```
with(LinearAlgebra);
```

```
[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm,
    BilinearForm, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension,
    ColumnOperation, ColumnSpace, CompanionMatrix, ConditionNumber, ConstantMatrix,
    ConstantVector, Copy, CreatePermutation, CrossProduct, DeleteColumn, DeleteRow,
    Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct,
    EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute,
    FrobeniusForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic,
    GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix,
    HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix,
    IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary,
    JordanBlockMatrix, JordanForm, KroneckerProduct, LA_Main, LUDecomposition,
    LeastSquares, LinearSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction,
    MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply,
    MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm,
    Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm,
    QRDecomposition, RandomMatrix, RandomVector, Rank, RationalCanonicalForm,
    ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix,
    ScalarMultiply, ScalarVector, SchurForm, SingularValues, SmithForm,
    StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis, SylvesterMatrix,
    ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix,
    VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply,
    ZeroMatrix, ZeroVector, Zip]                                                    (1)
```

```
> lucas := proc(n)
    local M, i, L:
    M := 2^n - 1 : L := 4 :
    for i from 1 to n - 2 do
    L := L^2 - 2 mod M: od;
    evalb(L = 0);
    end;
```

```
lucas := proc(n)                                                                    (2)
    local M, i, L;
    M := 2^n - 1; L := 4; for i to n - 2 do L := mod(L^2 - 2, M) end do; evalb(L = 0)
end proc
```

```
> lucas(126);
                                    false                                           (3)
```

```
> isprime(2^127 - 1);
                                    true                                            (4)
```

```
> M := RandomMatrix(2, 2);
```

(5)

$$M := \begin{bmatrix} 50 & -16 \\ 10 & -9 \end{bmatrix} \quad (5)$$

```
> Power(M, 10);
```

$$Power\left(\begin{bmatrix} -32 & -4 \\ -74 & 27 \end{bmatrix}, 10\right) \quad (6)$$

```
> ?Power
> puissance := proc(M, n, p)
    local x, y, m;
    m := n;
    x := copy(M);
    y := IdentityMatrix(2);
    while m ≠ 0 do
        if m mod 2 = 1 then y := x.y mod p :fi:
        x := x^2 mod p;
        m := floor(m/2);

    od:
    y;
    end:
```

```
> L := Matrix([[1, 0], [0, 2]]);
```

$$L := \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \quad (7)$$

```
> puissance(L, 5, 7);
```

$$\begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \quad (8)$$

```
> IdentityMatrix(2); a := rand(1..7); a;
```

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$a := \mathbf{proc}()\ \mathbf{proc}()\ \mathbf{option}\ builtin = RandNumberInterface;\ \mathbf{end\ proc}(6, 7, 3) + 1\ \mathbf{end\ proc}$$

$$a \quad (9)$$

```
>
> M^87484845;
Warning,  computation interrupted
> %mod 7;
```

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \quad (10)$$

```
> puissance(M, 8747784578777584845, 47);
```

$$\begin{bmatrix} 11 & 2 \\ 34 & 36 \end{bmatrix} \tag{11}$$

```
> lucastest := proc(N, k, L)
  local S, D, P, W, i, j, C, n, A, B, v, X, V;
  i := 0 : n := nops(L) : W := Transpose(Matrix([1, 0])) :
   while i < k do:
  S := RandomTools[Generate](integer(range = 1 ..N)) : P
       := RandomTools[Generate](integer(range = 1 ..N)) :
  D := S^2 - 4·P: A := Matrix([[S, 1], [-P, 0]]) : X := puissance(A, N, N) : V := X.W:
  if igcd(N, D·P) ≠ 1 or V[1, 1] mod N ≠ 0 then i := i + 1 :
  else j := 1 : while j ≤ n do B := puissance(A, (N + 1)/L[j] - 1, N) : C := B.W:
      if igcd(C[1, 1], N) = 1 then j := j + 1 : else j := n + 15 :fi:od:
  if j = n + 1 then return(premier);
  else i := i + 1 :fi:
  fi:
  od:
  if i = k then return(pas premier);
  fi:
  end:
```

```
> L := [2, 3, 7, 5, 47] : isprime((3·7·27·25·47·81·16) - 1);
```

$$false \tag{12}$$

```
> debug(lucastest); lucastest((3·7·27·25·47·81·16) - 1, 4, L);
```

$$lucastest$$

```
{--> enter lucastest, args = 863427599, 4, [2, 3, 7, 5, 47]
```

$$i := 0$$

$$n := 5$$

$$W := \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$S := 102723287$$

$$P := 316773881$$

$$D := 10552072424988845$$

$$A := \begin{bmatrix} 102723287 & 1 \\ -316773881 & 0 \end{bmatrix}$$

$$X := \begin{bmatrix} 120879045 & 781481943 \\ 728907116 & 639254715 \end{bmatrix}$$

$$V := \begin{bmatrix} 120879045 \\ 728907116 \end{bmatrix}$$

$$i := 1$$

$$S := 388203733$$

$$P := 544418953$$

$$D := 150702136137459477$$

$$A := \begin{bmatrix} 388203733 & 1 \\ -544418953 & 0 \end{bmatrix}$$

$$X := \begin{bmatrix} 351196014 & 783481289 \\ 264156957 & 269067724 \end{bmatrix}$$

$$V := \begin{bmatrix} 351196014 \\ 264156957 \end{bmatrix}$$

$$i := 2$$

$$S := 366222185$$

$$P := 249861828$$

$$D := 134118687786726913$$

$$A := \begin{bmatrix} 366222185 & 1 \\ -249861828 & 0 \end{bmatrix}$$

$$X := \begin{bmatrix} 189690468 & 418613806 \\ 778814241 & 334473618 \end{bmatrix}$$

$$V := \begin{bmatrix} 189690468 \\ 778814241 \end{bmatrix}$$

$$i := 3$$

$$S := 76338301$$

$$P := 5441983$$

$$D := 5827536177798669$$

$$A := \begin{bmatrix} 76338301 & 1 \\ -5441983 & 0 \end{bmatrix}$$

$$X := \begin{bmatrix} 165311200 & 7023668 \\ 394526487 & 7540548 \end{bmatrix}$$

$$V := \begin{bmatrix} 165311200 \\ 394526487 \end{bmatrix}$$

$$i := 4$$

```
<-- exit lucastest (now at top level) = `*`(pas, `*`(premier))}
```

$$pas\ premier \tag{13}$$

```
> L := [2, 3, 17] : lucastest(101, 5, L);
{--> enter lucastest, args = 101, 5, [2, 3, 17]
```

$$i := 0$$

$$n := 3$$

$$W := \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$S := 100$$

$$P := 70$$

$$D := 9720$$

$$A := \begin{bmatrix} 100 & 1 \\ -70 & 0 \end{bmatrix}$$

$$X := \begin{bmatrix} 100 & 1 \\ 31 & 0 \end{bmatrix}$$

$$V := \begin{bmatrix} 100 \\ 31 \end{bmatrix}$$

$$i := 1$$

$$S := 36$$

$$P := 36$$

$$D := 1152$$

$$A := \begin{bmatrix} 36 & 1 \\ -36 & 0 \end{bmatrix}$$

$$X := \begin{bmatrix} 0 & 100 \\ 36 & 36 \end{bmatrix}$$

$$V := \begin{bmatrix} 0 \\ 36 \end{bmatrix}$$

$$j := 1$$

$$B := \begin{bmatrix} 0 & 17 \\ 95 & 95 \end{bmatrix}$$

$$C := \begin{bmatrix} 0 \\ 95 \end{bmatrix}$$

$$j := 18$$

$$i := 2$$

$$S := 10$$

$$P := 40$$

$$\mathrm{D} := -60$$

$$A := \begin{bmatrix} 10 & 1 \\ -40 & 0 \end{bmatrix}$$

$$X := \begin{bmatrix} 0 & 100 \\ 40 & 10 \end{bmatrix}$$

$$V := \begin{bmatrix} 0 \\ 40 \end{bmatrix}$$

$$j := 1$$

$$B := \begin{bmatrix} 43 & 18 \\ 88 & 65 \end{bmatrix}$$

$$C := \begin{bmatrix} 43 \\ 88 \end{bmatrix}$$

$$j := 2$$

$$B := \begin{bmatrix} 19 & 82 \\ 53 & 7 \end{bmatrix}$$

$$C := \begin{bmatrix} 19 \\ 53 \end{bmatrix}$$

$$j := 3$$

$$B := \begin{bmatrix} 19 & 4 \\ 42 & 80 \end{bmatrix}$$

$$C := \begin{bmatrix} 19 \\ 42 \end{bmatrix}$$

$$j := 4$$

```
<-- exit lucastest (now at top level) = premier}
```

$$premier \qquad \textbf{(14)}$$

>