Lifting $(\mathbb{Z}/2\mathbb{Z})^2$-actions

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1 Introduction

Let $G = (\mathbb{Z}/p\mathbb{Z})^2 \in Aut_k k[[z]]$ a faithfull action over (char. $k = p > 0$), let $\chi : G \to \mathbb{Z}/p\mathbb{Z}$ a non trivial character then its kernel defines by Galois theory an Artin-Schreier equation with conductor m_χ. In [G-M1], we proved that this action can be lifted to an action of $G \in$ on $Aut_R R[[Z]]$ (R is a DVR finite over $W(k)$) then $p|(m_0 + 1)$, where $m_0 = \min m_\chi$. As conductors are prime to p, this gives a numerical obstruction for the existence of liftings as soon as $p > 2$.

It is then a natural question to ask for lifting $(\mathbb{Z}/2\mathbb{Z})^2$-actions and more generally $(\mathbb{Z}/2\mathbb{Z})^n$-actions.

An easy global example shows that problems can arise as soon as $n \geq 3$. Namely for any n, $(\mathbb{Z}/2\mathbb{Z})^n$ can be realized as a group of transvections on the projective line over k for k an algebraically closed field with char. 2; but in char.0, this is the case iff $n = 1, 2$ and following a local-global principle [G-M1], [B-M], it follows that some action of $(\mathbb{Z}/2\mathbb{Z})^n$ on $k[[z]]$ can't be lifted if $n > 2$.

In the first part of this note we give numerical obstructions of local nature for the lifting. This is a translation of a numerical obstruction given by Bertin [B] in terms of ramification numbers.

In particular we get a numerical obstruction as soon as $p = 2$ and $n > 2$ in terms of the conductors attached to characters, i.e. the conductors of the Artin-Schreier equations corresponding to their kernel.

In his thesis [P2], G. Pagot gives two proofs of the existence of lifting in characteristic zero for any $(\mathbb{Z}/2\mathbb{Z})^2$-action over $k[[z]]$. The first one which also appears in [P1] concerns only the case where the conductors are equal. The second, which is given in chapter 5 in his thesis [P2], treats the general case. Some considerations on the branch locus show that in order to lift the case of distinct conductors one needs generically non equidistant geometry for the branch loci; this makes the proof more delicate.

Our aim in the second part of this note is to comment and report on his proof.

We would like to encourage the reader to look at [P 1,2] for numerical obstructions for the lifting which are of differential geometry nature contrarily to those in [B],[G-M1], which are of ramification nature.

2 Ramification in $(\mathbb{Z}/p\mathbb{Z})^n$-actions

Let k be an algebraically closed field of characteristic $p > 0$.

Let $G = (\mathbb{Z}/p\mathbb{Z})^n \subset Aut_k k[[z]]$ be a faithfull action. The following is a particular case of a theorem by J. Bertin ([B], Exemple 4.1). Let G_i (resp. G^j) be the ramification subgroups in lower (resp. upper) notation; recall that:

$G_i = \{\sigma \in G \mid v(\sigma(z) - z) \geq i + 1\}$,

Let $i_0 > i_1 > ...$, be the sequence of ramification numbers, then one can write $i_j = \mu_0 + \mu_1 + ... + \mu_j$ and

$G_0 = ... = G_{\mu_0} = G(0) = G$,
$G_{\mu_0+1} = ... = G_{\mu_0+\mu_1} = G(1)$,
$G_{\mu_0+\mu_1+1} = ... = G_{\mu_0+\mu_1+\mu_2} = G(2)$,
...

$G_{\mu_0+\mu_1+...+\mu_{t-2}+1} = ... = G_{\mu_0+\mu_1+...+\mu_{t-1}} = G(t-1) = \{1\}$,
where $|G(j)| = p^{n_j}$ and $n = n_0 > ... > n_{t-1} = 0$.

The Hasse-Arf theorem in this case asserts that: $p^{n-n_j}|\mu_j$, so we can write: $\nu_j := \mu_j/p^{n-n_j} \in \mathbb{N}$ and we have the upper numbering:

$G = G(0) = G^0 = ... = G^{\nu_0}$,
$G(1) = G^{\nu_0+1} = ... = G^{\nu_0+\nu_1}$,
$G(2) = G^{\nu_0+\nu_1+1} = ... = G^{\nu_0+\nu_1+\nu_2}$,
...
$\{1\} = G(t-1) = G^{\nu_0+...+\nu_{t-1}+1}$.

Now let us examine Bertin's condition: let $\sigma \in G(j) - G(j+1)$ then $p^{n-1}|(\mu_0 + ... + \mu_j + 1)$ which is equivalent to: $p^{n-1}|(\mu_0+1)$ and for $0 < j < t$, $p^{n-1}|\mu_j$ i.e. $p^{n_j-1}|\nu_j$.

Note that these conditions are stronger than those given by Hasse-Arf theorem.

Now we want concrete examples. In order to give a $G = (\mathbb{Z}/p\mathbb{Z})^n$-action on $k[[z]]$ we use Galois theory of extensions, namely we give equations for the generic extension $k((z))/k((z))^G = k((t))$. More precisely we consider n, $\mathbb{F}_p$-independant characters $\chi_j : G \to \mathbb{Z}/p\mathbb{Z}$ for $j = 0,...,n-1$. An equation for the extension coresponding to $H_j := Ker\chi_j$ is:

$w_j^p - w_j = f_j(1/t) = a_{j,0} + a_{j,1}/t + ... + a_{j,m_j}/t^{m_j}$

where $(m_j, p) = 1$ and $a_{j,m_j} \neq 0$; then m_j is the conductor of the extension and is equal to its ramification number.

By [S] we learn that the higher ramification notation is compatible with quotients so for a given character χ_j we have

$(G/H_j)^\nu = G^\nu H_j/H_j$.

So the number of ramification m_j is $\nu_0 + ... + \nu_{\varphi(j)}$ and $m_j := \max\{\nu \mid G^{\nu+1} \subset H_j\}$.

Bertin's conditions then give: $(*)p^{n_{\varphi(j)}-1}|m_j + 1$ for $0 \leq j$.

We can assume that $m_0 \leq m_1... \leq m_{n-1}$. The $\mathbb{F}_p$-independance can be expressed in terms of the f_j.

Let us first consider the case where $m_0 < m_1... < m_{n-1}$; then the $\mathbb{F}_p$-independance is automatic and $\varphi(.)$ is strictly monotonic so $\varphi(j) = j$ and $n_j = n - j$.

Now Bertin's conditions are $(*)p^{n-j-1}|m_j + 1$ for $0 \leq j$.

In the general case the same conditions hold if we choose for the sequence $m_0 \leq m_1... \leq m_{n-1}$ a "minimal" one under the condition that the conductors m_i are associated to n independant characters.

We remark that in the case $n = 2$ we get the condition $p|m_0 + 1$ in [G-M1].

If $p = 2$ and $n = 2$ the condition is empty and for $p = 2$ and $n > 2$ the conditions are $2^{n-j-1}|m_j+1$ for $0 \leq j \leq n-1$.

Remark 2.1. *Another way to find these congruences is to compare the degree of differents in $(\mathbb{Z}/p\mathbb{Z})^n$-extensions in characteristic 0 and p just like done in [G-M1].*

3 Lifting $(\mathbb{Z}/2\mathbb{Z})^2$-actions

Let k be an algebraically closed field of characteristic $p > 0$, R be a DVR finite over $W(k)$ and π a uniformizing parameter. Let σ be an order p-automorphism of the p-adic open disc $SpecR[[Z]]$ which induces mod π an order p-automorphism. The geometry of the generic fiber is induced by the p-adic valuation and the σ-geometric fix points obey to metric conditions which are given by the minimal semi-stable model of $SpecR[[Z]]$ in which these points specialize to distinct smooth points on a tree of projective lines; moreover these points give a configuration which obeys to differential datas (see [G-M 2], introduction for more details and [H] for a construction of order p-automorphisms from Hurwitz trees). In [G-M1] we consider two p-cyclic covers of a same open disc by open discs which induce modulo π Artin-Schreier covers with respective conductors $m_0 \leq m_1$ as in the previous paragraph; we give a necessary and sufficient condition on the two branch loci in order for the compositum of these two covers to induce a (p,p)-cover of the disc by a disc. This condition is that the number of common branch points is $(p-1)(m_0+1)/p$ (which should be an integer in order to lift!). It is known (cf. [G-M2] that if the $m_0 + 1$ (resp. $m_1 + 1$) branch points of the first (resp.

second) cover are equidistant then this distance is $\frac{p}{(p-1)m_i}v(p)$ where $i=0$ (resp. $i=1$). This metric condition imposes in the case where $m_0 < m_1$ to look for liftings with non equidistant geometry.

3.1 Pagot's construction when $m_0 = m_1$, [P1]

We consider two covers

$w_0^p - w_0 = f_0(1/t) = a_0 + a_1/t + ... + a_{m_0}/t^{m_0}$

$w_1^p - w_1 = f_1(1/t) = b_0 + b_1/t + ... + b_{m_1}/t^{m_1}$

where $m_0 = m_1$ is prime to p and a_{m_0}, b_{m_1} are $\mathbb{F}_p$-independant.

We try to lift these two covers with equidistant geometry; if it is the case then the $(m_0+1)+(m_1+1)-(p-1)(m_0+1)/p$ branch points will specialize to distinct points on a same projective line $\mathbb{P}^1_k$. Moreover then there is a 2-dimensional $\mathbb{F}_p$-vector space of logarithmic differentials on this line, whose non zero elements have only one zero; it is in the same point (say ∞) and same of order $m_0 - 1$, moreover their poles are the specialisations of the branch points. Such a vector space is called in Pagot's thesis a $L_{m_0+1,n=2}$ and he studied the existence of such spaces. One main point in his thesis is the reciprocal: each such space gives rise to a lifting!!(see [P2] theorem 3.1.1.). The proof mimics one given in [M] and should be considered as the first attempt in order to generalize to (p,p)-actions Henrio's lifting results for p-cyclic actions.

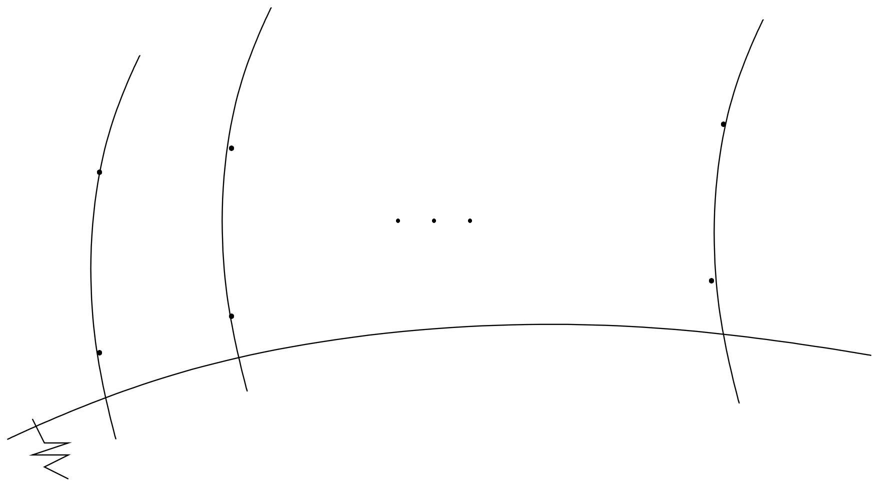
If $p > 2$, theorem 2.2.5 in [P2] gives numerical obstructions for the existence of spaces $L_{m+1,n}$ and theorem 3.2.3 deduces new numerical obstructions for the lifting.

If $p = 2$ there are many $L_{m+1,2}$. Pagot [P2] thm. 2.2.4) proves that if $m+1 = 2n$, and if $x_i, 1 \le i \le n$ are n distinct points on $\mathbb{A}^1_k$, there is an $L_{m+1,2} = \mathbb{F}_2\omega_0 \oplus \mathbb{F}_2\omega_2$ such that the $x_i, 1 \le i \le n$ are the common poles to ω_0 and ω_2 and this is exactly what we need for the lifting.

We will not go further here (see [P1] for the details) and we concentrate more on the general case.

3.2 Pagot's construction when $m_0 \le m_1$, [P1], chapter 5).

The idea is to build order 2-automorphisms whose minimal semi-stable model has special fiber which looks like this.



In fact this semi-stable curve is equipped with 2-torsors namely an α_2 torsor on the horizontal component and μ_2-torsors on each vertical component. These torsors, then induce differential datas namely the double points on the horizontal component are the poles of an additive differential with only one zero at ∞; moreover the order of these poles is equal to 2 equal to the number of fix points which specialise to the other component at this point. And the specialisation of branch points on a vertical component are the poles of a logarithmic differential (like in the previous subsection).

Following Henrio one can associate to such datas a metric tree named "Hurwitz tree"(note that we haven't considered a metric here; it comes from the thickness of double points). The key point here is then lemma 5.1.1. part 1 in [P1]; it says that such a differentials exist. In fact there is such a result adapted to each p; namely

Lemma 3.1. *Let k an algebraically closed field with characteristic $p>0$. Let $t_1,...,t_r$, r distinct elements in k. Let $q(x):=\prod_{1\leq i\leq r}(x-t_i)$ and $\tilde{q}(x):=\sum_{1\leq i\leq r}\frac{1}{q'(t_i)^p}\prod_{j\neq i}(x-t_j)^{p-1}$, then $\omega:=\frac{dx}{q(x)^p}=d(\frac{\tilde{q}}{q^{p-1}})$.*

In fact one needs a deformation of this lemma and this is given in part 2 of lemma 5.1.1. We need to be more precise then Pagot's thesis; the proof simply should have been written a bit more carefully; we try to do so.

G. Pagot's first remark is that the above equality can be written $d(\tilde{q}q-x)=0$; i.e. $\tilde{q}q-x\in(k[x])^p$. THis can be deform in the following:

Lemma 3.2. *Let R be a DVR finite over $W(k)$. Let $T_1,...,T_r\in R$ with $v(T_i-T_j)=0$ for $i<j$. Let $Q(X):=\prod_{1\leq i\leq r}(X-T_i)$ and $\tilde{Q}(x):=\sum_{1\leq i\leq r}\frac{1}{Q'(T_i)^p}\prod_{j\neq i}(X-T_j)^{p-1}$. Let $H(X)=-\sum_{1\leq i\leq r}\frac{T_i^{1/p}}{Q'(T_i)}\frac{Q(X)}{X-T_i}$, then $\tilde{Q}(X)Q(X)-X-H(X)^p\in pR[T_1^{1/p},...,T_r^{1/p}]$.*

Proof. : One has $\tilde{Q}(X)Q(X)=XQ(X)^p\sum_{1\leq i\leq r}\frac{1}{Q'(T_i)^p}\frac{1}{(X-T_i)^p}-\sum_{1\leq i\leq r}\frac{T_i}{Q'(T_i)^p}\frac{Q(X)^p}{(X-T_i)^p}$; now the result follows from the identity $\frac{1}{Q(X)}=\sum_{1\leq i\leq r}\frac{1}{Q'(T_i)}\frac{1}{X-T_i}$. $\square$

Lemma 5.1.2. in [P2] gives a construction for $p\geq 2$ of an order p-automorphism of the disc with a given "Hurwitz tree" which generilizes to $p>2$ the one we considered in figure above. We will simply check Pagot's construction in case $p=2$, and we encourage the reader to look at Pagot's construction for $p>2$.

Proposition 3.3. *Let k be an algebraically closed field of characteristic 2 and R a DVR finite over $W(k)$. Let $\rho\in W(k)^{alg}$ such that $\frac{1}{2}v(2)\leq v(\rho)<v(2)$. Let $T_1,...,T_r\in R$ with $v(T_i-T_j)=0$ for $i<j$ and $Q(X):=\prod_{1\leq i\leq r}(X-T_i)$. Let $\alpha\in R$ with $v(\alpha)=0$. Let $F(X):=\prod_{1\leq i\leq r}[(X-T_i)^2-2\rho\alpha\frac{T_i^{1/2}}{Q'(T_i)}(X-T_i)+\rho^2\alpha^2\frac{1}{Q'(T_i)^2}(X-T_i)]=\prod_{1\leq i\leq r}(X-T_i)(X-\tilde{T}_i)$. Then $v(T_i-\tilde{T}_i)=v(\rho^2)$ and $F(X)-(Q(X)+\rho\alpha H(X))^2-\rho^2\alpha^2X+\in 4R^{alg}[X]$ where $H(X)$ is defined in the lemma. The cover $Y^2=F(X)$ has good reduction over $R[(4/\rho^2)^{1/(2r-1)}]$ relatively to the coordinate $T:=(\frac{2}{\rho})^{\frac{2}{2r-1}}X$. An equation for the cover at the special fibre is $w^2-w=\overline{\alpha}^2\frac{1}{t^{2r-1}}$; moreover the minimal semi-stable model in which the branch points specialize to smooth and distinct points has a special fibre like that in the figure above.*

Proof. We can write $F(X)=Q(X)\prod_{1\leq i\leq r}(X-T_i-2\rho\alpha\frac{T_i^{1/2}}{Q'(T_i)}+\rho^2\alpha^2\frac{1}{Q'(T_i)^2})=Q(X)(Q(X)+\rho^2\alpha^2\tilde{Q}(X)+2\rho\alpha H(X))+O(4)$. Then $F(X)=(Q(X)+\rho\alpha H(X))^2+\rho^2\alpha^2(Q(X)\tilde{Q}(X)-H(X)^2)+O(4)=(Q(X)+\rho\alpha H(X))^2+\rho^2\alpha^2X+4P(X)$, where $P(X)\in W(k)^{alg}[X]$ and degree $<2r$ by the lemma. Now let $Y:=2V+Q(X)+\rho\alpha H(X)$ then $V^2+(Q(X)+\rho\alpha H(X))V=\frac{1}{4}\rho^2\alpha^2X+P(X)$. We write $X^{-1}=(\frac{2}{\rho})^{\frac{2}{2r-1}}T^{-1}$ and $V=X^rW$; as $v(\frac{2}{\rho})>0$ and $\rho^2X^{1-2r}=4T^{1-2r}$; the result follows. $\square$

Remark 3.4. *The same result is still valid if we translate the $\tilde{T}_i$ by elements in $4R$.*

Pagot uses the following corollary

Corollary 3.5. *Same hypothesis as in the proposition. Let $0<m_0\leq m_1$ odd integers and $r:=\frac{m_0+1}{2}$. Let $\rho_0:=2^{1/m_1}$, $\rho_r=2^{2-\frac{m_0}{m_1}}$ and $\rho=\rho_r^{1/2}$; then $\frac{1}{2}v(2)\leq v(\rho)<v(2)$. Let $C_j\in R$ for $1\leq j\leq r-1$ and $C_0=\alpha^2$. Set $F_1(X):=F(X)+\rho^2\sum_{1\leq j\leq r-1}C_j\rho_0^{2j}X^{2j+1}$; then $F_1(X)=\prod_{1\leq i\leq r}(X-X_i)(X-\tilde{X}_i)$. The cover $Y^2=F_1(X)$ has potentially good reduction relatively to the coordinate $T:=(\frac{2}{\rho})^{\frac{2}{2r-1}}X=\rho_0X$. An equation for the cover at the special fibre is $w^2-w=\sum_{0\leq j\leq r-1}\overline{C}_j/t^{m_0-2j}$; moreover the minimal semi-stable model in which the branch points specialize to smooth and distinct points has a special fibre like that in the figure above.*

Proof. We use the same notations as in the proof of the proposition. Let $Y := 2V + Q(X) + \rho\alpha H(X)$ then $V^2 + (Q(X) + \rho\alpha H(X))V = \frac{1}{4}\rho^2[\sum_{0\leq j\leq r-1} C_j \rho_0^{2j} X^{2j+1}] + P(X)$. We write $X^{-1} = \rho_0 T^{-1}$ and $V = X^r W$; in reduction we get $w^2 - w = \sum_{0\leq j\leq r-1} \overline{C}_j / t^{m_0 - 2j}$. Now let us look at the geometry of the branch locus. As $F_1(X)$ and $F(X)$ are equal mod π; we see that $v(X_i - X_j) = 0$ for $i < j$ where X_i is one of the two roots of $F_1(X)$ such that $v(X_i - T_i) > 0$. We have seen that $F(X) = (Q(X) + \rho\alpha H(X))^2 + \rho^2\alpha^2 X + O(4)$, then $F_1'(X) = \rho^2\alpha^2 + 2(Q(X) + \rho\alpha H(X))(Q'(X) + \rho\alpha H'(X)) + 0(4)$. It follows that $v(F_1'(X_i)) = v(X_i - \tilde{X}_i) > 0$. □

Now we can prove Pagot's theorem 5.2.1.

Theorem 3.6. *Let k be an algebraically closed field of characteristic 2 and $G = (\mathbb{Z}/2\mathbb{Z})^2 \subset Aut_k k[[z]]$ be a faithfull action then there is R a DVR finite over $W(k)$ and an action $G = (\mathbb{Z}/2\mathbb{Z})^2 \subset Aut_R R[[Z]]$ which lifts the given action.*

Proof. We consider two covers

$w_0^2 - w_0 = f_0(1/t) = a_0 + a_1/t + ... + a_{m_0}/t^{m_0}$

$w_1^2 - w_1 = f_1(1/t) = b_0 + b_1/t + ... + b_{m_1}/t^{m_1}$

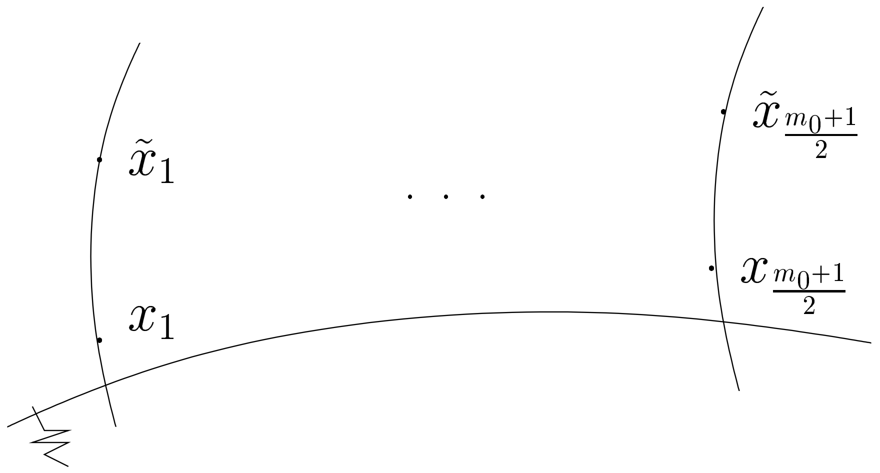
where $m_0 \leq m_1$ are prime to 2 and a_{m_0}, b_{m_1} are $\mathbb{F}_2$-independant corresponding to the kernel of two independant characters; moreover we can assume that the pair $m_0 \leq m_1$ is minimal. After a suitable change of parameter we can assume that

$f_0(1/t) = \sum_{0\leq j\leq \frac{m_0-1}{2}} \frac{c_j}{t^{m_0-2j}}$

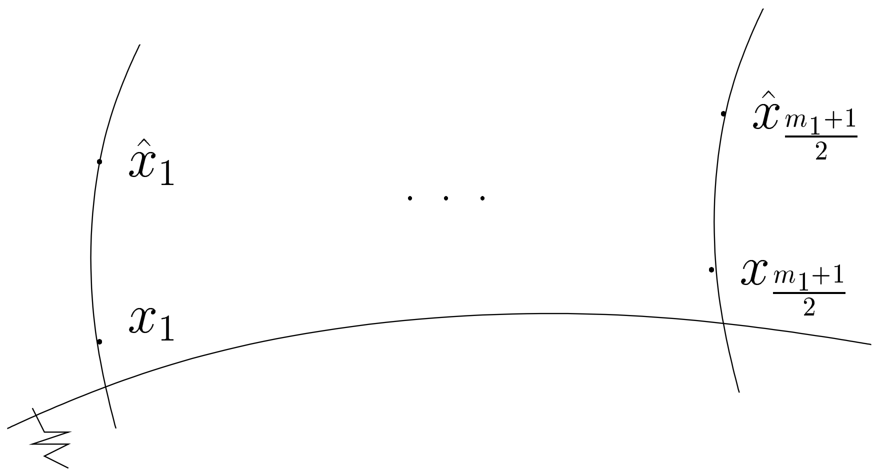
and

$f_1(1/t) = \frac{1}{t^{m_1}}$

We apply the corollary above with C_j a lifting in $W(k)$ of c_j and $\rho_0 := 2^{1/m_1}$: we get a cover $F_0(X) = \prod_{1\leq i\leq \frac{m_0+1}{2}} (X - X_i)(X - \tilde{X}_i)$ with $v(X_i - \tilde{X}_i) = v(2^{2-\frac{m_0}{m_1}})$ which has potentially good reduction relatively to the coordinate $T := \rho_0 X$ and induces at the special fiber the cover $w_0^2 - w_0 = f_0(1/t)$; moreover the geometry of the branch locus is given by the following:



Now we lift the cover $w_1^2 - w_1 = \frac{1}{t^{m_1}}$. Let $X_{r+1}, ..., X_{\frac{m_1+1}{2}} \in W(k)$ such that $v(X_i - X_j) = 0$ for $i \neq j$. We apply the proposition above with $\alpha = 1$, $\rho = 2^{1/2}$ and $r = \frac{m_0+1}{2}$ and $T_i = X_i$ for $1 \leq i \leq \frac{m_0+1}{2}$; then there exist $\hat{X}_1, ..., \hat{X}_{\frac{m_1+1}{2}} \in W(k)^{alg}$ such that $v(X_i - \hat{X}_i) = v(2)$ and such that the cover $Z^2 = F_1(X) = \prod_{1\leq i\leq \frac{m_1+1}{2}} (X - X_i)(X - \hat{X}_i)$ has potentially good reduction relatively to the coordinate $T := (\frac{2}{\rho})^{\frac{2}{2r-1}} X = \rho_0 X$ and induces at the special fiber the cover $w_1^2 - w_1 = f_1(1/t)$; moreover the geometry of the branch locus is given by the following: If $m_0 < m_1$; then $v(X_i - \tilde{X}_i) = v(2^{2-\frac{m_0}{m_1}})$ and $v(X_i - \hat{X}_i) = v(2)$, then the two covers have simultaneously potentially good reduction and their branch loci meet in $\frac{m_0+1}{2}$ points so it follows from [G-M1] that their compositum has potentially good reduction relatively to the same coordinate.



If $m_0 = m_1$, then $v(X_i - \tilde{X}_i) = v(X_i - \hat{X}_i) = v(2)$. Pagot uses the remark after the proposition in order to adjust the number of common points in the two branch locus; in fact it follows from Kato's formula [G-M1], that the maximum number of common points between the two branch locus is $(p-1)\frac{m_0+1}{p}$ and so in all cases necessarily $\hat{X}_i \neq \tilde{X}_i$. □

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