Erratum to Smooth curves having a large automorphism p-group in characteristic $p > 0$.

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The proof of the last statement in Lemma 2.4. part 2 is wrong. Namely the error concerns the proof of the equality $G_2/H = (G/H)_2$. For this we use the equality $G_2 = G^2$ as a general fact for p-groups which is wrong. Indeed for a p-group G, the Herbrand function φ_G satisfies $G_2 = G^{\varphi_G(2)}$ and $\varphi_G(2) = 1 + \frac{|G_2|}{|G|}$. Moreover $\varphi_G(2) = 2$ iff $G = G_2$ which is not the case in particular for big actions!

The equality $G_2/H = (G/H)_2$ as stated in part 2 is still true but its proof is postponed after Theorem 2.7.

So replace Lemma 2.4 by the following:

Lemma 2.4. *Let G be a finite p-subgroup of $\mathrm{Aut}_k(C)$. We assume that the quotient curve C/G is isomorphic to $\mathbb{P}^1_k$ and that there is a point of C (say ∞) such that G is the wild inertia subgroup G_1 of G at ∞. We also assume that the ramification locus of the cover $\pi : C \to C/G$ is the point ∞, and the branch locus is $\pi(\infty)$. Let G_2 be the second ramification group of G at ∞ and H a subgroup of G. Then*

1. *C/H is isomorphic to $\mathbb{P}^1_k$ if and only if $H \supset G_2$.*
2. *In particular, if (C, G) is a big action with $g \geq 2$ and if H is a normal subgroup of G such that $H \subsetneq G_2$, then $g_{C/H} > 0$ and $(C/H, G/H)$ is also a big action.*

Proof:

1. Applied to the cover $C \to C/G \simeq \mathbb{P}^1_k$, the Hurwitz genus formula (see for instance [Stichtenoth 93]) yields $2(g-1) = 2|G|\,(g_{C/G} - 1) + \sum_{i\geq 0}(|G_i| - 1)$. When applied to the cover $C \to C/H$, it yields $2(g-1) = 2|H|\,(g_{C/H} - 1) + \sum_{i\geq 0}(|H \cap G_i| - 1)$. Since $H \subset G = G_0 = G_1$, it follows that
$$2|H|g_{C/H} = -2(|G| - |H|) + \sum_{i\geq 0}(|G_i| - |H \cap G_i|) = \sum_{i\geq 2}(|G_i| - |H \cap G_i|).$$
Therefore, $g_{C/H} = 0$ if and only if for all $i \geq 2$, $G_i = H \cap G_i$, i.e. $G_i \subset H$, which is equivalent to $G_2 \subset H$, proving 1.
2. Together with part 1, Proposition 2.2.4 shows that $(C/H, G/H)$ is a big action. □

Now in the proof of Theorem 2.7 replace the sentence

"The first assertion now follows from Lemma 2.4.2." by the following:

"As $(C/H, G/H)$ is a big action and $(C/H)/(G_2/H) \simeq \mathbb{P}^1_k$ it follows from Lemma 2.4.1 that $(G/H)_2 \subset G_2/H$. Here $|G_2/H| = p$ and the equality $|(G/H)_2| = 1$ is in contradiction with proposition 2.2.1. The equality $G_2/H = (G/H)_2$ then follows."
The end of the proof of Theorem 2.7 works the same.

Now one can complete Lemma 2.4. by the following:

Remark 2.8. *Under the same hypothesis as in Lemma 2.4.2 we have the equality $G_2/H = (G/H)_2$. Namely by Theorem 2.7.4 we have $G_2 = D(G)$ and $(G/H)_2 = D(G/H)$. It is a general fact that for H a normal subgroup of G one has the equality $D(G/H) \simeq D(G)/(H \cap D(G))$. The equality then follows as $H \subset G_2 = D(G)$.*