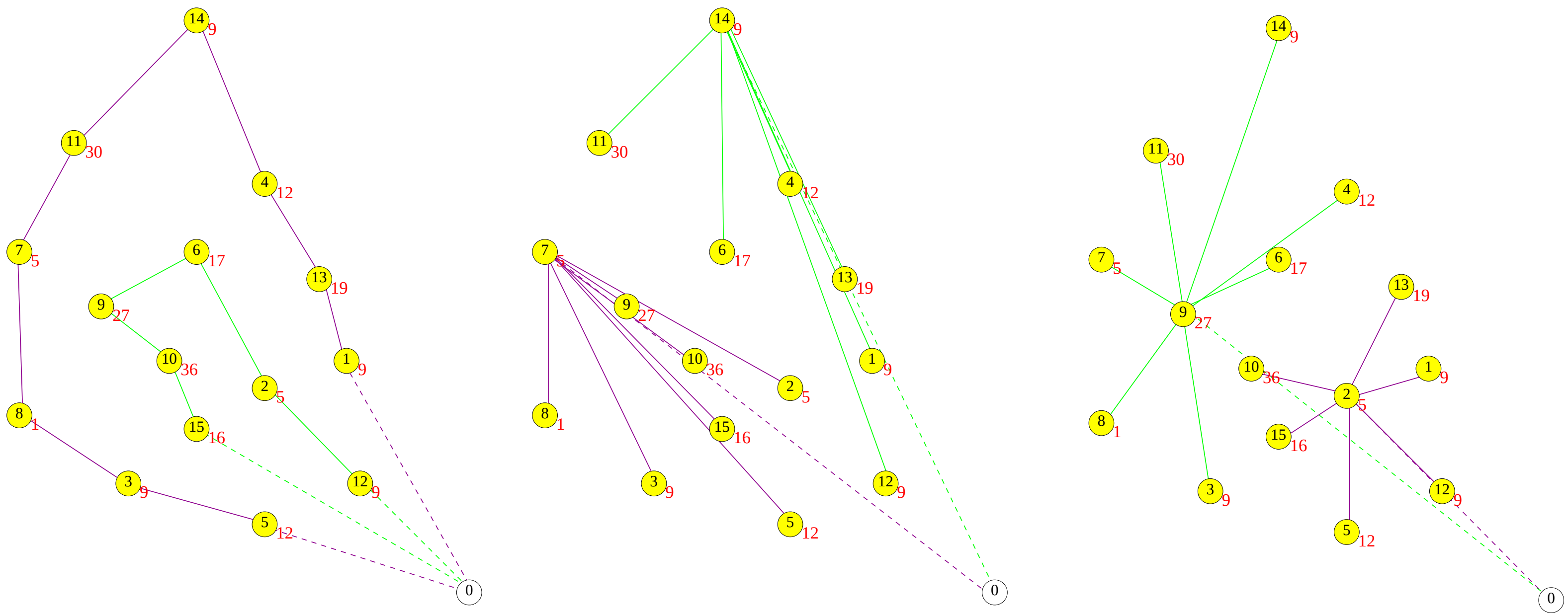
Surrogate transportation costs



$$c_{ik} = d_{i,k} + min\{d_{0,i} - d_{0,k}, d_{i,n+1} - d_{k,n+1}\}$$



Surrogate transportation costs



$$c_{ik} = d_{i,k} + min\{d_{0,i} - d_{0,k}, d_{i,n+1} - d_{k,n+1}\}$$

costs	routing	cluster	facility loc
routing sol	<u>190.9</u>	214.1	184.5
cluster sol	196.8	<u>200.4</u>	147.0
facility loc sol	198.8	208.7	<u>135.5</u>

Compact Formulation

$x_{i\ell vps} = 1$ if cust. i is collected ℓ periods worth of stock by vehicle v, every p periods, starting in s;
$y_{vps} = 1$ if vehicle v ... $z_{ikvps} = 1$ if cust. i is in a cluster of seed k ...

$$\min \quad V_{\max} + \alpha \sum_{v,p,s} \frac{1}{p} \Big(\sum_k f_k z_{kkvps} + \sum_{i,k:i \neq k} c_{ik} z_{ikvps} \Big) \qquad (1)$$

$$\sum_{\ell,v,p,s} \theta_t^{\ell ps} x_{i\ell vps} = 1 \qquad \forall i \in N', t = 1, \ldots, T \qquad (2)$$

$$\sum_{k \in N'} z_{ikvps} = \sum_{\ell} x_{i\ell vps} \quad \forall i \in N', v, p, s \qquad (3)$$

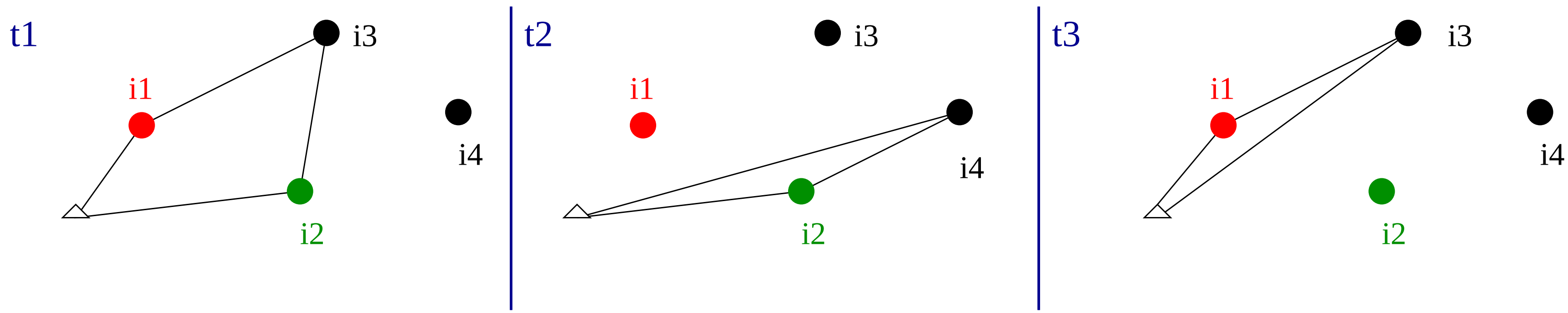
$$z_{ikvps} \leq z_{kkvps} \qquad \forall i \in N', k \in N', v, p, s \qquad (4)$$

$$\sum_{i \in N', \ell} \ell \, r_i \, x_{i\ell vps} \leq W y_{vps} \qquad \forall v, p, s \qquad (5)$$

$$\sum_{v,p,s} \delta_t^{ps} y_{vps} \leq V_{\max} \qquad \forall t = 1, \ldots, T \qquad (6)$$

Decomposition into Vehicle Tasks

A Complete planning can be decomposed in periodic vehicle tasks:



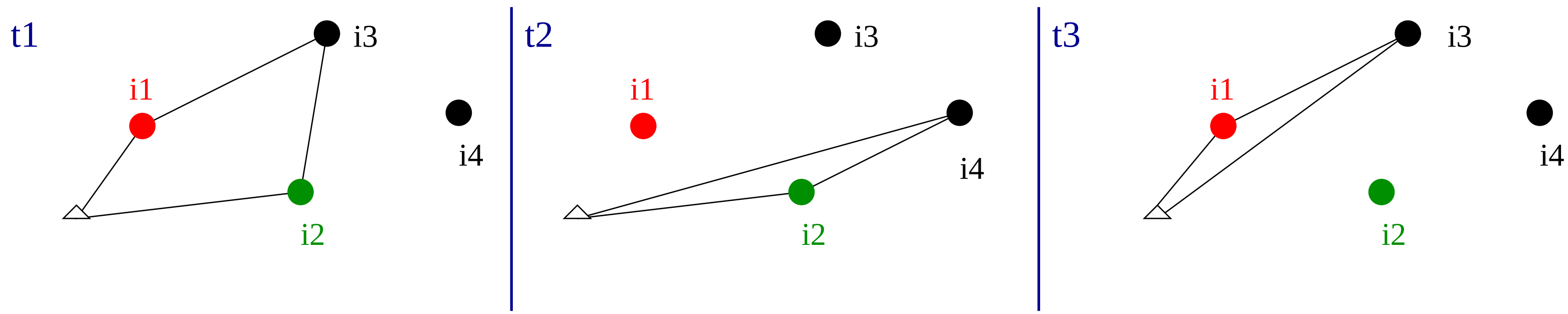
3 vehicle tasks of periodicity 3 $\rightarrow$ 1 vehicle needed

A periodic vehicle task q is defined by:

- its pickup pattern x^q: $x^q_{il} = 1$ if l period worth of stock is pickup from cust i.
- its cluster cost c^q
- its first occurrence (starting time) s^q
- its periodicity p^q

Column Generation Formulation

Each column defines a periodic vehicle task: $\{(c^q, s^q, p^q, x^q)\}_{q \in Q}$



The master models inventory planning

$$[DM] \equiv \min \; V_{\max} \; + \; \alpha \sum_{q \in Q} \frac{c^q}{p^q} \lambda_q \tag{7}$$

$$\sum_q \theta_{it}^q \lambda_q \; \geq \; 1 \qquad \forall i \in N', \; t = 1, \ldots, T \tag{8}$$

$$\sum_q \delta_t^q \lambda_q \; \leq \; V_{\max} \qquad \forall t = 1, \ldots, T \tag{9}$$

$$\lambda_q \; \in \; \{0, 1\} \qquad \forall \, q \in Q \tag{10}$$

Pricing Subproblem

For each starting time s periodicity p cluster center k, solve a multiple choice knapsack problem:

$$\max \sum_{i,\ell} g_{i\ell} x_{i\ell} \tag{12}$$

$$\sum_{\ell} x_{k\ell} = 1 \tag{13}$$

$$\sum_{\ell} x_{i\ell} \leq 1 \qquad \forall\, i \neq k \tag{14}$$

$$\sum_{i,\ell} \ell\, r_i\, x_{i\ell} \leq W \tag{15}$$

$$x_{i\ell} \in \{0,1\} \qquad \forall\, i,\ell. \tag{16}$$

State Space Relaxation

Aggregate columns that differ only by their starting dates:

$$\{(c^q, s^q, p^q, x^q)\}_{q \in Q} \quad \rightarrow \quad \{(c^r, p^r, x^r)\}_{r \in R}.$$

Summing over t, leads to a master modeling an average behavior:

$$Z^A = \min \; V\text{aver} \; + \; \alpha \sum_{r \in R} \frac{c^r}{p^r} \lambda_r \tag{17}$$

$$\sum_{r \in R, \ell} \frac{\ell}{p^r} x^r_{i\ell} \lambda_r \; \geq \; 1 \qquad \forall\, i \in N' \tag{18}$$

$$\sum_{r \in R} \frac{1}{p^r} \lambda_r \; \leq \; V\text{aver} \tag{19}$$

$$\lambda_r \; \in \; I\!N \qquad \forall\, r \in R \tag{20}$$

$$V \geq V\text{aver} \; \in \; I\!N, \tag{21}$$

Discrete versus Aggregate Time Formulation

- LP equivalence:

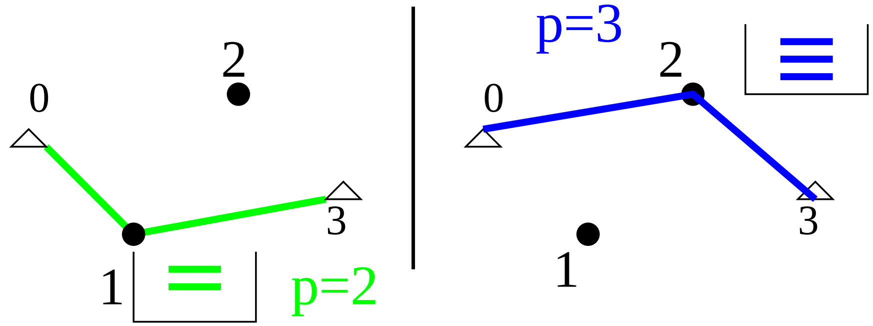
$$\lambda_r = \sum_{q \in Q(r)} \lambda_q \ , \tag{22}$$

$$\lambda_q = \frac{1}{p^r} \lambda_r, \quad q \in Q(r) \tag{23}$$

- No IP equivalence
- $\neq$ LP computing time

	discrete form.		aggregate form.	
Instance	Col	Time	Col	Time
IND8	71	1.75s	20	0.25s
IND27	-	$>$1h	123	5.00s
RAND100	-	$>$1h	701	3.52s

Discrete $\neq$ Aggregate Time IP Formulation



aggreg. sol. λ_r	discrete sol. λ_q		t1	t2	t3	t4	t5	t6
$\lambda_A = 1$	$\lambda_{A_1} = \frac{1}{2}$	$s_{A_1} = 1$	■	✓	■	✓	■	✓
	$\lambda_{A_2} = \frac{1}{2}$	$s_{A_2} = 2$	✓	■	✓	■	✓	■
$\lambda_B = 1$	$\lambda_{B_1} = \frac{1}{3}$	$s_{B_1} = 1$	■	✓	✓	■	✓	✓
	$\lambda_{B_2} = \frac{1}{3}$	$s_{B_2} = 2$	✓	■	✓	✓	■	✓
	$\lambda_{B_3} = \frac{1}{3}$	$s_{B_3} = 3$	✓	✓	■	✓	✓	■

- Task A: $0 - 1(2) - 3$ with $p_A = 2$,
- Task B: $0 - 2(3) - 3$ with $p_B = 3$,

$$V\text{aver} = 1 \geq \frac{1}{2} + \frac{1}{3} \quad \text{while} \quad V_{\max} = 2$$

Discrete $\neq$ Aggregate Time IP Formulation

- task A: $0 - \cdots - 1(3) - \cdots - n+1$ with $p_A = 6$, and
- task B: $0 - \cdots - 1(2) - \cdots - n+1$ with $p_B = 4$.

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12
θ_{1t}^{A}			✓	✓	■				✓	✓	■	
θ_{1t}^{B}	✓	■			✓	■			✓	■		

customer pick-up date conflict

Solving the Aggregate Master

- Include artificial columns; their cost defines an UB on dual var.
- Use a dual heuristic to “warm start” the col. gen. procedure.
- Solve multiple choice knapsacks using Dynamic Prog. (Pisinger, 95).
- Partial Pricing: consider
  - periodicities in decreasing order,
  - most attractive seed first.
- Re-optimisation of
  - seed location,
  - periodicity: $p = \max_i \{\ell : x_{i\ell} = 1\}$

Adding Cutting Planes

In Mast LP sol: a pattern that covers $\frac{5}{6}$ of customer i's demand whose $t_{\max} = 5$, can be selected 1.2 time.

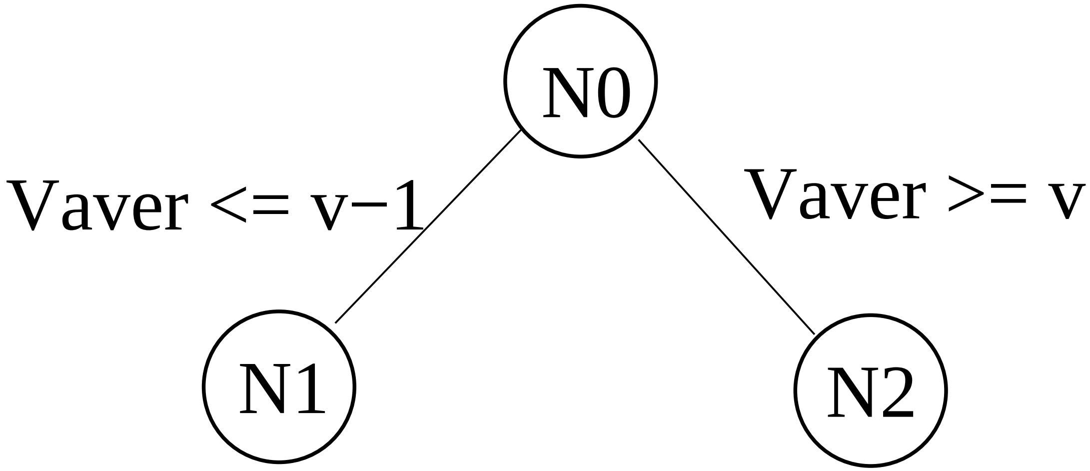
aggreg. sol. λ_r	t1	t2	t3	t4	t5	t6
$\lambda_r = 1.2$	✓	✓	✓	✓	■	

To cut this LP solution is:

$$\sum_{r,\ell:\, \ell = p^r} x_{i\ell}^r \lambda_r + \frac{1}{2} \sum_{r,\ell:\, \ell \neq p^r} x_{i\ell}^r \lambda_r \geq 1,$$

- small improvement: 2% gap
- large increase in computing time: 70% of the time in re-opt.
- modifications to the structure of master LP solution: bad for primal heuristic

Branching on V_{aver}



- v is a lower bound on number of vehicles if N1 infeasible
- Improvement in dual bound in N2: since
  $Z^A = \min \; V_{\text{aver}} + \alpha \sum_{r \in R} \frac{c^r}{p^r} \lambda_r$
- Cutting Planes + Partial Branching

gaps

gap-root	gap-cut	gap-br	gap-br-cut
23.05	**21.80**	**10.63**	**9.67**

% time

PP	RM	CP	Sep	N0	N1	N2
10.84	8.14	73.49	4.14	5.87	16.17	77.91

Primal Heuristics

- Rounding:
  Round aggregate master solution to construct discrete master solution
  - Round-up mast col and select starting date : greedy selection, $\text{argmin}_{rs}\{(\delta_{rs} + \alpha \frac{c^r}{p^r})/(\sum_{i,\ell} \ell x_{i\ell})\}$
  - Solve the residual master: pricing requires enumerating over starting dates
  - Diversify the search: limited backtracking at the root node

- Local Search: starting from a master IP sol
  - Remove “worst” col : low load columns + complementary:
    - favour custo. exchange: i.e. close-by clusters;
    - favour vehicle saving: i.e. same (s, p).
  - Solve the residual master using the Rounding heur.

Computational results

260 customers, 60 periods, 10 vehicles

- Dual Bounds:

Algorithm	bound	time
AM LP + br + cut	838.17	5h21m

- Primal Bounds:

Algorithm	gap (in %)	$V_{\max}$	time
RH + Branch, $P = \{1, 2, 3, 4, 5, 6\}$	9.25	9	4h29m
RH + LS, $P = \{1, 2, 3, 4, 5, 6\}$	8.92	9	3h40m
RH + Branch, $P = \{1, 2, 3\}$ + post-opt	6.67	9	1h49m
RH + LS, $P = \{1, 2, 3\}$ + post-optim	6.23	9	2h53m

Summary: Planning for Inventory Routing

- Column generation + surrogate relaxation to avoid symmetries
- Rounding heuristic implicitly carried in discrete time formulation
- Solutions with quality warranty: $LB \overset{6\%}{\leftrightarrow} SOL \overset{10\%}{\leftrightarrow} INDUST$