

# FFTetFiltres

```
def fftrec(L,W,p,n) :          # L : polynome sous forme de liste
    if n==1 :                 # W : liste des w^k
        return L
    m=n//2
    L0=[L[2*i] for i in range(0,m)]
    L1=[L[2*i+1] for i in range(0,m)]
    V0=fftrec(L0,W,2*p,m)
    V1=fftrec(L1,W,2*p,m)
    U=[0 for i in range(0,n)]
    for i in range(0,m) :
        U[i]=V0[i]+W[p*i]*V1[i]
        U[i+m]=V0[i]-W[p*i]*V1[i]
    return U
```

```
def fft(L,w,n) :
    m=n//2
    W=[1,w]
    for i in range(2,m) :
        W.append(w*W[i-1])
    return fftrec(L,W,1,n)
```

```
PI=pi.numerical_approx()
```

```
f(t)=sin(18*PI*t)+3*cos(10*PI*t)
```

```
PI
```

```
3.14159265358979
```

```
f(1)
```

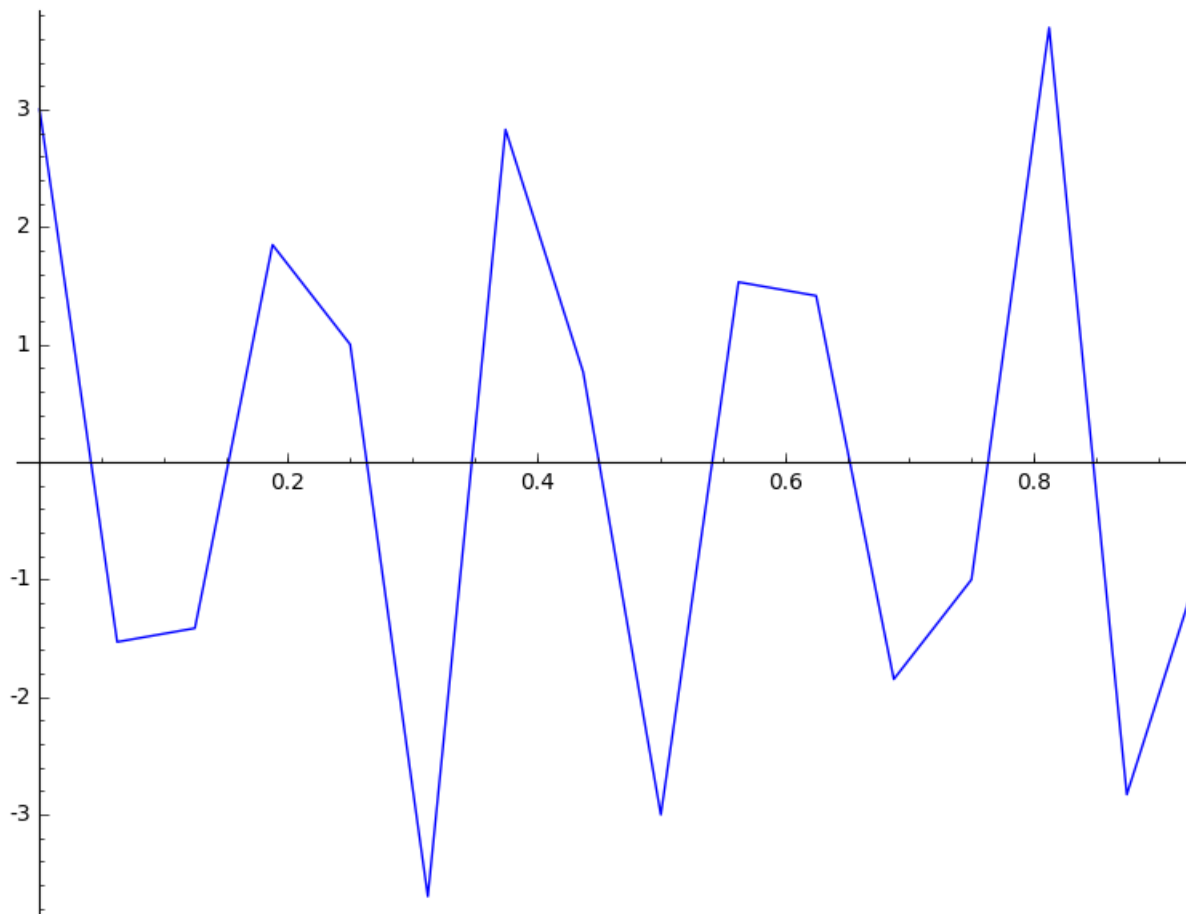
```
3.000000000000000
```

```
f(17)
```

```
3.000000000000000
```

```
F=[f(i/16) for i in range(16)]
```

```
line([(i/16,F[i]) for i in range(16)])
```



```
lF=line([(i/16,F[i]) for i in range(16)])
```

```
w=exp(2*I*pi/16).numerical_approx()
```

```
Fc=fft(F,w^(-1),16)
```

```
Fc
```

```
[-7.77156117237610e-16,
 3.10862446895044e-15 - 1.77635683940025e-15*I,
 5.87398095807803e-15 + 9.46749892661973e-15*I,
 -1.24344978758018e-14 + 7.99360577730113e-15*I,
 -9.32587340685131e-15 + 1.88737914186277e-15*I,
 24.00000000000000 - 3.15303338993544e-14*I,
 1.01132305965242e-14 + 3.25024998871885e-15*I,
 5.10702591327572e-15 + 8.000000000000001*I,
 -5.44009282066327e-15,
 -3.10862446895044e-15 - 8.000000000000001*I,
 1.01132305965242e-14 - 3.25024998871885e-15*I,
 24.00000000000000 + 7.99360577730113e-15*I,
 -9.32587340685131e-15 - 1.88737914186277e-15*I,
 -1.06581410364015e-14 + 4.44089209850063e-16*I,
 5.87398095807802e-15 - 9.46749892661973e-15*I,
 1.99840144432528e-15 + 7.10542735760100e-15*I]
```

```
m(t)=t/16
```

```
Fc=map(m,Fc)
```

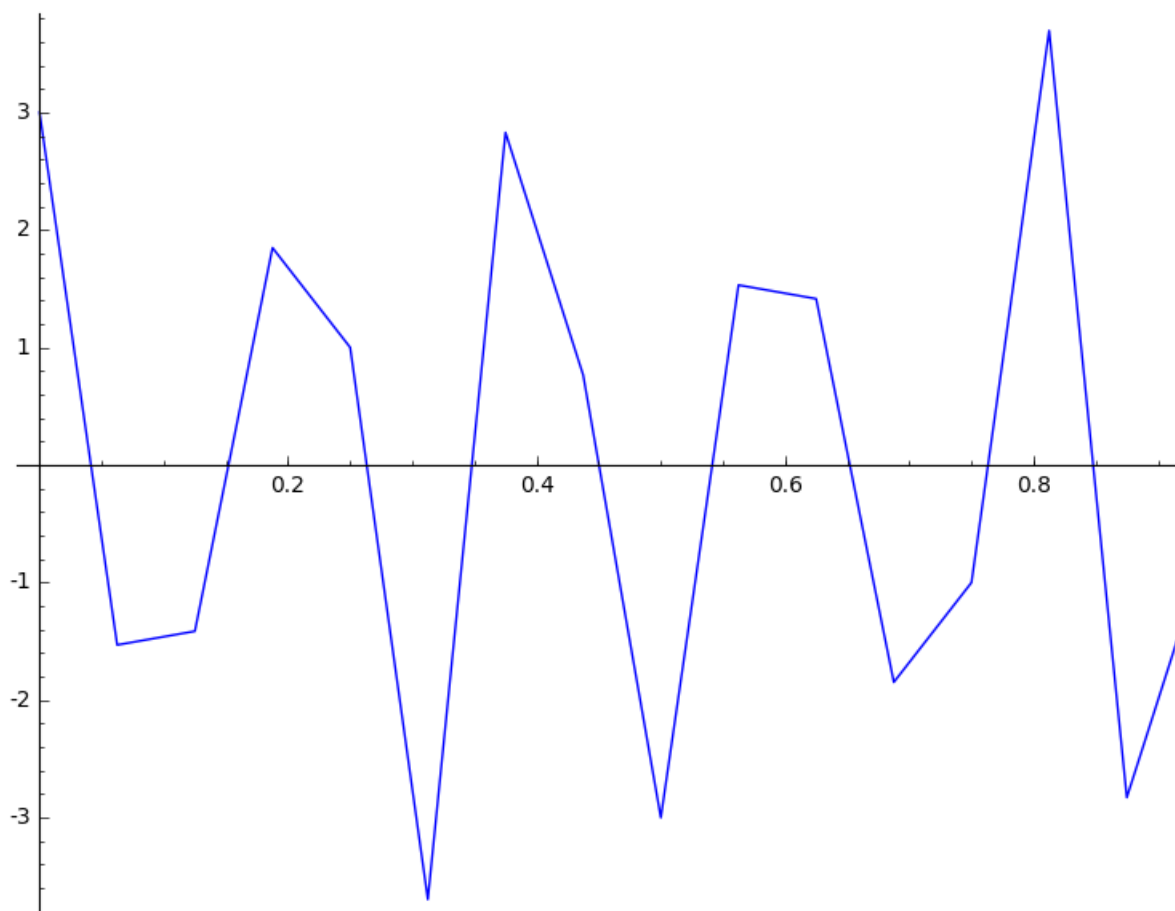
```
Fc
```

```
[-4.85722573273506e-17,
 1.94289029309402e-16 - 1.11022302462516e-16*I,
 3.67123809879877e-16 + 5.91718682913733e-16*I,
 -7.77156117237610e-16 + 4.99600361081320e-16*I,
 -5.82867087928207e-16 + 1.17961196366423e-16*I,
 1.500000000000000 - 1.97064586870965e-15*I,
 6.32076912282764e-16 + 2.03140624294928e-16*I,
 3.19189119579733e-16 + 0.500000000000001*I,
 -3.40005801291454e-16,
 -1.94289029309402e-16 - 0.500000000000000*I,
 6.32076912282764e-16 - 2.03140624294928e-16*I,
 1.500000000000000 + 4.99600361081320e-16*I,
 -5.82867087928207e-16 - 1.17961196366423e-16*I,
 -6.66133814775094e-16 + 2.77555756156289e-17*I,
 3.67123809879877e-16 - 5.91718682913733e-16*I,
 1.24900090270330e-16 + 4.44089209850063e-16*I]
```

```
tFc=fft(Fc,w,16)
```

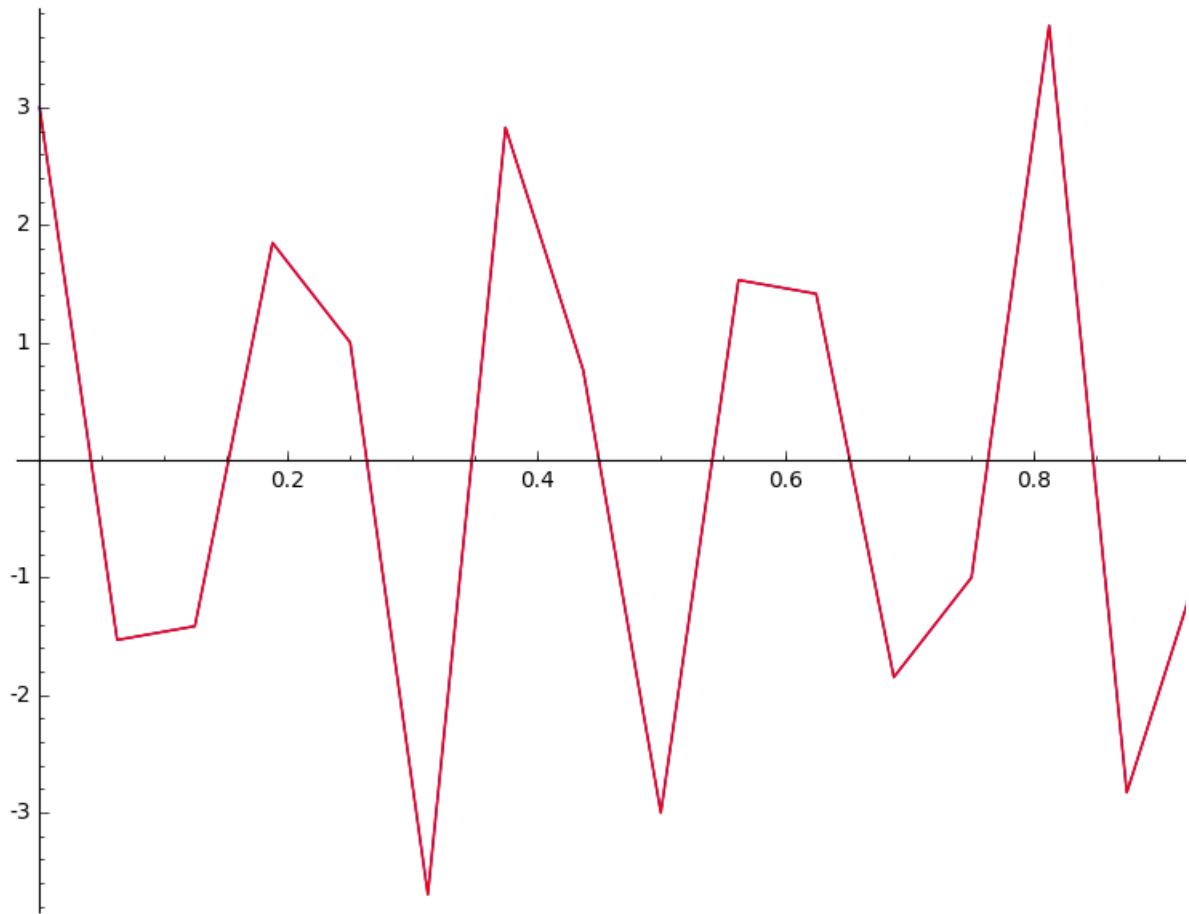
```
tFc=map(real,tFc)
```

```
line([(i/16,tFc[i]) for i in range(16)])
```

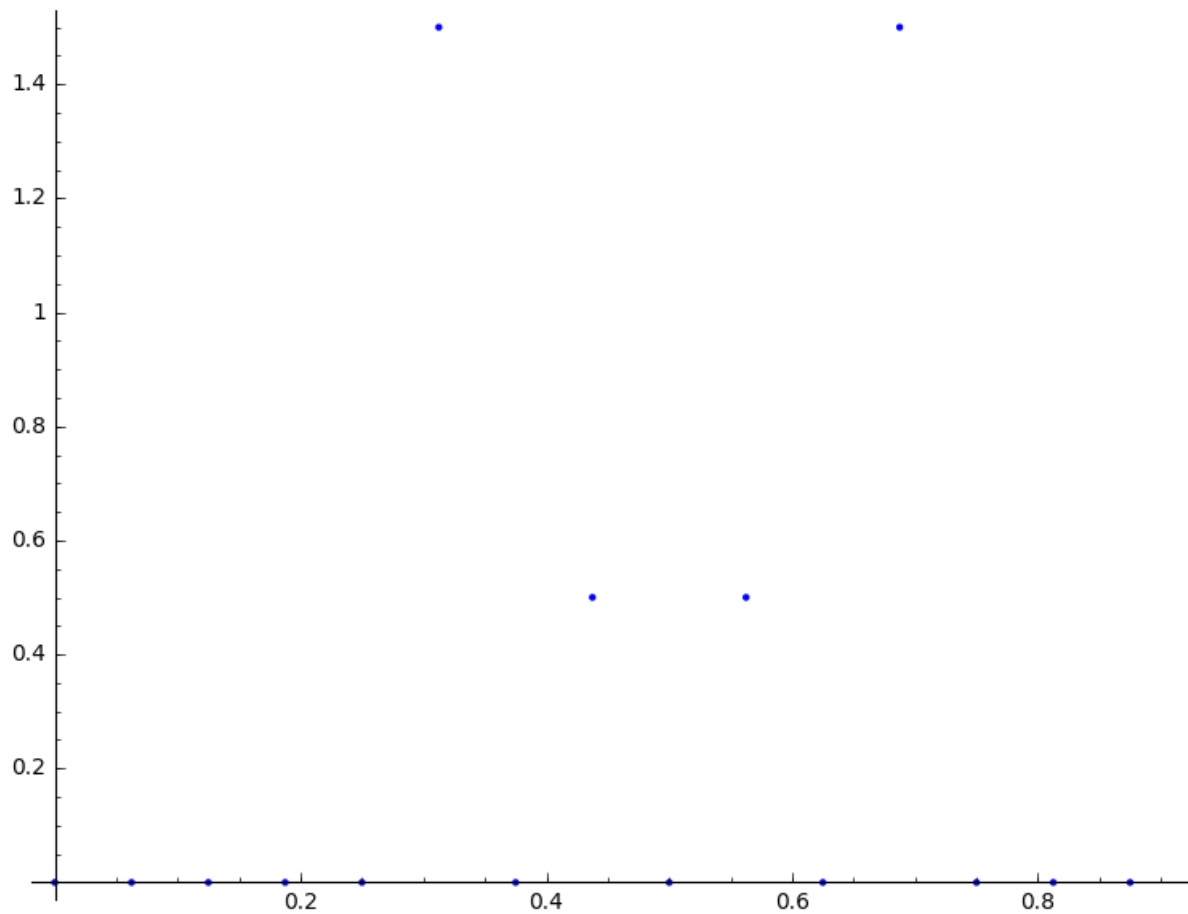


```
ltFc=line([(i/16,tFc[i]) for i in range(16)],color='red')
```

```
lF+ltFc
```



```
point([(i/16,map(abs,Fc)[i]) for i in range(16)])
```



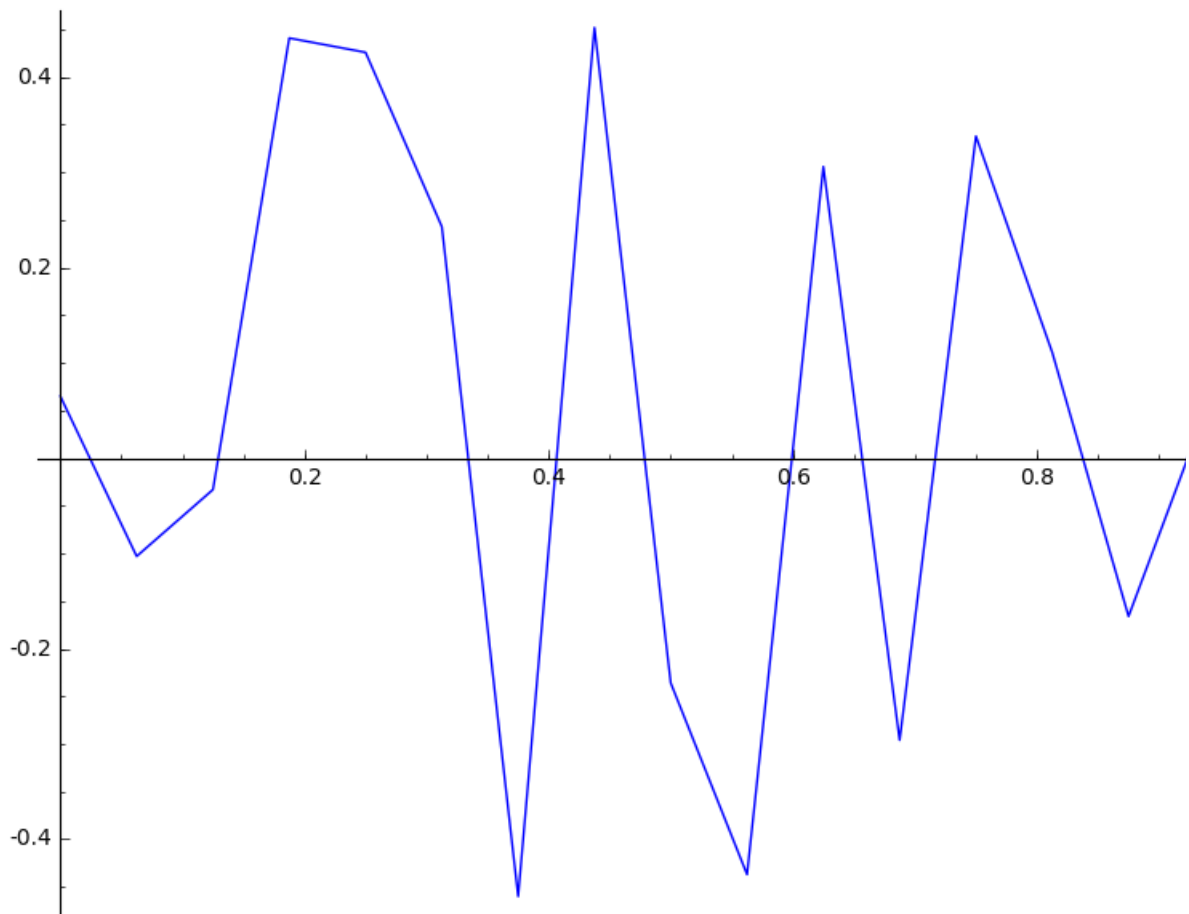
```
def h():
    return (ZZ.random_element(-500,500)/1000).numerical_approx()
```

```
B=[h() for i in range(16)]
```

```
B
```

```
[0.06500000000000000,
-0.10300000000000000,
-0.03300000000000000,
0.44100000000000000,
0.42600000000000000,
0.24300000000000000,
-0.46000000000000000,
0.45200000000000000,
-0.23600000000000000,
-0.43700000000000000,
0.30600000000000000,
-0.29600000000000000,
0.33800000000000000,
0.11100000000000000,
-0.16600000000000000,
0.04800000000000000]
```

```
line([(i/16,B[i]) for i in range(16)])
```

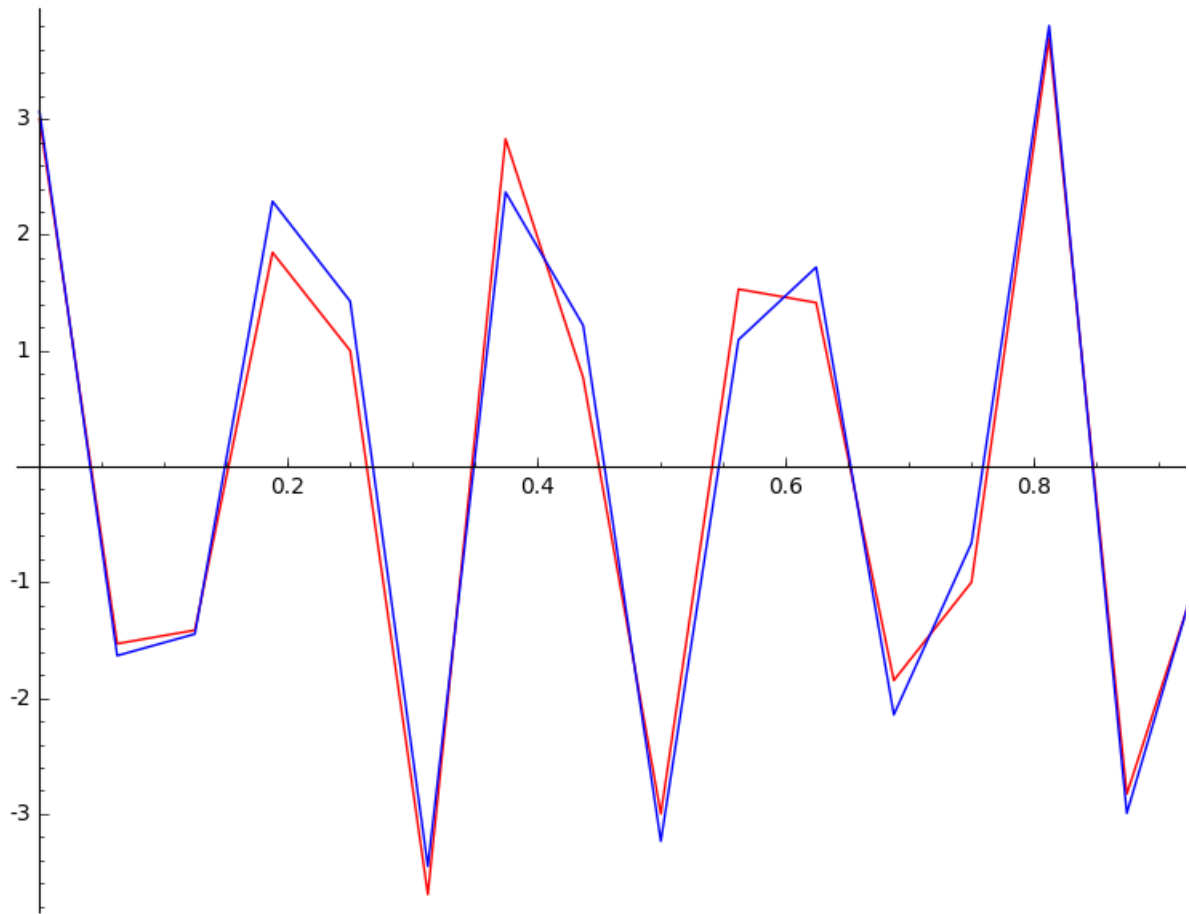


```
FB=[F[i]+B[i] for i in range(16)]
```

```
lFB=line([(i/16,FB[i]) for i in range(16)])
```

```
lF=line([(i/16,F[i]) for i in range(16)],color='red')
```

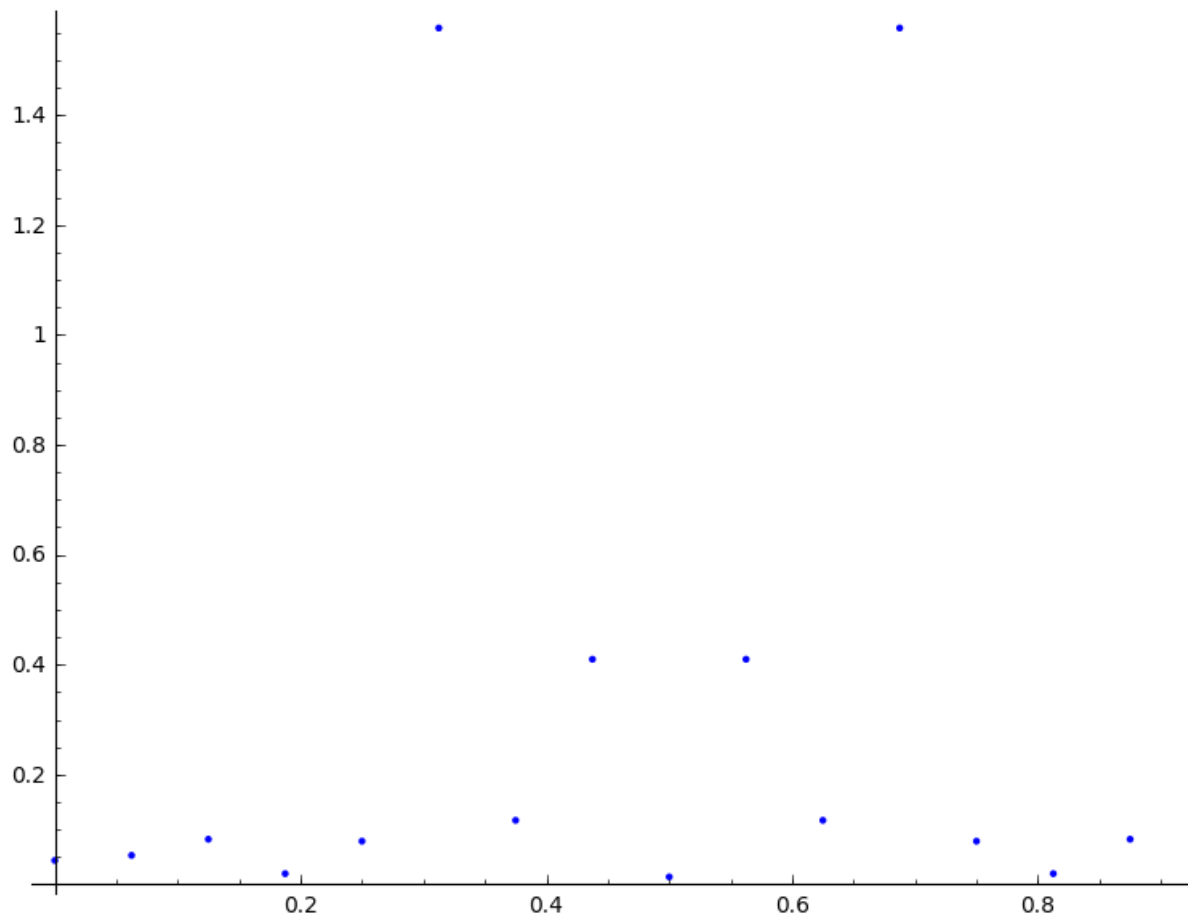
```
plot(lF+lFB)
```



```
FBC=fft(FB,w^(-1),16)
```

```
FBC=map(m,FBC)
```

```
point([(i/16,map(abs,FBC)[i]) for i in range(16)])
```

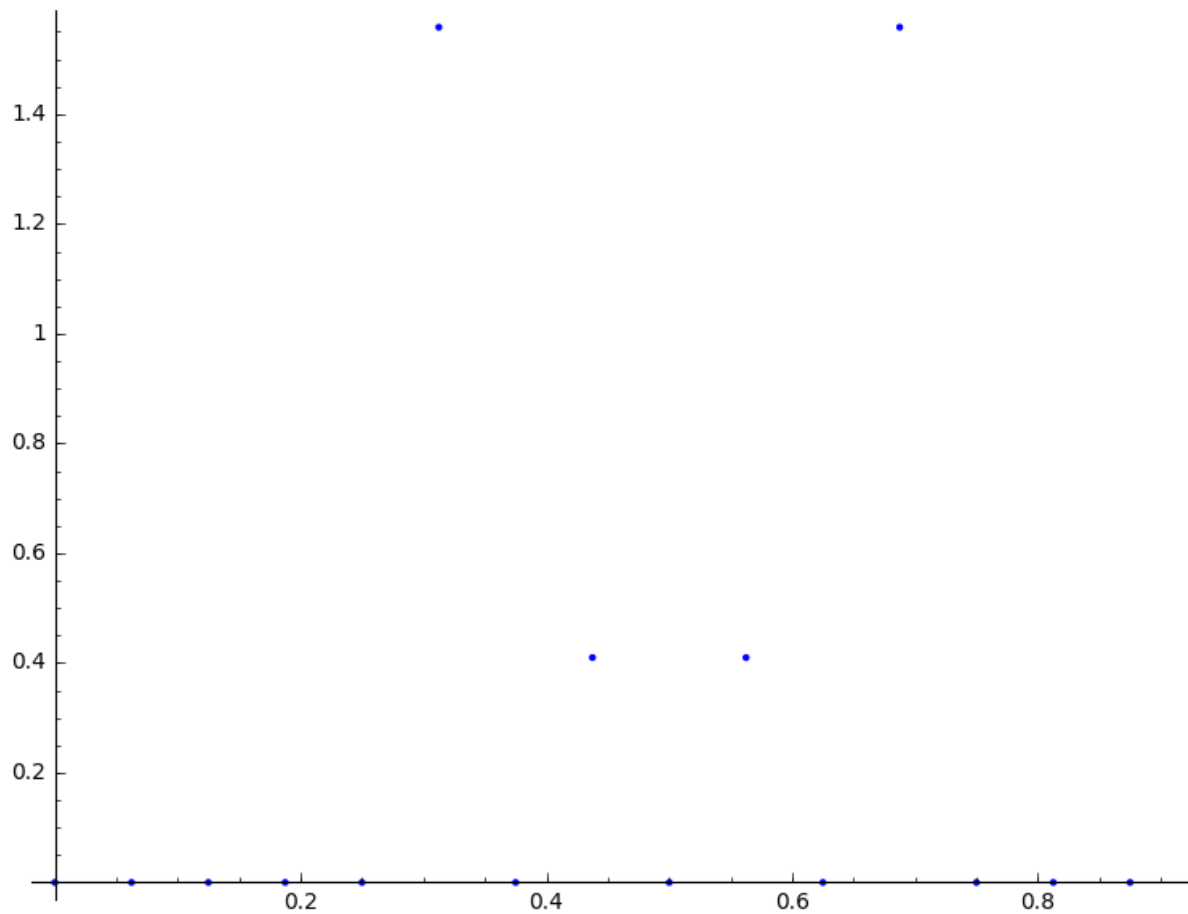


```
clean=FBc
```

```
for i in range(16):  
    if abs(FBc[i])<=1/4:  
        clean[i]=0
```

```
point([(i/16,map(abs,clean)[i]) for i in range(16)])
```





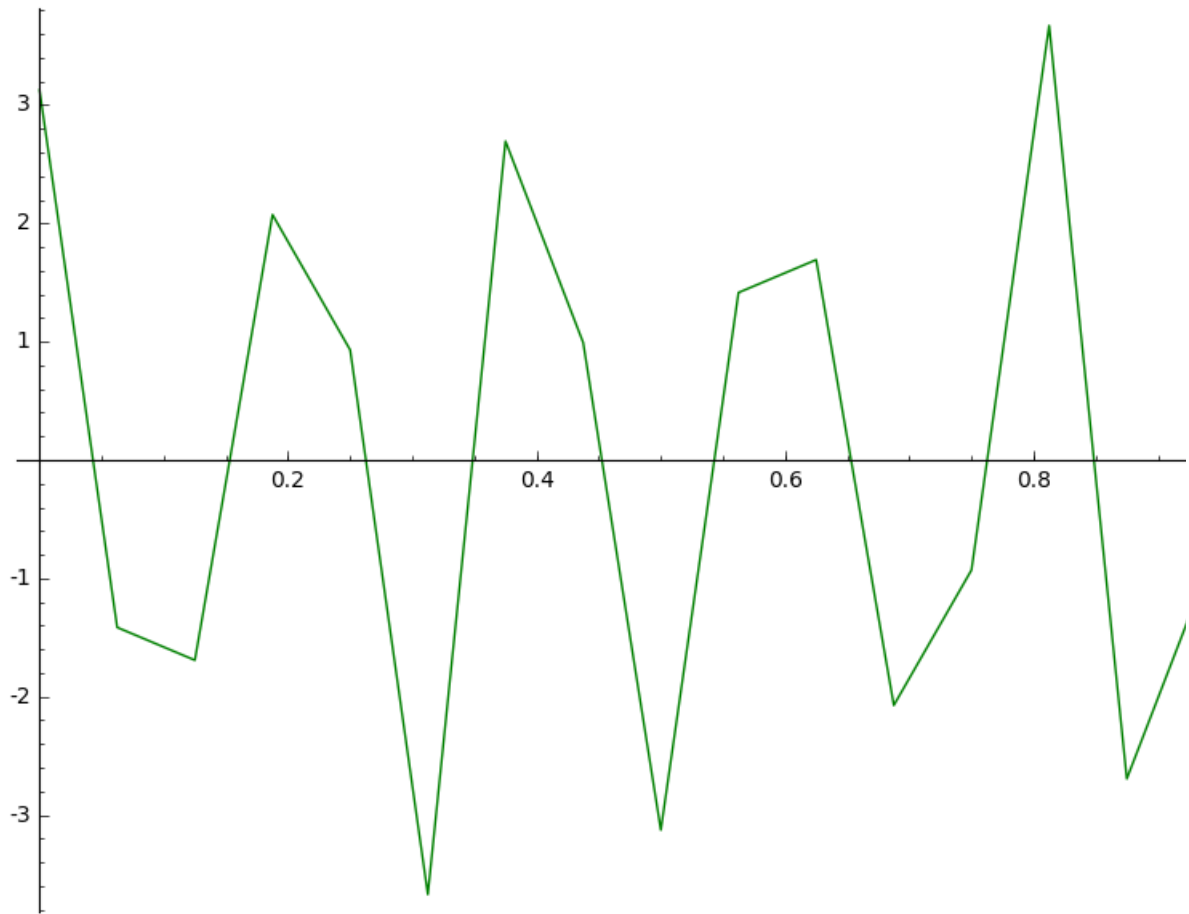
```
filtre=fft(clean,w,16)
```

```
filtre
```

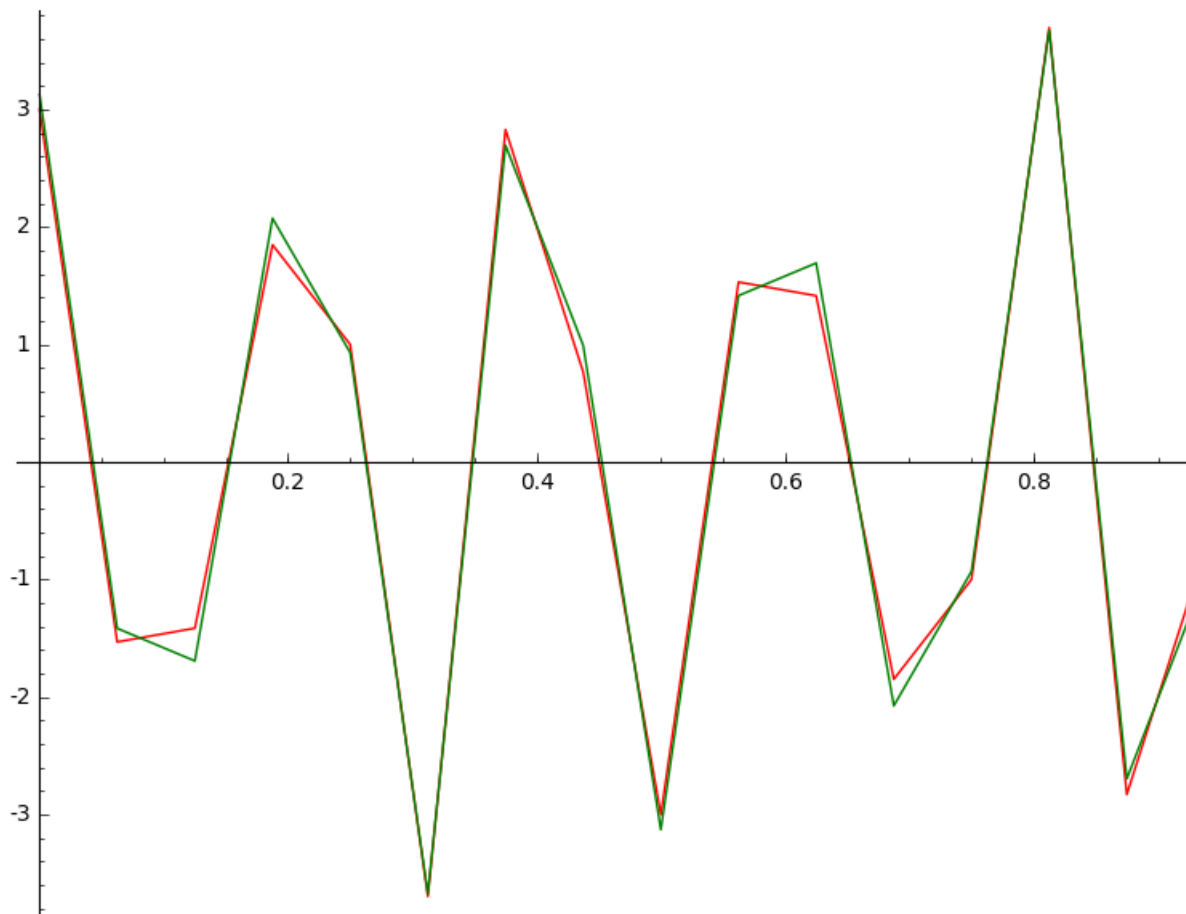
```
[3.12761038101623 - 9.99200722162641e-16*I,
-1.41518544159521 - 2.22044604925031e-16*I,
-1.69228034670089 + 1.11022302462516e-15*I,
2.07347602463320 + 2.22044604925031e-16*I,
0.929999938178794 - 1.34337006570242e-16*I,
-3.67216982171128 - 6.66133814775094e-16*I,
2.69465015964434 + 8.88178419700125e-16*I,
0.992428827088166 + 9.43689570931383e-16*I,
-3.12761038101623 + 9.99200722162641e-16*I,
1.41518544159521 + 2.22044604925031e-16*I,
1.69228034670089 - 1.11022302462516e-15*I,
-2.07347602463320 - 2.22044604925031e-16*I,
-0.929999938178794 + 1.34337006570242e-16*I,
3.67216982171128 + 6.66133814775094e-16*I,
-2.69465015964434 - 8.88178419700125e-16*I,
-0.992428827088166 - 9.43689570931383e-16*I]
```

```
lfiltre=line([(i/16,map(real,filtre)[i]) for i in range(16)],color='green')
```

```
lfiltre
```



lF+lfiltre



```
clean[5]
```

```
1.54687933252823 - 0.0680611490556840*I
```

```
Fc[5]
```

```
1.500000000000000 - 1.97064586870965e-15*I
```

```
f(t)=sin(18*PI*t)+3*cos(10*PI*t)
```

```
x=[(i/1024).numerical_approx() for i in range(1024)]
```

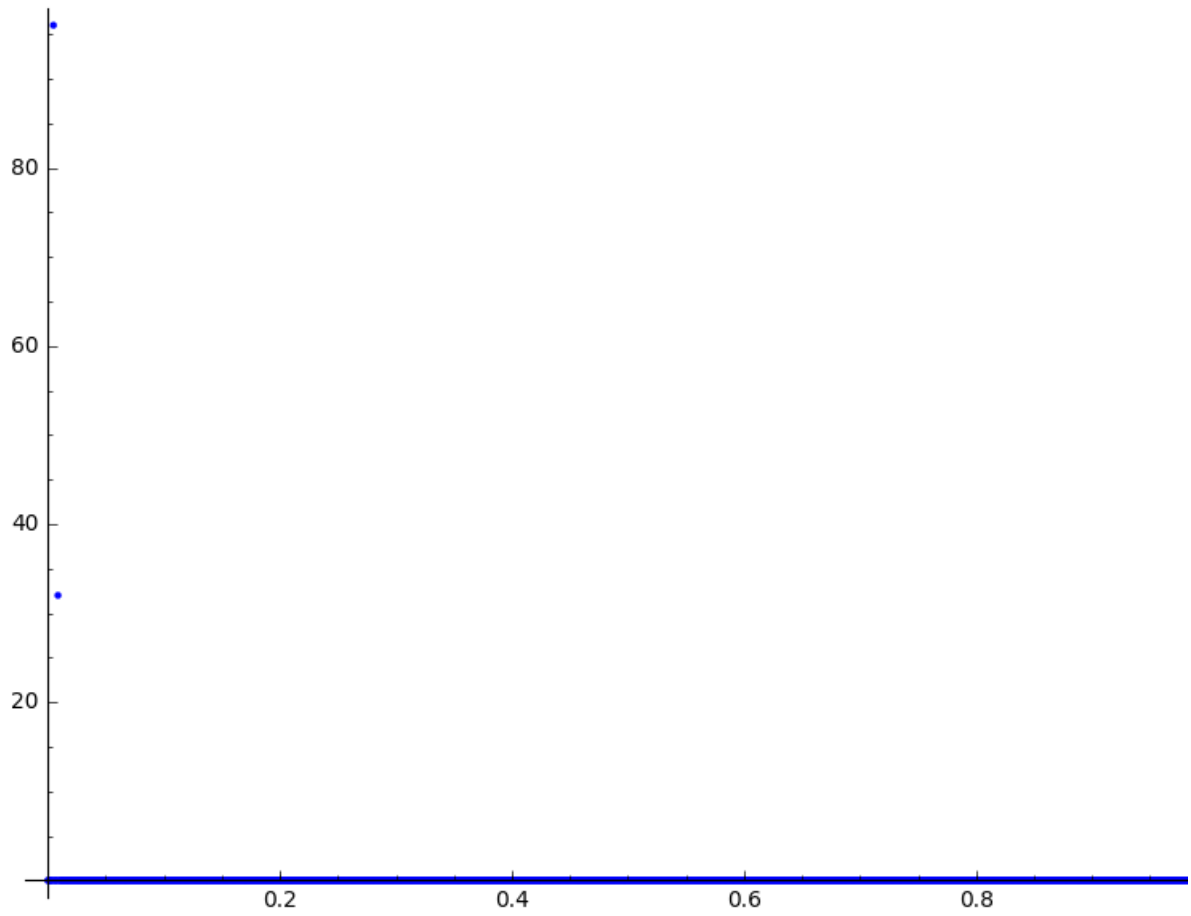
```
F=map(f,x)
```

```
w=exp(2*I*PI/1024)
```

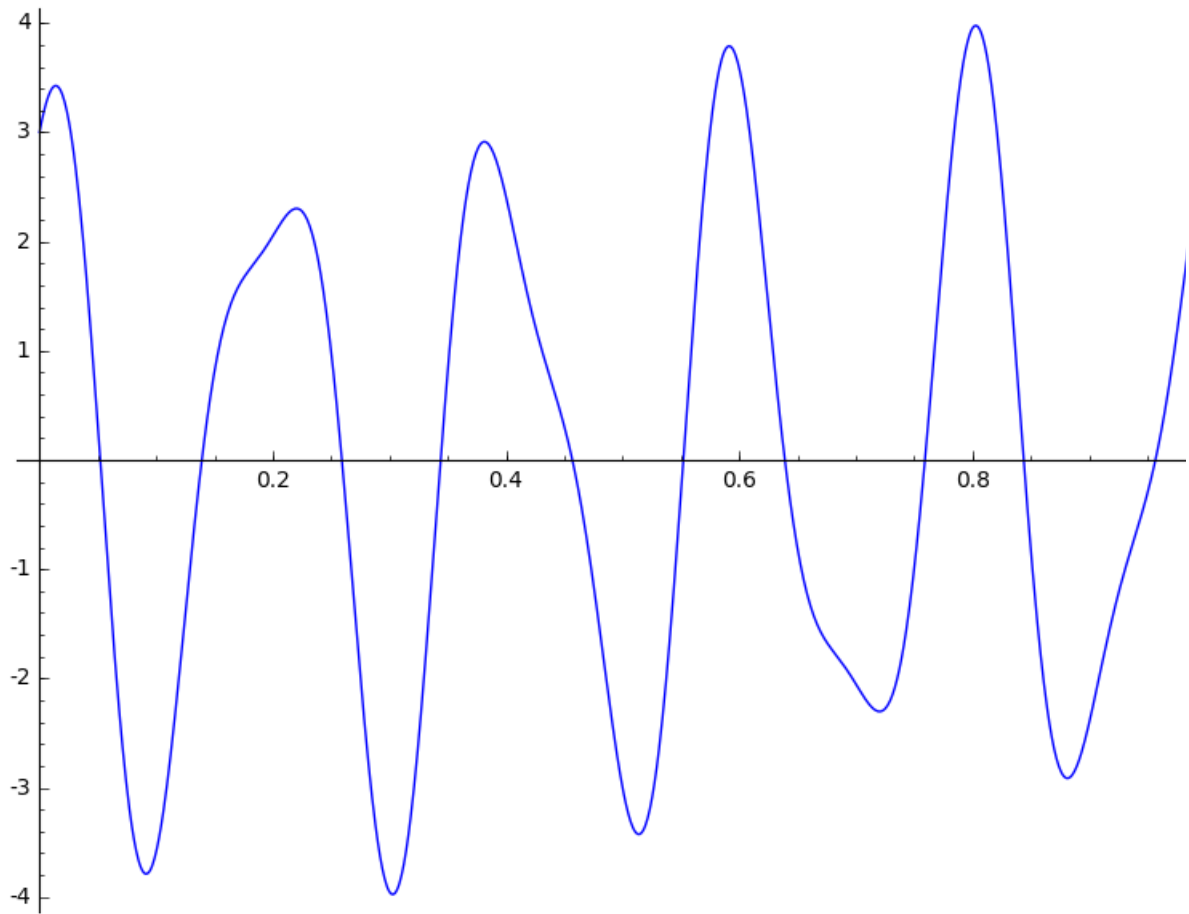
```
Fc=fft(F,w^-1,1024)
```

```
Fc=map(m,Fc)
```

```
point(zip(x,map(abs,Fc)))
```

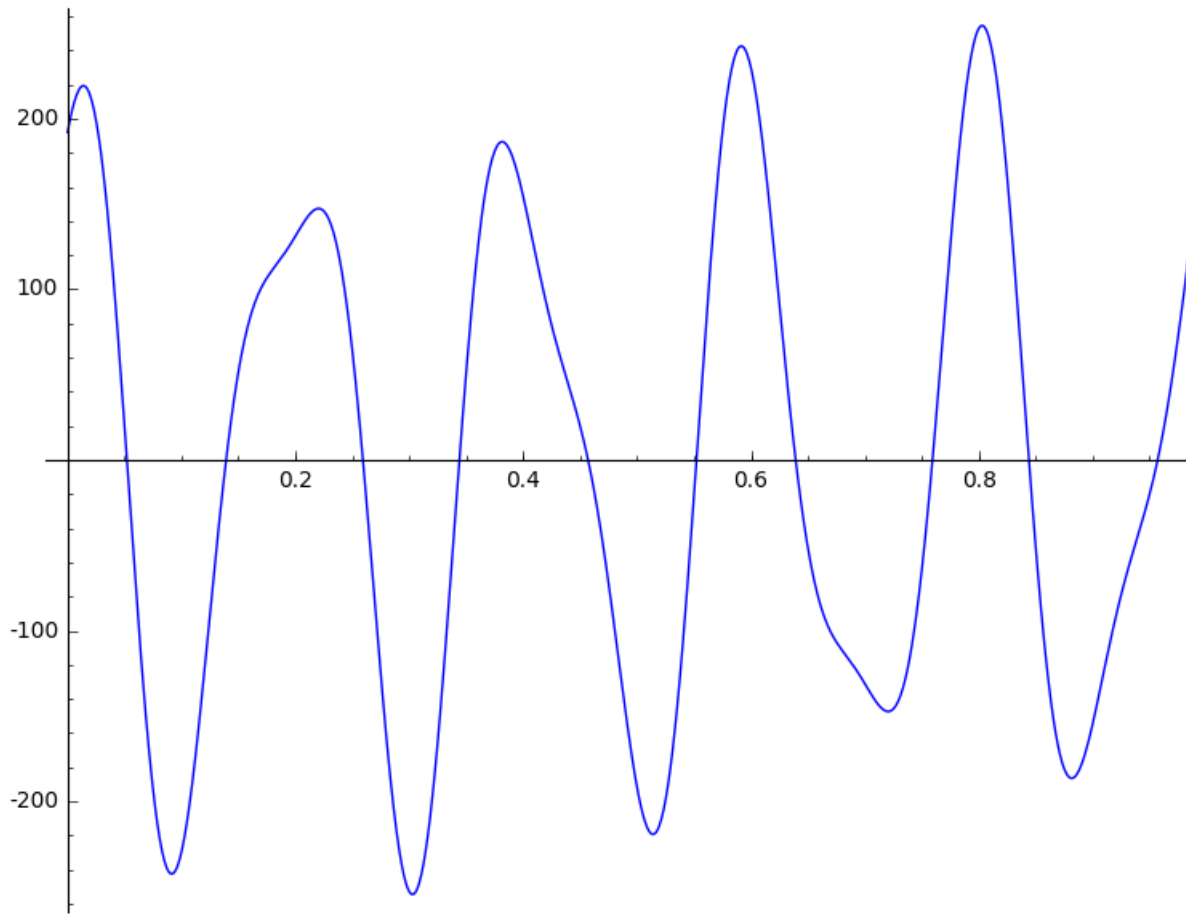


```
line(zip(x, F))
```



```
tFc=fft(Fc,w,1024)
```

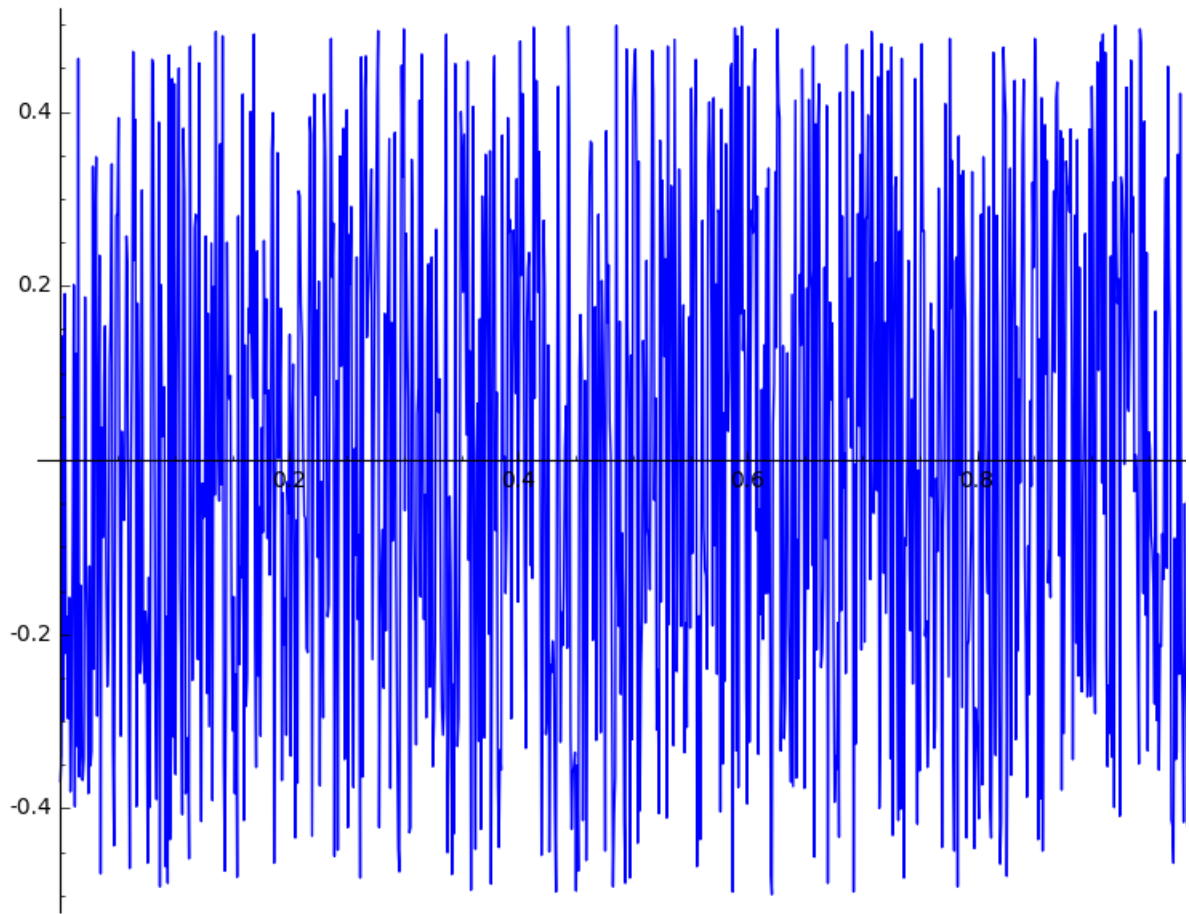
```
line(zip(x,map(real,tFc)))
```



```
m(t)=t/1024
```

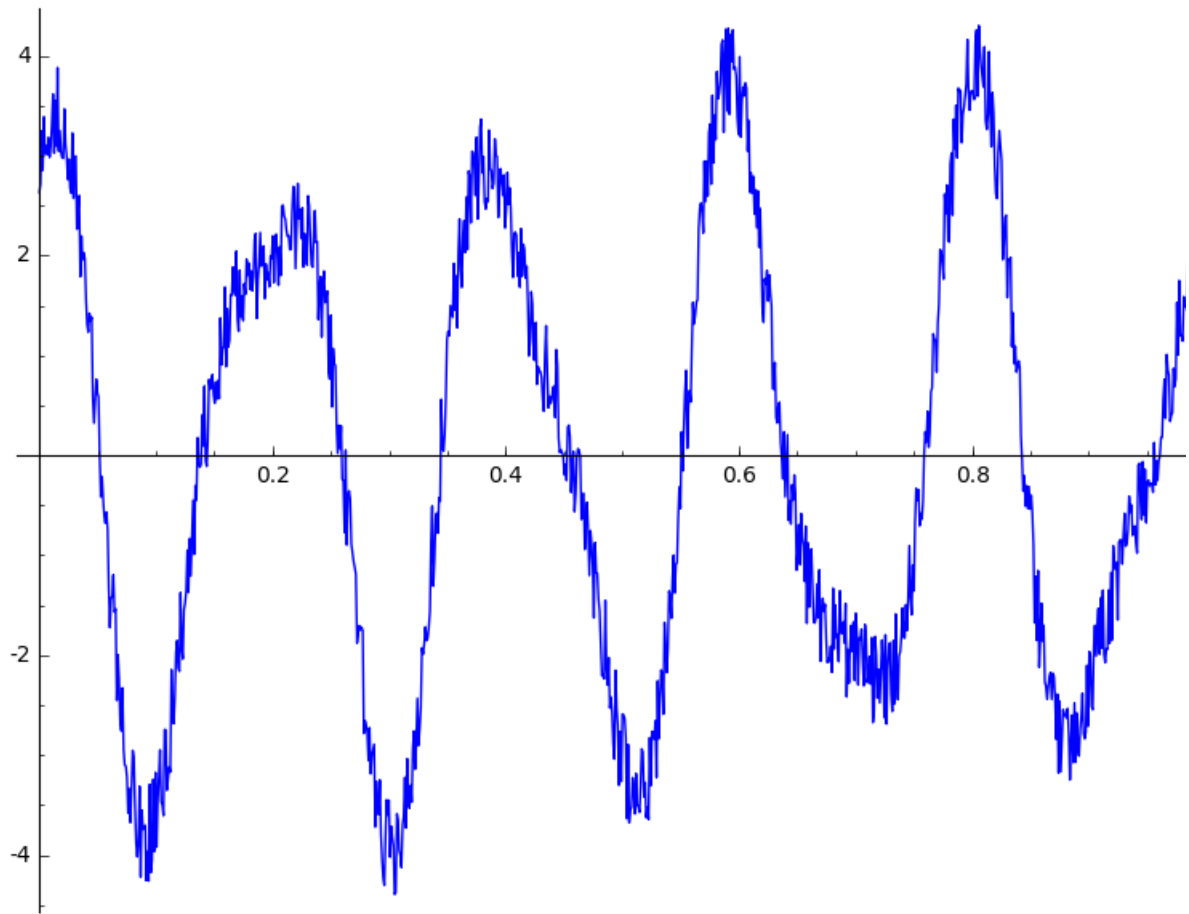
```
B=[h() for i in range(1024)]
```

```
line(zip(x,B))
```



```
FB=[F[i]+B[i] for i in range(1024)]
```

```
line(zip(x,FB))
```

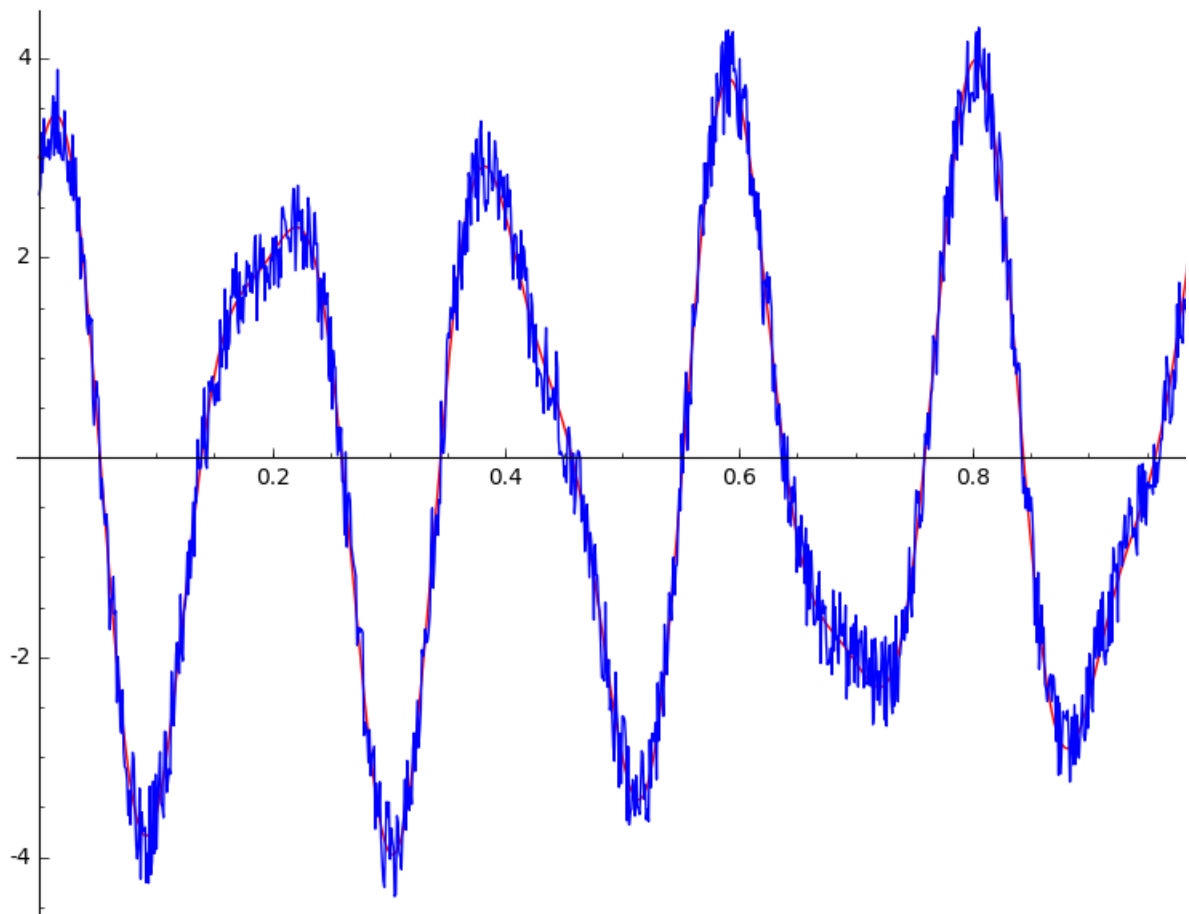


```
GrapheF=line(zip(x,F),color='red')
```

```
GrapheFB=line(zip(x,FB))
```

```
GrapheF+GrapheFB
```

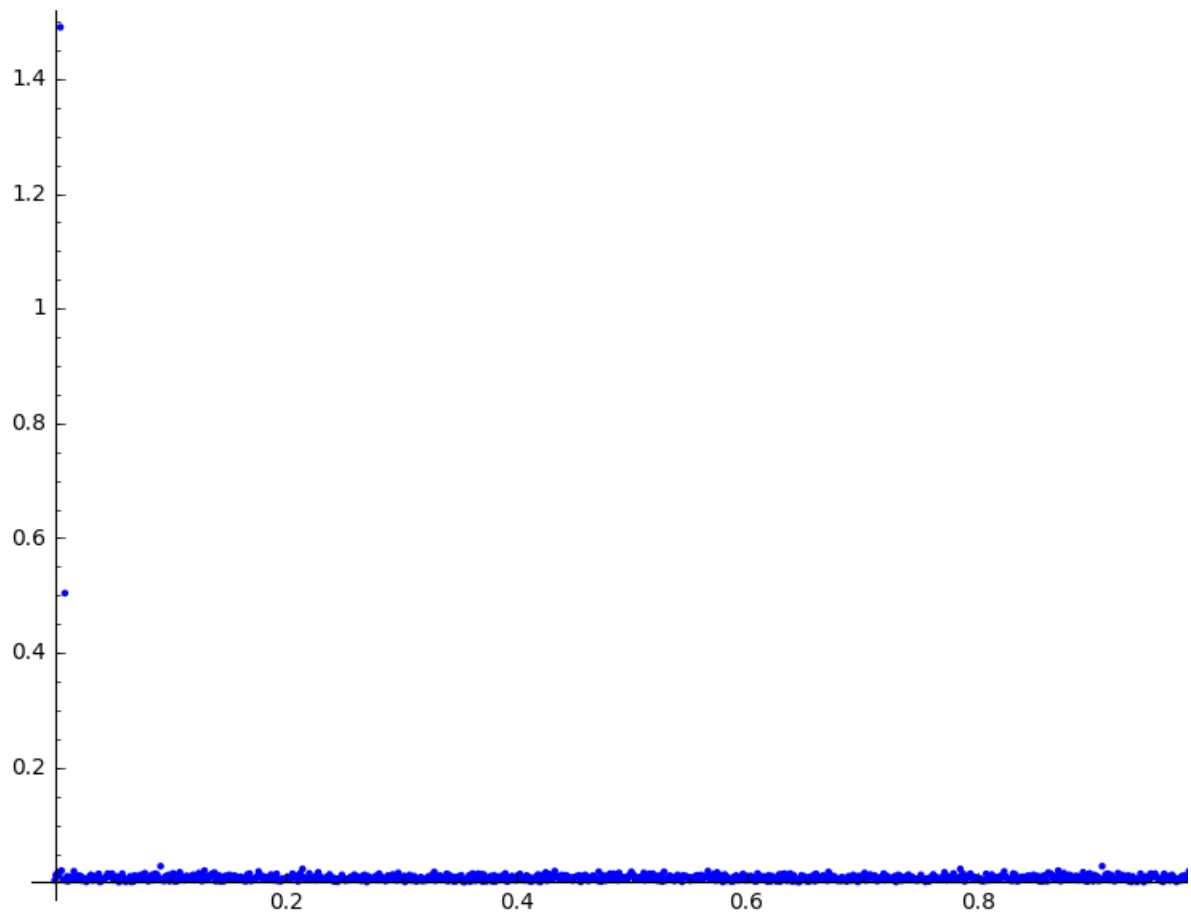




```
m(t)=t/1024
```

```
FBc=map(m,fft(FB,w^-1,1024))
```

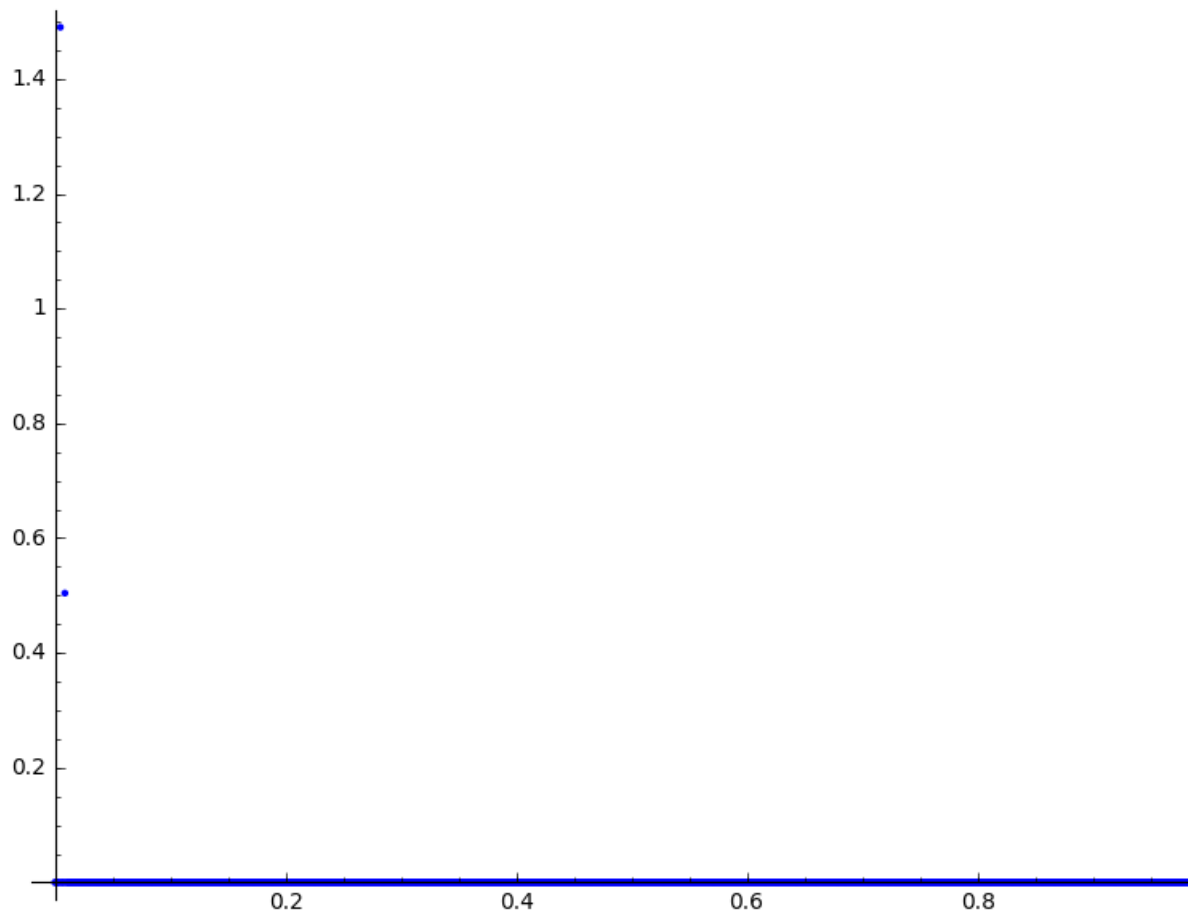
```
point(zip(x,map(abs,FBc)))
```



```
nettoie=FBc
```

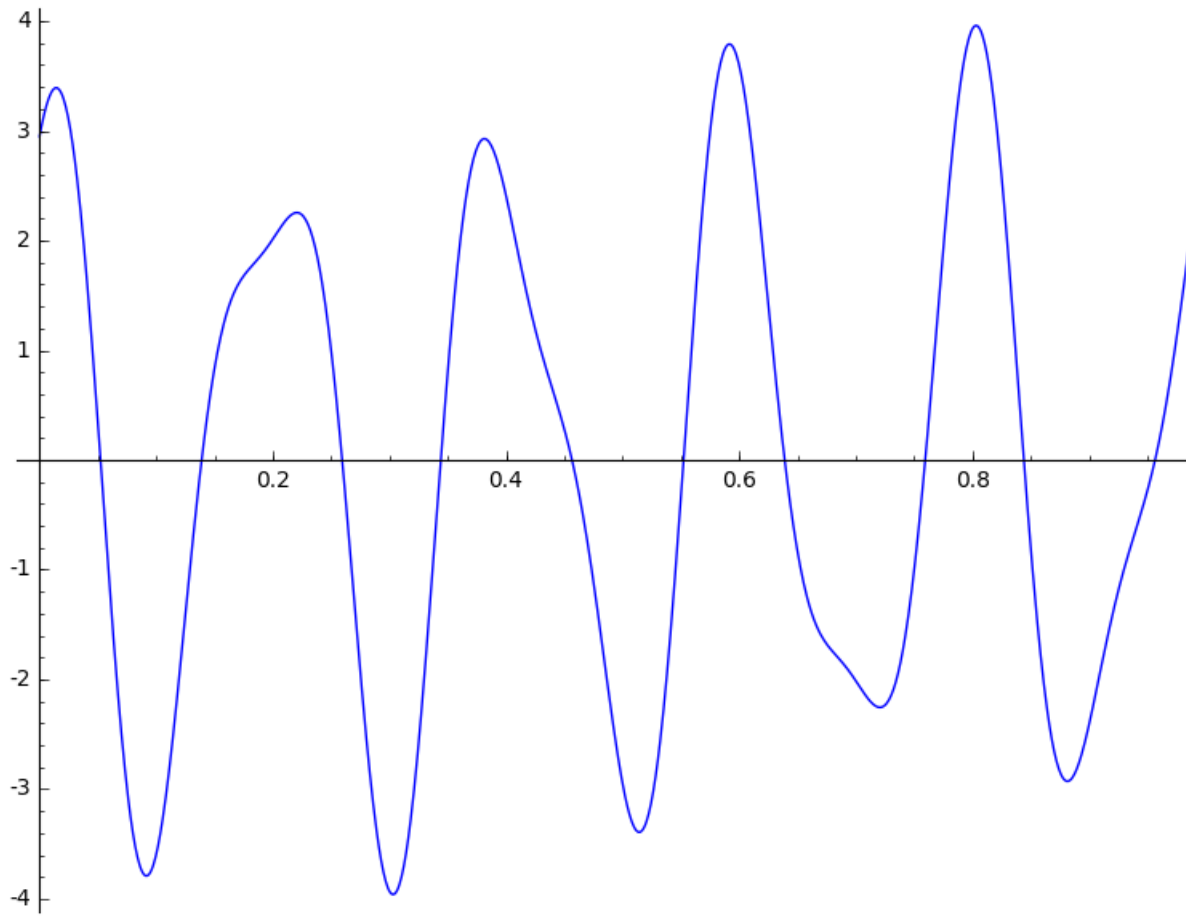
```
for i in range(1024):  
    if abs(FBc[i])<=1/4:  
        nettoie[i]=0
```

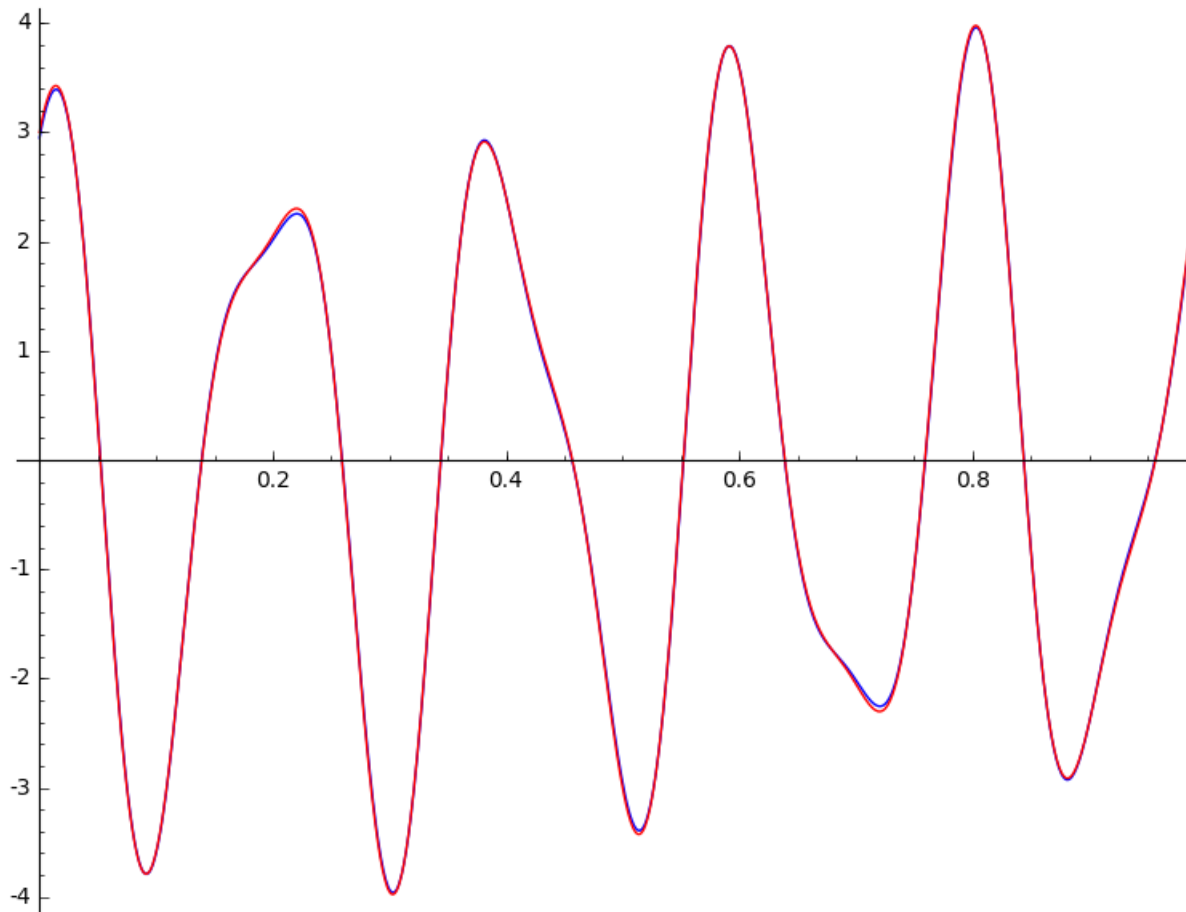
```
point(zip(x,map(abs,nettoie)))
```



```
filtre=map(real,fft(nettoie,w,1024))
```

```
line(zip(x,filtre))
```





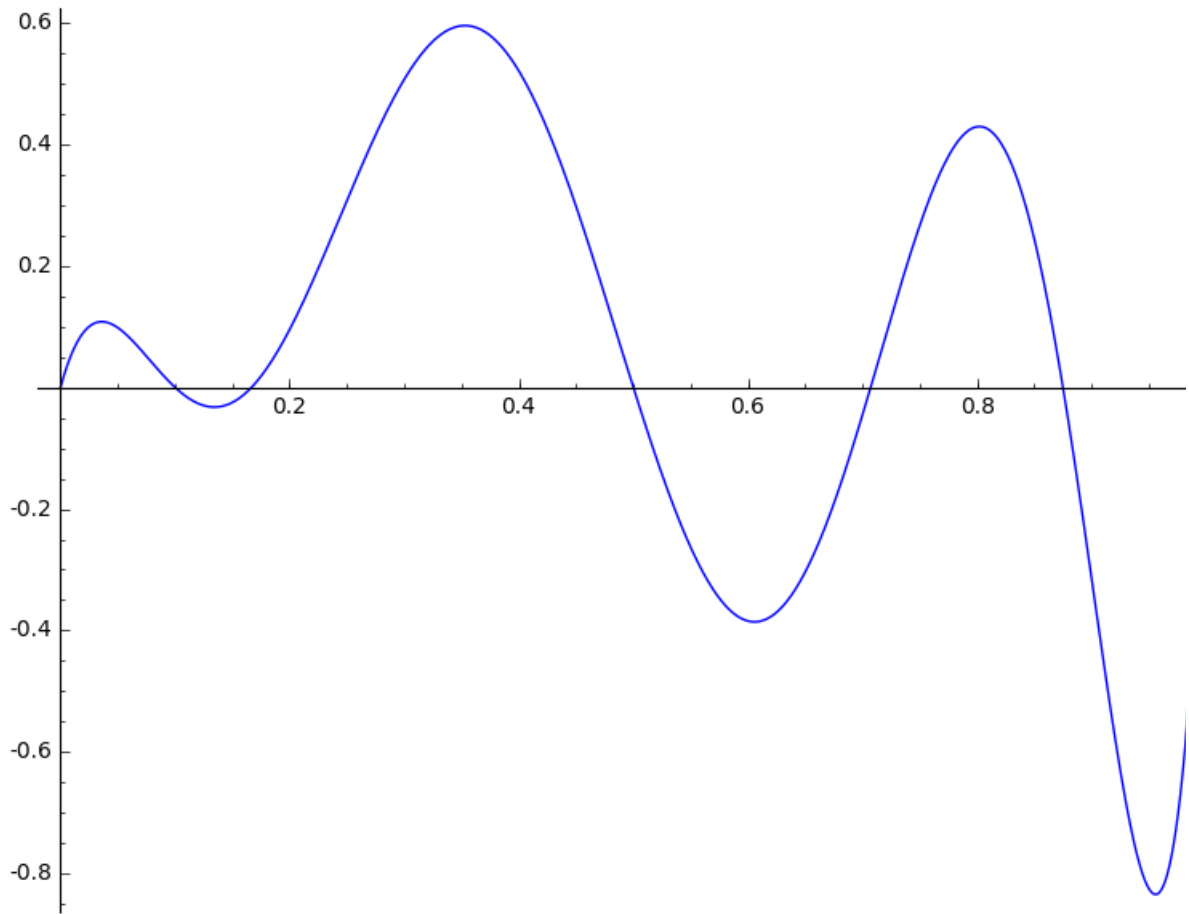
```
f(t)=1920*(t-1/6)*(t^2-1/2)*(t-1/2)*(t-7/8)*(t-1/10)*(t-1)*t
```

```
expand(f(t))
```

```
1920*t^8 - 5072*t^7 + 3768*t^6 + 692*t^5 - 2082*t^4 + 908*t^3 -  
141*t^2 + 7*t
```

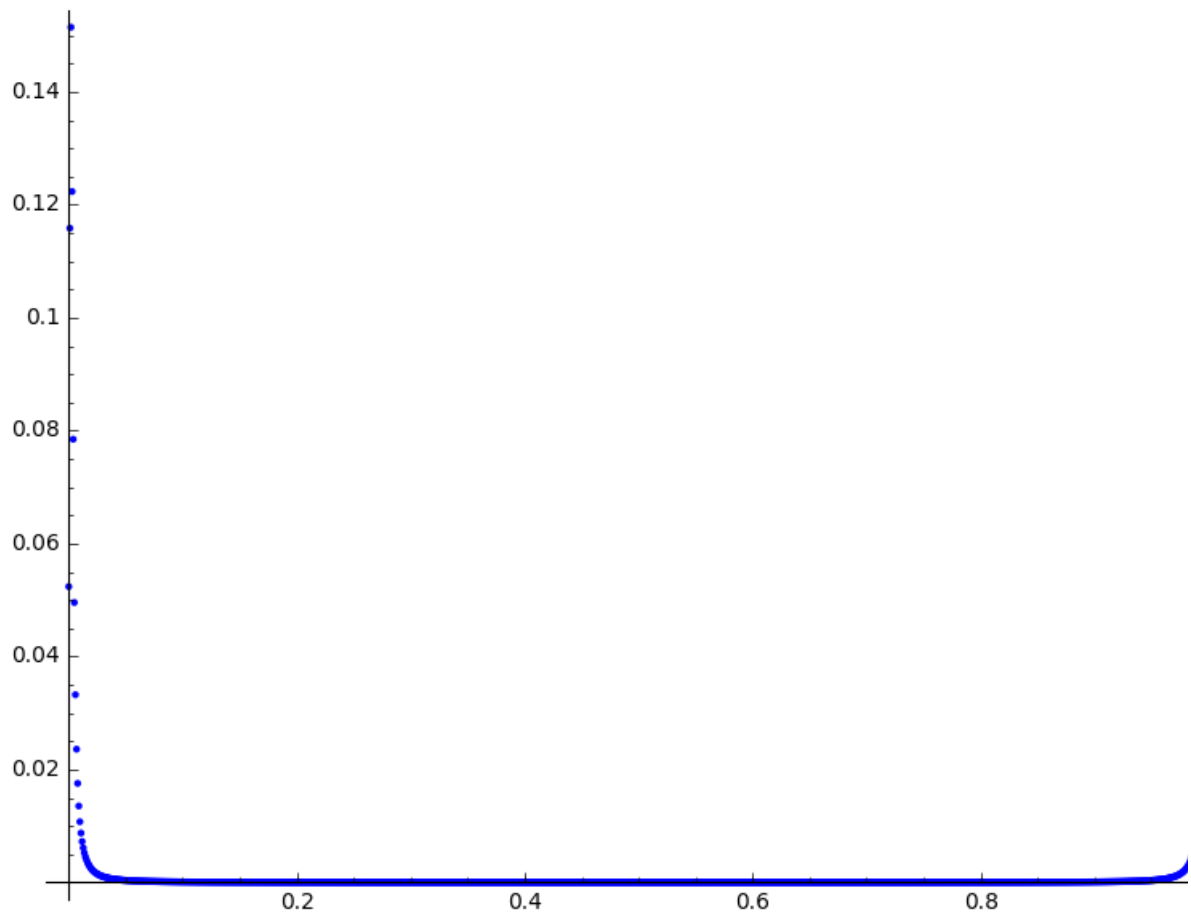
```
F=map(f,x)
```

```
line(zip(x,F))
```



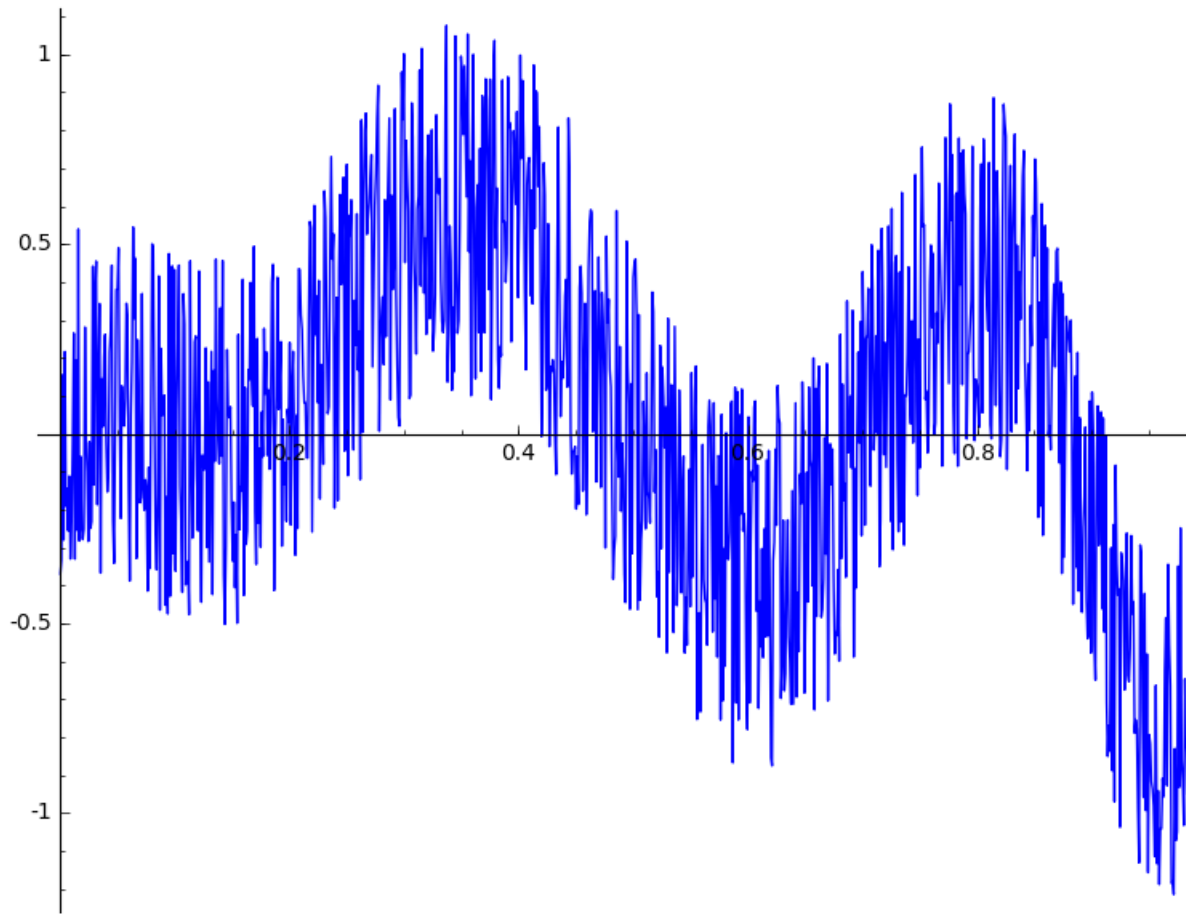
```
Fc=map(m,fft(F,w^-1,1024))
```

```
point(zip(x,map(abs,Fc)))
```



```
FB=[F[i]+B[i] for i in range(1024)]
```

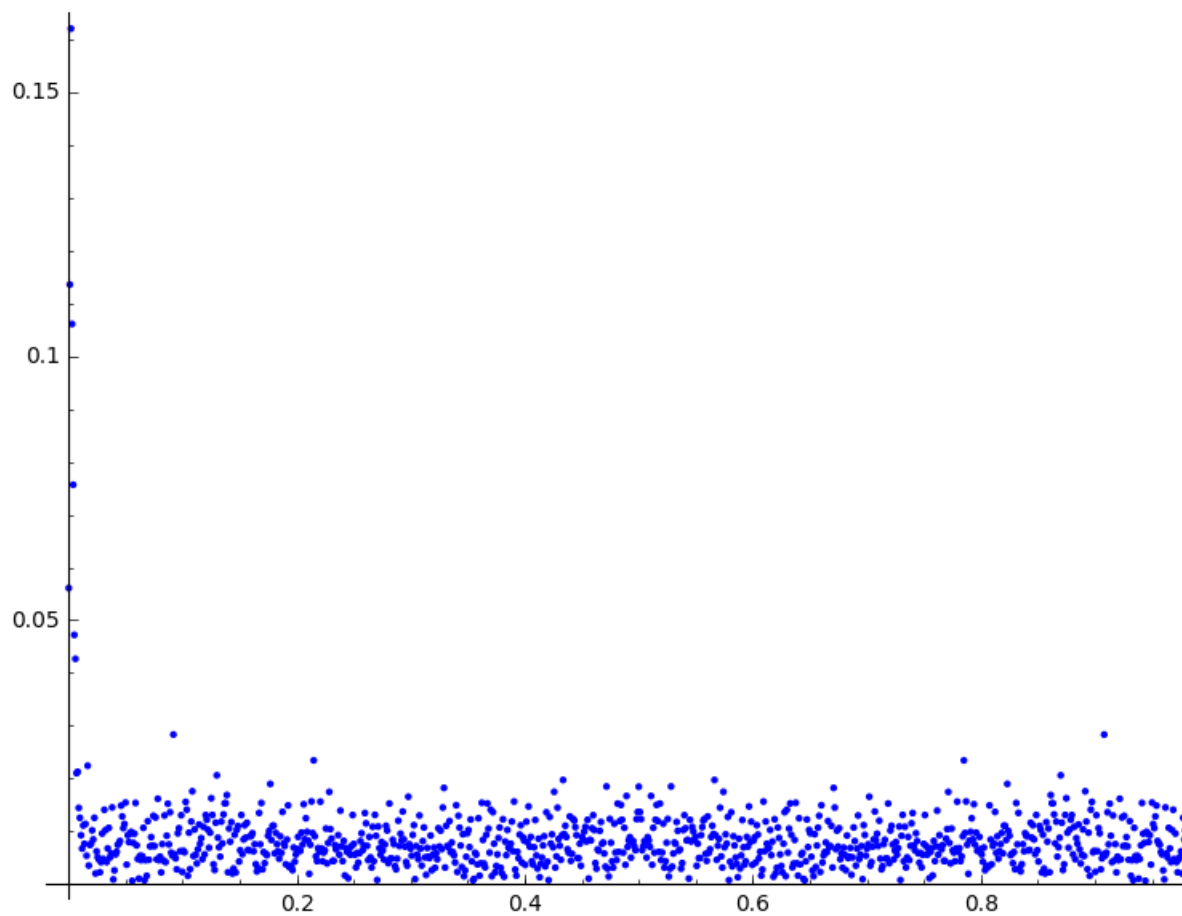
```
line(zip(x,FB))
```



```
FBC=map(m,fft(FB,w^-1,1024))
```

```
point(zip(x,map(abs,FBC)))
```



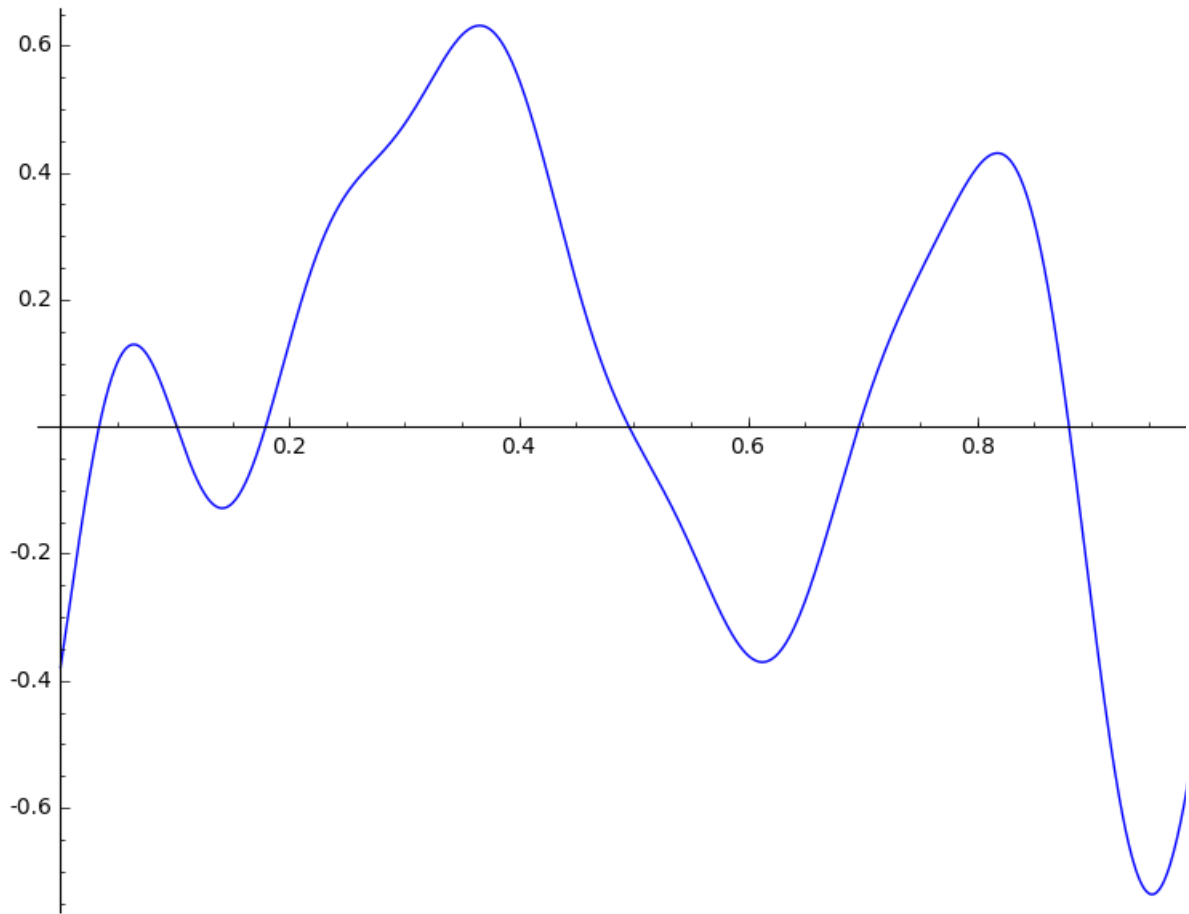


```
Net=copy(FBc)
```

```
for i in range(1024):  
    if abs(Net[i])<.03:  
        Net[i]=0
```

```
Filtre=fft(Net,w,1024)
```

```
line(zip(x,map(real,Filtre)))
```



```
GrapheFiltre=line(zip(x,map(real,Filtre)))
```

```
GrapheF=line(zip(x,F),color='red')
```

```
GrapheFiltre+GrapheF
```

