Feuille de TD 2 : Matrices

Ex. 1. $A = \begin{pmatrix} 0 & 1 \\ -3 & 2 \end{pmatrix}$ $B = \begin{pmatrix} -1 & 3 \\ 0 & 2 \end{pmatrix}$ $C = \begin{pmatrix} -1 & 3 \\ -3 & 2 \end{pmatrix}$

$$A + B = \begin{pmatrix} 0 & 1 \\ -3 & 2 \end{pmatrix} + \begin{pmatrix} -1 & 3 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 0-1 & 1+3 \\ -3+0 & 2+2 \end{pmatrix} = \begin{pmatrix} -1 & 4 \\ -3 & 4 \end{pmatrix}$$

$$5A - C = \begin{pmatrix} 0 & 5 \\ -15 & 10 \end{pmatrix} - \begin{pmatrix} -1 & 3 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -12 & 8 \end{pmatrix}$$

$$A - B + C = \begin{pmatrix} 0+1-1 & 1-3+3 \\ -3-0-3 & 2-2+2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -6 & 2 \end{pmatrix}$$

Ex. 2. $A = \begin{pmatrix} 0 & x_1 \\ x_2 & 1 \end{pmatrix}$ $B = \begin{pmatrix} -1 & x_3 \\ 0 & 2 \end{pmatrix}$ $C = \begin{pmatrix} x_4 & 3 \\ -3 & 0 \end{pmatrix}$

$$A - B = \begin{pmatrix} 1 & x_1 - x_3 \\ x_2 & -1 \end{pmatrix}$$

$$2A - C = \begin{pmatrix} 0 & 2x_1 \\ 2x_2 & 2 \end{pmatrix} - \begin{pmatrix} x_4 & 3 \\ -3 & 0 \end{pmatrix} = \begin{pmatrix} -x_4 & 2x_1 - 3 \\ 2x_2 + 3 & 2 \end{pmatrix}$$

$$A - B + 2C = \begin{pmatrix} 0+1+2x_4 & x_1 - x_3 + 6 \\ x_2 - 0 - 6 & 1 - 2 + 0 \end{pmatrix} = \begin{pmatrix} 1+2x_4 & x_1 - x_3 + 6 \\ x_2 - 6 & -1 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 0 & x_1 \\ x_2 & 1 \end{pmatrix}\begin{pmatrix} 0 & x_1 \\ x_2 & 1 \end{pmatrix} = \begin{pmatrix} 0\cdot 0 + x_1 x_2 & 0\cdot x_1 + x_1 \cdot 1 \\ x_2 \cdot 0 + 1\cdot x_2 & x_2 \cdot x_1 + 1\cdot 1 \end{pmatrix} = \begin{pmatrix} x_1 x_2 & x_1 \\ x_2 & x_1 x_2 + 1 \end{pmatrix}$$

$$\begin{array}{cc} & \begin{pmatrix} 0 & x_1 \\ x_2 & 1 \end{pmatrix} A \\ A & \\ \begin{pmatrix} 0 & x_1 \\ x_2 & 1 \end{pmatrix} & \begin{pmatrix} x_1 x_2 & x_1 \\ x_2 & x_1 x_2 + 1 \end{pmatrix} A^2 \end{array}$$

Ex. 3. $A = \begin{pmatrix} 1 & 2 \end{pmatrix}$ $B = \begin{pmatrix} 0 & -1 \\ -3 & 2 \end{pmatrix}$ $C = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$

A est 1×2, B est 2×2 et C est 2×1

donc AB est possible : $(1\times \underbrace{2)(2}\times 2)$ et le résultat est 1×2 : $\begin{pmatrix} 1 & 2 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} -6 & 3 \end{pmatrix}$

~~[illegible]~~ BA n'est pas possible : $(2\times2)\cdot(1\times2)$

AC est possible : $(1\times2)(2\times1)$ et le résultat est (-2) matrice 1×1

CA est possible aussi $(2\times1)(1\times2)$: $CA = \begin{pmatrix} 0 \\ -1 \end{pmatrix}\begin{pmatrix} 1 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ -1 & -2 \end{pmatrix}$

BC ~~et CB sont~~ est possible, $(2\times2)(2\times1) = (2\times1)$

$BC = \begin{pmatrix} 0 & -1 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$. CB n'est pas possible : $(2\times1)(2\times2)$

On remarque que même si AC et CA existent tous les deux, $AC \neq CA$, ce qui montre que le produit matriciel est non commutatif.

<u>Ex. 4.</u> $A = \begin{pmatrix} n_1 & n_2 \\ 0 & n_3 \end{pmatrix}$ $B = \begin{pmatrix} d_1 & 0 & d_2 \\ 0 & d_3 & d_4 \\ -1 & d_5 & d_6 \end{pmatrix}$ $C = \begin{pmatrix} 1 & 0 & n_4 \end{pmatrix}$

AB est $(2\times2)(3\times3)$, pas possible, BA non plus.

AC n'est pas possible, CA non plus.

$BC = \begin{pmatrix} d_1 & 0 & d_2 \\ 0 & d_3 & d_4 \\ -1 & d_5 & d_6 \end{pmatrix}\begin{pmatrix} 1 & 0 & n_4 \end{pmatrix}$ n'est pas possible

$CB = \begin{pmatrix} 1 & 0 & 4 \end{pmatrix}\begin{pmatrix} d_1 & 0 & d_2 \\ 0 & d_3 & d_4 \\ -1 & d_5 & d_6 \end{pmatrix} = \begin{pmatrix} d_1-4 & 4d_5 & d_2+4d_6 \end{pmatrix}$

$\underset{(1\times3)}{C}\ \underset{(3\times3)}{B} = (1\times3)$

<u>Ex. 5.</u> $A = \begin{pmatrix} n_1 & n_2 \\ 0 & n_3 \\ -n_4 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 3 & 1 \end{pmatrix}$ $C = \begin{pmatrix} 1 & 0 & n_4 \\ d_1 & 0 & 0 \end{pmatrix}$

(3×2) (3×3) (2×3)

Les produits possibles sont AC, BA, CA et CB

$AC = \begin{pmatrix} n_1+d_1n_2 & \cancel{0} & n_1n_4 \\ n_3d_1 & \cancel{0} & 0 \\ -n_4 & 0 & -n_4^2 \end{pmatrix}$ $BA = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 3 & 1 \end{pmatrix}\begin{pmatrix} n_1 & n_2 \\ 0 & n_3 \\ -n_4 & 0 \end{pmatrix} = \begin{pmatrix} n_1+n_4 & n_2 \\ 0 & 2n_3 \\ -n_1-n_4 & -n_2+3n_3 \end{pmatrix}$

$$CA = \begin{pmatrix} 1 & 0 & x_4 \\ d_1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 & x_2 \\ 0 & x_3 \\ -x_4 & 0 \end{pmatrix} = \begin{pmatrix} x_1 - x_4^2 & x_2 \\ d_1 x_1 & d_1 x_2 \end{pmatrix}$$

$$CB = \begin{pmatrix} 1 & 0 & x_4 \\ d_1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 1-x_4 & 3x_4 & -1+x_4 \\ d_1 & 0 & -d_1 \end{pmatrix}$$

On peut calculer le carré seulement des matrices carrées, donc de B.

$$B^2 = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 2 & -3 & -2 \\ 0 & 4 & 0 \\ -2 & 9 & 2 \end{pmatrix}$$

Ex. 6.

$$\begin{cases} 3x + 5y = 11 \\ 2x + 3y = 7 \end{cases} \qquad \left(\begin{array}{cc|c} 3 & 5 & 11 \\ 2 & 3 & 7 \end{array}\right) \quad \text{c-à-d} \quad \begin{pmatrix} 3 & 5 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ 7 \end{pmatrix}$$

$$\begin{array}{c} 2\mathrm{I} \\ 3\mathrm{II} \end{array} \left(\begin{array}{cc|c} 6 & 10 & 22 \\ 6 & 9 & 21 \end{array}\right) \Rightarrow \begin{array}{c} \\ \mathrm{II}-\mathrm{I} \end{array} \left(\begin{array}{cc|c} 6 & 10 & 22 \\ 0 & -1 & -1 \end{array}\right) \Rightarrow \begin{array}{c} \mathrm{I}+10\mathrm{II} \\ \\ \end{array} \left(\begin{array}{cc|c} 6 & 0 & 12 \\ 0 & -1 & -1 \end{array}\right) \Rightarrow$$

$$\Rightarrow \begin{array}{c} \frac{1}{6}\mathrm{I} \\ -\mathrm{II} \end{array} \left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array}\right) \quad \text{c-à-d} \quad \begin{cases} x = 2 \\ y = 1 \end{cases}$$

$$\begin{cases} 3x + 5y + 4z = 11 \\ 2x + 3y - z = 7 \end{cases} \qquad \left(\begin{array}{ccc|c} 3 & 5 & 4 & 11 \\ 2 & 3 & -1 & 7 \end{array}\right) \Rightarrow \begin{array}{c} 2\mathrm{I} \\ 3\mathrm{II} \end{array} \left(\begin{array}{ccc|c} 6 & 10 & 8 & 22 \\ 6 & 9 & -3 & 21 \end{array}\right) \Rightarrow$$

$$\Rightarrow \begin{array}{c} \\ \mathrm{II}-\mathrm{I} \end{array} \left(\begin{array}{ccc|c} 6 & 10 & 8 & 22 \\ 0 & -1 & -11 & -1 \end{array}\right) \Rightarrow \begin{array}{c} \mathrm{I}+10\mathrm{II} \\ \\ \end{array} \left(\begin{array}{ccc|c} 6 & 0 & -102 & 12 \\ 0 & -1 & -11 & -1 \end{array}\right) \Rightarrow \begin{array}{c} \frac{1}{6}\mathrm{I} \\ -\mathrm{II} \end{array} \left(\begin{array}{ccc|c} 1 & 0 & -17 & 2 \\ 0 & 1 & 11 & 1 \end{array}\right)$$

$$\begin{cases} x = 17z + 2 \\ y = -11z + 1 \end{cases} \quad \text{soit} \quad \begin{cases} x = 17t + 2 \\ y = -11t + 1 \\ z = t \end{cases} \quad \text{soit} \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 17 \\ -11 \\ 1 \end{pmatrix}$$

$$\begin{cases} -9x + 9y + 6z = 114 \\ 4x - 7z = -91 \\ -x - 2z = -26 \end{cases} \qquad \left(\begin{array}{ccc|c} -9 & 9 & 6 & 114 \\ 4 & 0 & -7 & -91 \\ -1 & 0 & -2 & -26 \end{array}\right) \begin{array}{c} \mathrm{III} \\ \mathrm{I}-9\mathrm{III} \\ \mathrm{II}+4\mathrm{III} \end{array} \left(\begin{array}{ccc|c} -1 & 0 & -2 & -26 \\ 0 & 9 & 24 & 348 \\ 0 & 0 & -15 & -299 \end{array}\right)$$

etc.
unique solution.

$$\begin{cases} 2x + 3y = 0 \\ 3x + 7y = 0 \\ 4x + 3y = 0 \end{cases} \qquad \left(\begin{array}{cc|c} 2 & 3 & 0 \\ 3 & 7 & 0 \\ 4 & 3 & 0 \end{array}\right) \Rightarrow \begin{array}{c} 3\mathrm{I} \\ 2\mathrm{II} \\ \\ \end{array} \left(\begin{array}{cc|c} 6 & 9 & 0 \\ 6 & 14 & 0 \\ 4 & 3 & 0 \end{array}\right) \Rightarrow \begin{array}{c} \\ \mathrm{II}-\mathrm{I} \\ \\ \end{array} \left(\begin{array}{cc|c} 6 & 9 & 0 \\ 0 & 5 & 0 \\ 4 & 3 & 0 \end{array}\right) \Rightarrow \begin{array}{c} \frac{1}{3}\mathrm{I} \\ \frac{1}{5}\mathrm{II} \\ \\ \end{array} \left(\begin{array}{cc|c} 2 & 3 & 0 \\ 0 & 1 & 0 \\ 4 & 3 & 0 \end{array}\right)$$

$$\Rightarrow \begin{array}{c} \mathrm{I} \\ \mathrm{II} \\ \mathrm{III}-2\mathrm{I} \end{array} \left(\begin{array}{cc|c} 2 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 0 \end{array}\right) \Rightarrow \begin{array}{c} \\ \\ -\frac{1}{3}\mathrm{III} \end{array} \left(\begin{array}{cc|c} 2 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{array}\right) \Rightarrow \begin{array}{c} \mathrm{I}-3\mathrm{II} \\ \\ \mathrm{III}-\mathrm{II} \end{array} \left(\begin{array}{cc|c} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases}$$

unique solution.

Ex. 7. $\begin{cases} x + n_1 y = 0 \\ 3x - n_2 y = 0 \end{cases}$ $\left(\begin{array}{cc|c} 1 & n_1 & 0 \\ 3 & -n_2 & 0 \end{array}\right) \underset{\text{II}-3\text{I}}{\sim} \left(\begin{array}{cc|c} 1 & n_1 & 0 \\ 0 & -3n_1-n_2 & 0 \end{array}\right)$

$\begin{cases} x + n_1 y = 0 \\ (-3n_1 - n_2) y = 0 \end{cases}$

si $-3n_1 - n_2 \neq 0 \Rightarrow 3n_1 + n_2 \neq 0$

$\begin{cases} x = 0 \\ y = 0 \end{cases}$ $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ unique solution

si $3n_1 + n_2 = 0 \iff n_1 = n_2 = 0$

$\begin{cases} x = 0 \\ 0 = 0 \end{cases}$ sol: la droite d'équation $x = 0$.

$\begin{cases} x + n_3 y - z = 1 \\ 2x + n_1 y = 1 \end{cases}$ $\left(\begin{array}{ccc|c} 1 & n_3 & -1 & 1 \\ 2 & n_1 & 0 & 1 \end{array}\right) \underset{\text{II}-2\text{I}}{\sim} \left(\begin{array}{ccc|c} 1 & n_3 & -1 & 1 \\ 0 & -2n_3+n_1 & 2 & -1 \end{array}\right)$

$n_1 - 2n_3 \neq 0$ $\begin{cases} x = -n_3 \left(\dfrac{-1-2z}{-2n_3+n_1}\right) + z + 1 \\ y = \dfrac{-1-2z}{-2n_3+n_1} \end{cases}$ etc.

$n_1 - 2n_3 = 0$ $\begin{cases} x + n_3 y - z = 1 \\ 2z = -1 \end{cases}$ $\begin{cases} x = -n_3 y + \frac{3}{2} \\ z = \frac{1}{2} \end{cases}$ etc.

Ex. 8 $A = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$ $\det A = 3 \cdot 2 - 5 \cdot 1 = 6 - 5 = 1$

A est inversible :

$A^{-1} = \dfrac{1}{\det A} \begin{pmatrix} +2 & -5 \\ -1 & +3 \end{pmatrix} = \begin{pmatrix} +2 & -5 \\ -1 & +3 \end{pmatrix}$ et en effet $\begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$B = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$ $\det B = 4 - 3 = 1 \Rightarrow B^{-1} = \frac{1}{1}\begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$

$C = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ $\det C = 0 + 1 + 1 - 0 - 0 - 0 = 2 \neq 0$

C est donc inversible

$$\underbrace{\begin{pmatrix} 0&1&1\\1&0&1\\1&1&0\end{pmatrix}}_{C}\left|\underbrace{\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}}_{\mathbb{1}}\right. \Rightarrow \begin{array}{c} \mathrm{II}\\ \mathrm{I}\\ \mathrm{III}-\mathrm{II}\end{array}\left(\begin{array}{ccc|ccc} 1&0&1&0&1&0\\0&1&1&1&0&0\\0&1&-1&0&-1&1\end{array}\right) \Rightarrow$$

$$\begin{array}{c} \\ \\ \mathrm{III}-\mathrm{II}\end{array}\left(\begin{array}{ccc|ccc} 1&0&1&0&1&0\\0&1&1&1&0&0\\0&0&-2&-1&-1&1\end{array}\right) \Rightarrow \begin{array}{c} \\ \\ -\frac{1}{2}\,\mathrm{III}\end{array}\left(\begin{array}{ccc|ccc} 1&0&1&0&1&0\\0&1&1&1&0&0\\0&0&1&\frac{1}{2}&\frac{1}{2}&-\frac{1}{2}\end{array}\right)$$

$$\begin{array}{c} \mathrm{I}-\mathrm{III}\\ \mathrm{II}-\mathrm{III}\\ \\ \end{array}\underbrace{\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}}_{\mathbb{1}}\left|\underbrace{\begin{pmatrix}-\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ \frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\\ \frac{1}{2}&\frac{1}{2}&-\frac{1}{2}\end{pmatrix}}_{C^{-1}}\right. \Rightarrow C^{-1} = \frac{1}{2}\begin{pmatrix}-1&1&1\\1&-1&1\\1&1&-1\end{pmatrix}$$

en effet, $CC^{-1} = \frac{1}{2}\begin{pmatrix}0&1&1\\1&0&1\\1&1&0\end{pmatrix}\begin{pmatrix}-1&1&1\\1&-1&1\\1&1&-1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}2&0&0\\0&2&0\\0&0&2\end{pmatrix} = \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$

$D = \begin{pmatrix}2&1&1\\-1&1&1\\1&-1&2\end{pmatrix}$ $\det D = 4+1+1-1+2+2 = 9 \neq 0 \Rightarrow D$ inversible

$$\left(\begin{array}{ccc|ccc} 2&1&1&1&0&0\\-1&1&1&0&1&0\\1&-1&2&0&0&1\end{array}\right) \Rightarrow \begin{array}{c} \mathrm{III}\\ \mathrm{I}-2\mathrm{III}\\ \mathrm{II}+\mathrm{III}\end{array}\left(\begin{array}{ccc|ccc} 1&-1&2&0&0&1\\0&3&-3&1&0&-2\\0&0&3&0&1&1\end{array}\right) \Rightarrow$$

$$\Rightarrow \begin{array}{c} \\ \frac{1}{3}\,\mathrm{II}\\ \frac{1}{3}\,\mathrm{III}\end{array}\left(\begin{array}{ccc|ccc} 1&-1&2&0&0&1\\0&1&-1&\frac{1}{3}&0&-\frac{2}{3}\\0&0&1&0&\frac{1}{3}&\frac{1}{3}\end{array}\right) \Rightarrow \begin{array}{c} \mathrm{I}-2\mathrm{III}\\ \mathrm{II}+\mathrm{III}\\ \\ \end{array}\left(\begin{array}{ccc|ccc} 1&-1&0&0&-\frac{2}{3}&\frac{1}{3}\\0&1&0&\frac{1}{3}&\frac{1}{3}&-\frac{1}{3}\\0&0&1&0&\frac{1}{3}&\frac{1}{3}\end{array}\right)$$

$$\Rightarrow \begin{array}{c} \mathrm{I}+\mathrm{II}\\ \\ \\ \end{array}\left(\begin{array}{ccc|ccc} 1&0&0&\frac{1}{3}&-\frac{1}{3}&0\\0&1&0&\frac{1}{3}&\frac{1}{3}&-\frac{1}{3}\\0&0&1&0&\frac{1}{3}&\frac{1}{3}\end{array}\right) \qquad D^{-1} = \frac{1}{3}\begin{pmatrix}1&-1&0\\1&1&-1\\0&1&1\end{pmatrix}$$

Ex. 9. $A = \begin{pmatrix}0&1\\0&0\end{pmatrix}$ $\det A = 0 - 1\cdot 0 = 0 \Rightarrow A$ n'est pas inversible.

$B = \begin{pmatrix}1&0&1\\0&-1&0\\1&0&0\end{pmatrix}$ $\det B = 0+0+0+1+0+0 = 1 \neq 0$

B est inversible.

$$\left(\underbrace{\begin{matrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{matrix}}_{B} \;\middle|\; \underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{\mathbb{1}}\right) \Longrightarrow \begin{array}{c} \mathrm{III} \\ -\mathrm{II} \\ \mathrm{I} \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \end{array}\right)$$

$$\Longrightarrow \begin{array}{c} \mathrm{I} \\ \\ \mathrm{III}-\mathrm{I} \end{array} \left(\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \;\middle|\; \underbrace{\begin{matrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{matrix}}_{B^{-1}}\right) \qquad B^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \qquad \det C = -1 + 0 + 0 + 1 + 0 + 0 = 0$$

$\Rightarrow$ C n'est pas inversible.

<u>Ex. 10</u> dépend des paramètres individuels.