Solving the robust CVRP under demand uncertainty

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1 Introduction

We consider the following classical Capacitated Vehicle Routing Problem (CVRP). Let $G = (V, A)$ be a complete digraph with nodes $V = \{0, 1, \ldots, n\}$ and arcs $\{(i, j) \in V \times V : i \neq j\}$. Node $0 \in V$ represents the unique depot, and each node $i \in V' = V \setminus \{0\}$ corresponds to a customer with demand $d_i \in \mathbb{R}_+$ (let $d_0 = 0$). The depot is the departure and return base for a fleet m homogeneous vehicles of capacity Q. The set of vehicles is denoted as $K = \{1, \ldots, m\}$. Each vehicle incurs a transportation cost $c_{ij} \in \mathbb{R}_+$ if it traverses the arc $(i, j) \in A$. We define a route as a depot-base-walk in graph G, i.e., a sequence of nodes visited by a vehicle such that the first and the last node in this sequence is the depot. Let a_i^r be equal to the number of times a node $i \in V'$ is visited in a route r. A route r is demand-feasible if the total demand of visited customers does not exceed the vehicle capacity: $a^r d \leq Q$. Let Ω be the set of all demand-feasible routes. We assume here that demand-feasible routes may visit a node more than once. The cost c^r of a route is the total transportation cost incurred by traversing the route arcs. The problem consists in finding a set of demand-feasible routes of the minimum total cost such that every customer is visited exactly once. By introducing a binary variable λ_r for every $r \in \Omega$, the problem can be modeled by the following set partitioning formulation.

$$\begin{aligned} \min \quad & \sum_{r\in\Omega} c^r \lambda_r, \\ \text{s.t.} \quad & \sum_{r\in\Omega} a_i^r \lambda_r = 1, \quad i \in V', \\ & \sum_{r\in\Omega} \lambda_r \leq m, \\ & \lambda_r \in \{0,1\}, \quad r \in \Omega. \end{aligned}$$

2 Demand uncertainty

We consider that the demand vector d is uncertain and can take any value in a given polytope $\mathcal{D}$ that is included in the box $[\overline{d}, \overline{d}+\hat{d}]$ defined by the vectors $\overline{d}, \hat{d} \in \mathbb{R}^n_+$, where the components of $\overline{d}$ represent the nominal values, while those of $\hat{d}$ are the deviations. Observe that considering downward deviations of d is not of any use: they would not impair the feasibility given that demand values only play a role in the capacity constraints. Our study focuses on two definitions of polytope $\mathcal{D}$. The first one is the budget polytope introduced in [1], and widely used in the robust combinatorial optimization literature since then. Given $\Gamma \in \mathbb{R}_+$, the budget polytope is given by

$$\mathcal{D}^{knap} = \left\{ d \in \mathbb{R}^n_+ : d_i = \overline{d}_i + \eta_i \hat{d}_i, i \in V^0, \sum_{i\in V^0} \eta_i \leq \Gamma, \; 0 \leq \eta \leq 1 \right\}.$$

The second polytope we consider is that of [2, 3]. Therein is defined a customer partition $V_C = V_1 \cup \dots \cup V_s$ and associated budgets $b_1, \dots, b_s \in \mathbb{R}_+$, yielding potytope:

$$\mathcal{D}^{prop} = \left\{ d \in \mathbb{R}^n_+ : d_i = \overline{d}_i + \xi_i, i \in V^0, \sum_{i\in V_k} \xi_i \leq b_k, k = 1, \dots, s, \; 0 \leq \xi \leq \hat{d} \right\}.$$

Considering the change of variable $\xi_i = \hat{d}_i \eta_i$ underlines that $\mathcal{D}^{knap}$ constrains the number of elements of d that deviate simultaneously from their nominal values, while $\mathcal{D}^{prop}$ constrains the total amount of deviation within each subset of customers.

The demand uncertainty only affects the feasibility of the routes r used in the set partitioning formulation. Therefore, the robust counterparts of the above formulation result in set partitioning formulations that involve the set of routes that are demand-feasible for all values of d in $\mathcal{D}^{knap}$ or $\mathcal{D}^{prop}$. We denote these sets of routes by $\Omega(\mathcal{D}^{knap})$ and $\Omega(\mathcal{D}^{prop})$, respectively.

3 Robust algorithm

The contribution of our work is to extend the classical algorithms developed for the deterministic CVRP to their robust counterpart. Specifically, Branch-Cut-and-Price algorithms are the state-of-the-art approaches to solve this set partitioning formulation [5]. Therein, the linear relaxation of the formulation is solved by column generation. The pricing problem is solved typically by dynamic programming using a forward labeling algorithm. To improve the quality of column generation lower bounds, set Ω is restricted to the set of ng-paths. Rounded capacity cuts and limited memory rank-1 cuts are used to further strengthen the root bound. In this work, we use the Branch-Cut-and-Price algorithm from [7]. It has the advantage to efficiently handle instances with real value demands, which is important in the aforementioned robust variant of the problem.

In adapting the deterministic algorithms one must essentially specialize the pricing oracle to generate routes in $\Omega(\mathcal{D}^{knap})$ or $\Omega(\mathcal{D}^{prop})$. This amounts to solving a minimum cost "robust" constrained shortest path problems. Using well-known results from the robust optimization literature (see [1, 4, 6] for details) these problems are equivalent to solving a set of deterministic constrained shortest path problems with different weights, and taking the best of them. Specifically, the sets $\mathcal{D}^{knap}$ and $\mathcal{D}^{prop}$ require to solve $H(\mathcal{D}^{knap}) = \lceil \frac{n-\Gamma}{2} \rceil + 1$ and $H(\mathcal{D}^{prop}) = 2^s$ deterministic problems, respectively. Let $\Omega_h(\mathcal{D}^{knap})$ and $\Omega_h(\mathcal{D}^{prop})$ denote the sets of routes that can be generated by the h-th deterministic problem associated to the sets $\mathcal{D}^{knap}$ and $\mathcal{D}^{prop}$, respectively. In the proposed set partitioning formulation, we just set $\Omega = \bigcup_{h=1}^{H(\mathcal{D})} \Omega_h(\mathcal{D})$, for both $\mathcal{D} = \mathcal{D}^{knap}$ and $\mathcal{D} = \mathcal{D}^{prop}$, transforming the robust problem into a deterministic equivalent CVRP.

Other important modifications to the algorithm of [7] are the extension of the valid inequalities to the robust context and the implementation of a dedicated heuristic to find primal solutions. We report on our iterated local search (ILS) heuristic with variable neighborhood search (VNS).

4 Results

We performed preliminary computational tests on the instances with up to 150 customers proposed by [2, 3] for uncertainty set $\mathcal{D}^{prop}$. Our results indicate that (i) our ILS-VNS heuristic algorithm obtains significantly better solutions than the AMP heuristic proposed by [2] using a small fraction of its CPU time, and (ii) our exact algorithm is orders of magnitudes faster than [3], solving all instances to optimality.

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