Algorithme de Gosper

```
> restart;with(sumtools,gosper);
```

[gosper]

Pas 2

```
>
> pas2:=proc(r)
  local L,f,g,N,M,G,R,S,s,a,b,i,l,k,u,c,j,p,q,d;
  f:=numer(r);g:=denom(r);
  G:=subs(n=n+h,g);
  L:=NULL;
  R:=resultant(f,G,n);
  S:={solve(R=0,h)};s:=nops(S);
  for i from 1 to s do
       if type(S[i],integer) and S[i]>=0 then L:=L,S[i];fi;
  od;
  L:=[L];
  l:=nops(L);
  p[0]:=f;q[0]:=g;
  for j from 1 to l do
       s[j]:=gcd(p[j-1],subs(n=n+L[j],q[j-1]));
       p[j]:=simplify(p[j-1]/s[j]);
       q[j]:=simplify(q[j-1]/subs(n=n-L[j],s[j]));od;
  a:=p[l];
  b:=q[l];
       for i from 1 to l do min(1,L[i]);
            d[i]:=product(subs(n=n-k,s[i]),k=1..L[i]);od;
       c:=product(d[m],m=1..l);
  return a,b,c;
  end proc:
> pas2((n+3)/(n*(n+1)));
```

$$1, n, (n+2)(n+1)$$

```
> pas2(((n+1)^2+n+2)/(n^2+n+1));pas2(n/(n+2));
```

$$1, 1, (n-1)^2+3n$$

$$n, n+2, 1$$

```
>
```

Pas 3

```
> degsol:=proc(a,b,c)
  local da,db,dc,d1,d2,d,A,B;
  da:=degree(a,n);
```

```
   db:=degree(b,n);
   dc:=degree(c,n);
   A:=coeff(a,n,da-1);
   B:=coeff(subs(n=n-1,b),n,db-1);
   d1:=-1;
   d2:=(B-A)/coeff(a,n,da);
   if (da<>db) or (coeff(a,n,da)<>coeff(b,n,db)) then
        d:=dc-max(da,db);
   else
        if (dc-da+1)*coeff(a,n,da)+A-B<>0 then
             d1:=dc-da+1;
        fi;
        d:=max(d1,d2);
   fi;
   return d;
   end proc:
> degsol(n,n+2,1);
```

$$1$$

```
> degsol(1, 1, (n-1)^2+3*n);
```

$$3$$

```
> pas3:=proc(a,b,c,d)
   local X,x,i,S,sol,k;
   X:=sum(x[i]*n^i,i=0..d);
   S:=solve(identity(a*subs(n=n+1,X)-subs(n=n-1,b)*X = c, n),
   {seq(x[i],i=0..d)});
   if nops([S])=0 then return(fail) fi;
   assign(S);
   for i from 0 to d do
        if type(x[i],rational) then x[i]:=x[i];
        else x[i]:=0;
        fi;
   od;
   return sum(x[k]*n^k,k=0..d);
   end proc:
>
> pas3(n,n+2,1,1);
```

$$-1$$

```
> pas3(1, 1, 3*n+(n-1)^2,3);
```

$$\frac{2}{3}n+\frac{1}{3}n^3$$

```
> gosp:=proc(t)
  local r,a,b,c,d,x,s,p;
  r:=simplify(subs(n=n+1,t)/t);
  a,b,c:=pas2(r);
  d:=degsol(a,b,c);
  if d<0 then return Sum(subs(n=k,t),'k'=1..n-1) ;
```

```
  else x:=pas3(a,b,c,d);
       if x=fail then return Sum(subs(n=k,t),'k'=1..n-1) fi;
       s:=subs(n=n-1,b)*x*t/c;
       return factor(simplify(s-subs(n=1,s)));
  fi;
  end proc:
> gosp(1/n);
```

$$\sum_{k=1}^{n-1} \frac{1}{k}$$

```
> pas3(n,n+1,1,0);
```

fail

```
> gosp(n^2+n+1);
```

$$\frac{1}{3}(n-1)(n^2+n+3)$$

```
> gosper(n^2+n+1,n);
```

$$\frac{1}{3}(n-1)(n^2+n+3)$$

```
> gosper(1/(n*(n+1)),n);
```

$$\frac{n-1}{n}$$

```
> gosp(1/(n*(n+1)));
```

$$\frac{n-1}{n}$$

```
> gosp(n!);
```

$$\sum_{k=1}^{n-1} k!$$

```
> simplify((n+1)*(n+1)!/(n*n!));
```

$$\frac{(n+1)^2}{n}$$

```
> binomial(n,k);
```

binomial(n, k)

```
> convert(%,factorial);
```

$$\frac{n!}{k!\,(n-k)!}$$

```
> gosp(convert(binomial(m,n),factorial));
```

$$\sum_{k=1}^{n-1} \frac{m!}{k!\,(m-k)!}$$

Exemple

```
> with(SumTools[Hypergeometric]):
> t:=(-1)^k*binomial(2*n,k)^3;
```

$$(-1)^k \operatorname{binomial}(2\,n,\,k)^3$$

```
> convert(t,factorial);
```

$$\frac{(-1)^k\,((2\,n)!)^3}{(k!)^3\,((2\,n-k)!)^3}$$

```
>
> a1:=simplify(convert(subs(n=n+1,t)/t,factorial));
```

$$\frac{8\,(2\,n+1)^3\,(n+1)^3}{(-2\,n+k-1)^3\,(-2\,n-2+k)^3}$$

```
>
> tt := simplify ( (b0 + b1 * a1 ) * ( denom (a1) ) ) ;
```

$$\begin{aligned}&378\,b0\,n\,k^2-192\,b0\,n^5\,k-160\,b0\,n^3\,k^3-1056\,b0\,n^3\,k+720\,b0\,n^3\,k^2-720\,b0\,n^4\,k\\&\quad+792\,b0\,n^2\,k^2-756\,b0\,n^2\,k-264\,b0\,n\,k-264\,b0\,n\,k^3+90\,b0\,n\,k^4+60\,b0\,n^2\,k^4\\&\quad+8\,b0+8\,b1+72\,b0\,n-36\,b0\,k+264\,b0\,n^2+504\,b0\,n^3-63\,b0\,k^3+64\,b0\,n^6\\&\quad+288\,b0\,n^5+528\,b0\,n^4+33\,b0\,k^4-9\,b0\,k^5+b0\,k^6+66\,b0\,k^2+64\,b1\,n^6\\&\quad+288\,b1\,n^5+528\,b1\,n^4+504\,b1\,n^3+264\,b1\,n^2+72\,b1\,n-12\,b0\,n\,k^5-360\,b0\,n^2\,k^3\\&\quad+240\,b0\,n^4\,k^2\end{aligned}$$

```
> p := map ( factor , collect ( tt , [ b0, b1 ] , distributed )
  ) ;
```

$$(-2\,n+k-1)^3\,(-2\,n-2+k)^3\,b0+8\,(n+1)^3\,(2\,n+1)^3\,b1$$

```
> s := s4 * k^4 + s3 * k^3 + s2 * k^2 + s1 * k + s0 ;
```

$$s4\,k^4+s3\,k^3+s2\,k^2+s1\,k+s0$$

```
>
> Eq :=map ( factor , collect (-(2*n+2-k)^3*subs ( k = k + 1 ,
  s )-k^3*s-p   , k )   ) ;
```

$$\begin{aligned}&-(6\,s4\,n+2\,s4+b0)\,k^6+3\,(4\,b0\,n+3\,b0+4\,s4\,n^2-2\,s4-2\,n\,s3-s3)\,k^5-(8\,s4\,n^3\\&\quad-24\,s4\,n^2-36\,s4\,n-8\,s4-12\,n^2\,s3-6\,n\,s3+3\,s3+6\,n\,s2+4\,s2+90\,b0\,n+60\,b0\,n^2\\&\quad+33\,b0)\,k^4+(-6\,n\,s1-24\,s4\,n^2+30\,n\,s3+24\,s4\,n+63\,b0-8\,n^3\,s3+264\,b0\,n\\&\quad+360\,b0\,n^2+160\,b0\,n^3-5\,s1+s2+12\,n\,s2-32\,s4\,n^3+12\,n^2\,s3+17\,s4+11\,s3\\&\quad+12\,n^2\,s2)\,k^3-2\,(n+1)\,(120\,b0\,n^3+240\,b0\,n^2+24\,s4\,n^2+12\,n^2\,s3+4\,n^2\,s2\\&\quad+24\,s4\,n-6\,n\,s1+6\,n\,s3+156\,b0\,n-4\,n\,s2-5\,s2+33\,b0+3\,s4-3\,s3+3\,s0-3\,s1)\\&\quad k^2+4\,(n+1)^2\,(48\,b0\,n^3+84\,b0\,n^2-2\,n\,s1-8\,s4\,n-6\,n\,s3+48\,b0\,n-4\,n\,s2+s1\\&\quad+3\,s0-s2-5\,s4+9\,b0-3\,s3)\,k-8\,(n+1)^3\,(s4+s0+s1+s3+s2)\\&\quad-8\,(2\,n+1)^3\,(n+1)^3\,b0-8\,(n+1)^3\,(2\,n+1)^3\,b1\end{aligned}$$

```
> l := seq ( coeff ( Eq , k , i ) = 0 , i = 0 .. 6 ) ;
```

$$-8\,(n+1)^3\,(s4+s0+s1+s3+s2)-8\,(2\,n+1)^3\,(n+1)^3\,b0-8\,(n+1)^3\,(2\,n+1)^3\,b1=0$$
$$,\ 4\,(n+1)^2\,(48\,b0\,n^3+84\,b0\,n^2-2\,n\,s1-8\,s4\,n-6\,n\,s3+48\,b0\,n-4\,n\,s2+s1+3\,s0$$
$$-s2-5\,s4+9\,b0-3\,s3)=0,\ -2\,(n+1)\,(120\,b0\,n^3+240\,b0\,n^2+24\,s4\,n^2+12\,n^2\,s3$$
$$+4\,n^2\,s2+24\,s4\,n-6\,n\,s1+6\,n\,s3+156\,b0\,n-4\,n\,s2-5\,s2+33\,b0+3\,s4-3\,s3+3\,s0$$
$$-3\,s1)=0,\ -6\,n\,s1-24\,s4\,n^2+30\,n\,s3+24\,s4\,n+63\,b0-8\,n^3\,s3+264\,b0\,n$$
$$+360\,b0\,n^2+160\,b0\,n^3-5\,s1+s2+12\,n\,s2-32\,s4\,n^3+12\,n^2\,s3+17\,s4+11\,s3$$
$$+12\,n^2\,s2=0,\ -8\,s4\,n^3+24\,s4\,n^2+36\,s4\,n+8\,s4+12\,n^2\,s3+6\,n\,s3-3\,s3-6\,n\,s2$$
$$-4\,s2-90\,b0\,n-60\,b0\,n^2-33\,b0=0,$$
$$12\,b0\,n+9\,b0+12\,s4\,n^2-6\,s4-6\,n\,s3-3\,s3=0,\ -6\,s4\,n-2\,s4-b0=0$$

```
> solve ( { l } , { b0 , b1 , s0 , s1 , s2 , s3 , s4 } ) ;
```

$$\left\{ b0=-6\,s4\,n-2\,s4,\ b1=-\frac{\frac{2}{3}\,s4\,(n^2+1+2\,n)}{3\,n+2},\right.$$
$$s0=\frac{\frac{4}{3}\,s4\,(112\,n^5+440\,n^4+682\,n^3+521\,n^2+196\,n+29)}{3\,n+2},$$
$$s1=-\frac{s4\,(208\,n^4+644\,n^3+738\,n^2+371\,n+69)}{3\,n+2},$$
$$\left. s2=\frac{s4\,(198\,n+49+116\,n^3+264\,n^2)}{3\,n+2},\ s3=-10\,s4\,n-8\,s4,\ s4=s4 \right\}$$

```
>
```