Beyond Vehicle Routing: a General Purpose Branch-Cut-and-Price Code for Applications with Resource Constrained Shortest Path (RCSP) Pricing

Artur Pessoa[3] **Ruslan Sadykov**[1,2] Eduardo Uchoa[3]
François Vanderbeck[2,1]

[1] Inria Bordeaux, France
[2] Université Bordeaux, France
[3] Universidade Federal Fluminense, Brazil

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Modern Branch-Cut-and-Price for Vehicle Routing

- Bucket graph-based labelling algorithm for the RCSP pricing [Righini and Salani, 2006] [Sadykov et al., 2017]
- (Dynamic) partially elementary path (*ng*-path) relaxation [Baldacci et al., 2011b] [Roberti and Mingozzi, 2014] [Bulhoes et al., 2018b]
- Automatic dual price smoothing stabilization [Wentges, 1997] [Pessoa et al., 2017]
- Reduced cost fixing of (bucket) arcs in the pricing problem [Ibaraki and Nakamura, 1994] [Irnich et al., 2010] [Sadykov et al., 2017]
- Rounded Capacity Cuts [Laporte and Nobert, 1983] [Lysgaard et al., 2004]
- Limited-Memory Rank-1 Cuts [Jepsen et al., 2008] [Pecin et al., 2017b] [Pecin et al., 2017c] [Pecin et al., 2017a]
- Enumeration of elementary routes [Baldacci et al., 2008]
- Multi-phase strong branching [Pecin et al., 2017b]
- Generic (strong) diving heuristic [Sadykov et al., 2018]

Motivation

An expert team needs several months of work to implement a state-of-the-art Branch-Cut-and-Price algorithm

Our objective

A framework and a modelling tool which can be used to develop modern Branch-Cut-and-Price algorithms for specific problems faster and more easily

Set partitioning formulation over constrained path variables, with additional constraints and variables

- $\lambda_p^k = 1$ iff resource constrained path $p \in \mathcal{P}^k$ in directed graph $G^k = (V^k, A^k)$, $k \in K$, participates in the solution.
- $x^{p,k} \in \{0,1\}^{|A^k|}$ — characteristic vector of path $p \in \mathcal{P}^k$

$$\min \sum_{k \in K} \sum_{p \in \mathcal{P}^k} (c^k x^{p,k}) \lambda_p^k + f\, y$$

$$\begin{aligned}
\sum_{k \in K} \sum_{p \in \mathcal{P}^k} \sum_{a \in E_j} x_a^{p,k} \lambda_p^k \quad &= (\leq) \quad 1, \quad j \in J \\
\sum_{k \in K} \sum_{p \in \mathcal{P}^k} (D^k x^{p,k}) \lambda_p^k + D^0 y \quad &\geq \quad d \\
L^k \leq \sum_{p \in \mathcal{P}^k} \lambda_p^k \quad &\leq \quad U^k \quad \forall k \in K \\
y \quad &\in \quad \mathbb{N}^{n_I} \times \mathbb{R}^{n_F} \\
\lambda_p^k \quad &\in \quad \mathbb{N} \quad \forall k \in K, p \in \mathcal{P}^k
\end{aligned}$$

Elementarity (or packing) sets

$$E_j \subseteq \bigcup_{k \in K} A^k, \quad \forall j \in J$$

For each $j \in J$, at most one arc $a \in E_j$ can appear in the global solution of the problem.

Knowledge about elementarity sets allows us to apply important techniques,
critical for obtaining the state-of-the-art performance:

- Partially elementary paths (*ng*-paths) relaxation
- Limited-memory rank-1 packing cuts
- Enumeration of elementary routes

Information about the graphs in the model

For each graph G^k, $k \in K$

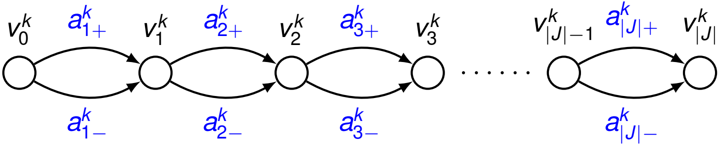
- Sets of vertices, arcs, resources V^k, A^k, R^k
- Source and the sink: v^k_{source}, v^k_{sink}
- Non-negative resource consumption of main resources $M^k \subseteq R^k$ on arcs: $q^k_{a,r} \in \mathbb{R}_+$, $a \in A^k$, $r \in M^k$
- Cycles of zero consumption of all main resources should not exist
- Unrestricted resource consumption of other resources on arcs: $q^k_{a,r} \in \mathbb{R}$, $a \in A^k$, $r \in R^k \setminus M^k$
- Resource consumption bounds on vertices: $[l^k_{v,r}, u^k_{v,r}]$, $v \in V^k$, $r \in R^k$

Example 1: Generalized Assignment Problem

The problem data

- set J of tasks, set K of machines of capacity Q^k, $k \in K$
- assignment cost c_j^k and size w_j^k, $j \in J$, $k \in K$

Graph G^k



- One resource with consumption: $q^k_{a^k_{j+}} = w^k_j$, $q^k_{a^k_{j-}} = 0$, $j \in J$
- Consumption bounds: $[l^k_{v^k_j}, u^k_{v^k_j}] = [0, Q^k]$, $j \in J \cup \{0\}$.

Generalized Assignment: formulation

$$\min \sum_{k \in K} \sum_{p \in \mathcal{P}^k} (c^k x^{p,k}) \lambda_p^k$$

$$\begin{aligned} \sum_{k \in K} \sum_{p \in \mathcal{P}^k} x_{a_{j+}^k}^{p,k} \lambda_p^k &= 1, \quad j \in J \\ \sum_{p \in \mathcal{P}^k} \lambda_p^k &\leq 1 \quad \forall k \in K \\ \lambda_p^k &\in \mathbb{N} \quad \forall k \in K, p \in \mathcal{P}^k \end{aligned}$$

Elementarity sets

$$E_j = \left\{ a_{j+}^k \right\}_{k \in K}, \quad j \in J$$

Example 2: Bin Packing and Vector Packing

The problem data

- set J of items, bin capacity Q_1 (and Q_2)
- item weights $w_{j,1}$ (and $w_{j,2}$)

Graph G

- $|J| + 2$ nodes : $v_{\text{source}} = v_0, v_1, \ldots, v_{|J|}, v_{\text{sink}} = v_{|J|+1}$
- $O(|J|^2)$ arcs : $a_{j'j} = (v_{j'}, v_j)$, $j', j \in J \cup \{0, |J| + 1\}$, $j' < j$.
- One (two) resources with consumption $q_{a_{j'j},1} = w_{j1}$ ($q_{a_{j'j},2} = w_{j2}$)
- Consumption bounds:

$$[l_{v_j,1}, u_{v_j,1}] = [0, Q_1], [l_{v_j,2}, u_{v_j,2}] = [0, Q_2], j \in J \cup \{0, |J| + 1\}$$

Bin Packing and Vector Packing: formulation

$$\min \sum_{p \in \mathcal{P}} \sum_{j \in J} x^p_{a_{0j}} \lambda_p$$

$$\begin{aligned} \sum_{p \in \mathcal{P}} \sum_{\substack{j' \in J: \\ j' < j}} x^p_{a_{j'j}} \lambda_p &= 1, \quad j \in J \\ \lambda_p &\in \mathbb{N}, \quad \forall p \in \mathcal{P} \end{aligned}$$

Elementarity sets

$$E_j = \{a_{j'j}\}_{j' \in J, j' < j}, \quad j \in J$$

Example 3: Team Orienteering Problem

The problem data

- Set J of locations, start and end points, m team members
- Profits c_j, $j \in J$, maximum tour length Q.

Graph G

- $|J| + 2$ nodes : $v_{\text{source}} = v_0, v_1, \ldots, v_{|J|}, v_{\text{sink}} = v_{|J|+1}$.
- $O(|J|^2)$ arcs : $a_{j'j} = (v_{j'}, v_j)$, $j' \in J \cup \{0\}$, $j \in J \cup \{|J|+1\}$.
- One resource: $q_{a_{j'j}} =$ Euclidean distance between j' and j
- Consumption bounds: $[l_{v_j}, u_{v_j}] = [0, Q]$, $j \in J \cup \{0, |J|+1\}$.

Variants

- Capacitated Team Orienteering Problem (CTOP)
- Capacitated Profitable Tour Problem (CPTP)

Team Orienteering Problem: formulation

$$\min \sum_{j \in J} c_j y_j$$

$$\begin{aligned}
\sum_{p \in \mathcal{P}} \sum_{j' \in J \cup \{0\}} x^p_{a_{j'j}} \lambda_p + y_j &= 1, \quad j \in J \\
\sum_{p \in \mathcal{P}} \lambda_p &= m, \\
y_j &\in \{0, 1\} \quad \forall j \in J \\
\lambda_p &\in \mathbb{N} \quad \forall p \in \mathcal{P}
\end{aligned}$$

Elementarity sets

$$E_j = \left\{a_{j'j}\right\}_{j' \in J \cup \{0\}}, \quad j \in J$$

Example 4: Pickup and Delivery with Time Windows

The problem data

- Set J of requests, set I of pickup (I^p), delivery (I^d) and depot ($j = 0, |I| - 1$) points, distances $d_{i'i}$, $i', i \in I$
- m vehicles with capacity Q and fixed cost f
- Request sizes w_j, $j \in J$.
- Time windows $[b_i, e_i]$ and service times s_i, $i \in I$.

Graph G

- $|I|$ nodes : $v_{\text{source}} = v_0, v_1, \ldots, v_{\text{sink}} = v_{|I|-1}$.
- $O(|I|^2)$ arcs : $a_{j'j} = (v_{j'}, v_j)$, $j' \in I \setminus \{|I| - 1\}$, $j \in J \setminus \{0\}$.
- $2 + |J|$ resources :
  - Time (main) resource: $q_{a_{i'i},1} = s_{i'} + d_{i'i}$
  - Capacity resource: $q_{a_{i'i},2} = \begin{cases} w_i, & i \in I^p, \\ -w_{i-|J|}, & i \in I^d. \end{cases}$
  - One binary resource for every request

Pickup and Delivery with Time Windows: formulation

$$\min \sum_{p \in \mathcal{P}} \left(\sum_{i \in I \setminus \{0\}} f \cdot x^p_{a_{0i}} + \sum_{i', i \in I} d_{i'i} x_{a_{i'i}} \right) \lambda_p$$

$$\begin{aligned} \sum_{p \in \mathcal{P}} \sum_{i' \in I \setminus \{i\}} x^p_{a_{i'i}} \lambda_p &= 1, \quad i \in I^p \\ \sum_{p \in \mathcal{P}} \lambda_p &\leq m, \\ \lambda_p &\in \mathbb{N} \quad \forall p \in \mathcal{P} \end{aligned}$$

Elementarity sets

$$E_i = \{a_{i'i}\}_{i' \in I \setminus \{i\}}, \quad i \in I^p$$

Non-robust rank-1 cuts [Jepsen et al., 2008] [Pecin et al., 2017c]

Each cut $\eta \in \mathcal{N}$ is obtained by a Chvátal-Gomory rounding of a set $\mathcal{C}_\eta \subseteq J$ of set packing constraints using a vector of multipliers ρ^η ($0 < \rho_j^\eta < 1, j \in \mathcal{C}_\eta$)

$$\sum_{k \in K} \sum_{p \in \mathcal{P}^k} \left\lfloor \sum_{j \in \mathcal{C}_\eta} \rho_j^\eta \sum_{a \in E_j} x_a^p \right\rfloor \lambda_p \leq \left\lfloor \sum_{j \in \mathcal{C}_\eta} \rho_j^\eta \right\rfloor$$

- Each active cut $\eta \in \mathcal{N}$ adds one resource in the RCSP pricing
- Limited-memory technique [Pecin et al., 2017b] is critical to reduce the impact on the pricing problem difficulty: for each E_j, $j \in \mathcal{C}_\eta$, a memory (on vertices or on arcs) is defined at the separation, making the resource local

Enumeration of elementary paths [Baldacci et al., 2008]

- We try to enumerate all elementary paths with reduced cost smaller than the current primal-dual gap in each graph G^k
- A labelling algorithm is used for enumeration
- If G^k is enumerated, the pricing can be done by inspection
- If all graphs are enumerated and the total number of paths is “small”, the problem can be finished by a MIP solver

Sufficient condition to apply

- Arcs in the same elementarity set should have the same coefficients in the “core” master constraints (excluding cuts and branching constraints)
- Arcs not in an elementarity set should not participate in the “core” master constraints

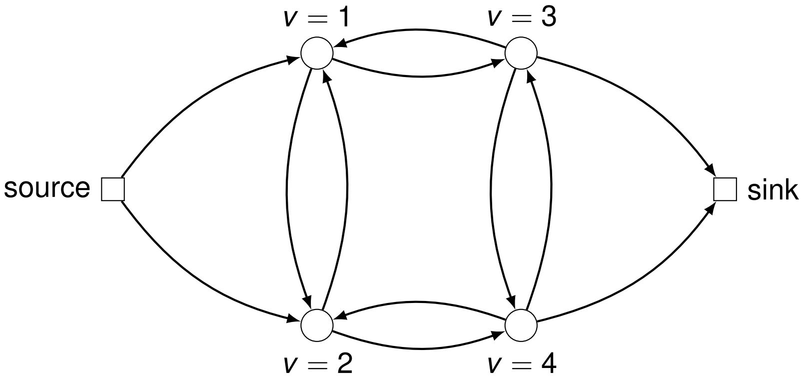
Pricing: structure of RCSP problem instances

- A directed graph $G = (V, A)$.
- Unrestricted in sign reduced costs $\bar{c}_a$ on arcs $a \in A$
- Main resources with non-negative resource consumption $d_{a,r} \in \mathbb{R}_+$, $a \in A$, $r \in M$
- Possibly other resources with unrestricted resource consumption $d_{a,r} \in \mathbb{R}$, $a \in A$, $r \in R \setminus M$.
- Up to $\approx 500 - 1000$ of (more or less) local binary or (small) integer resources

We want to

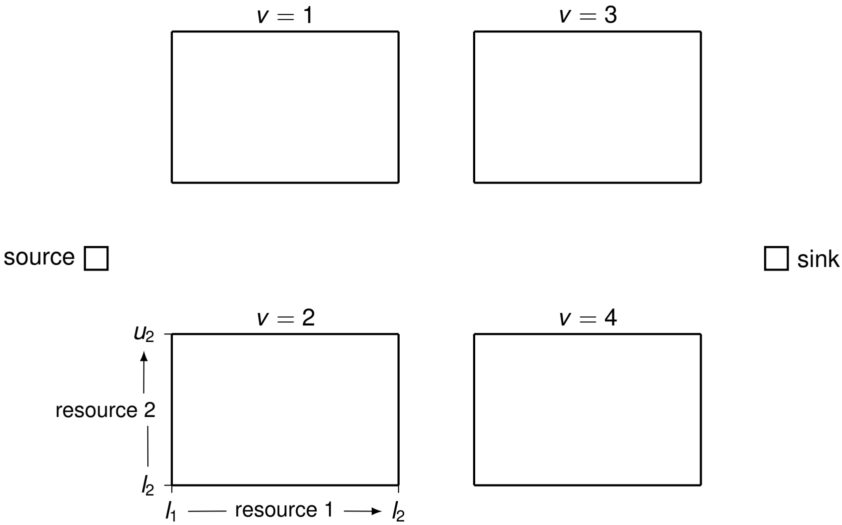
Find a walk from the source to the sink minimizing the total reduced cost respecting the resource constrains, as well as many other (up to 1000) different near-optimal feasible walks

Pricing: original graph



Pricing : the bucket graph [Sadykov et al., 2017]

Example with two main resources



Pricing : the bucket graph [Sadykov et al., 2017]

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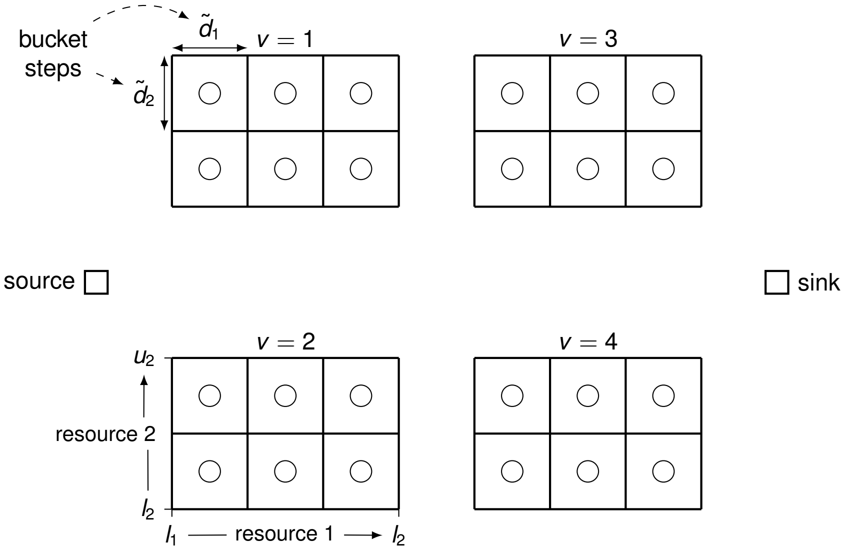

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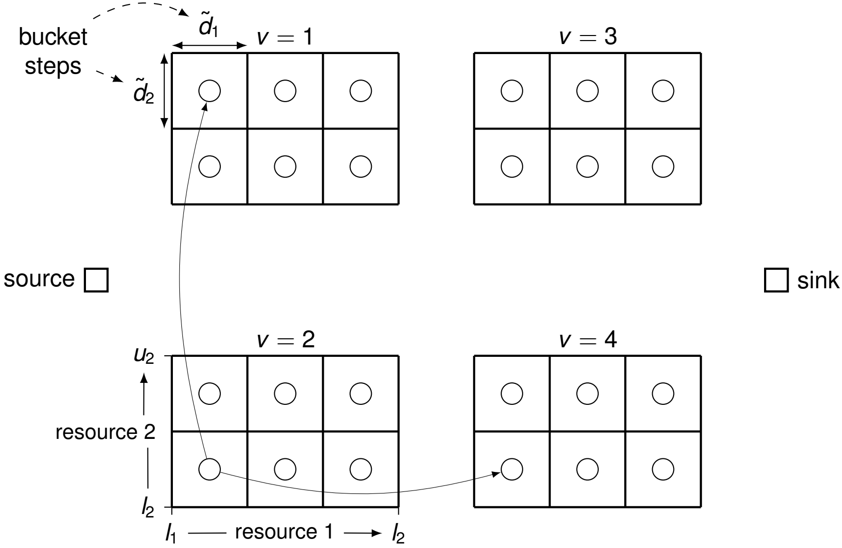
Example with two main resources



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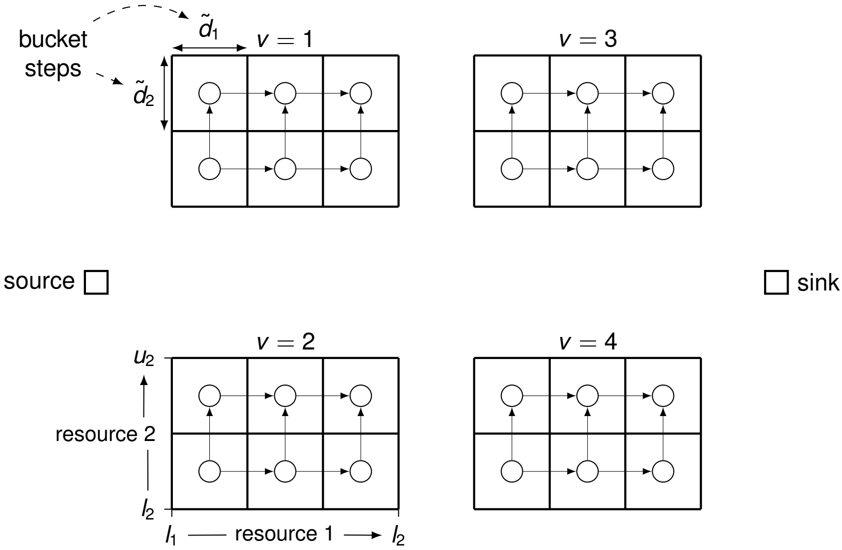

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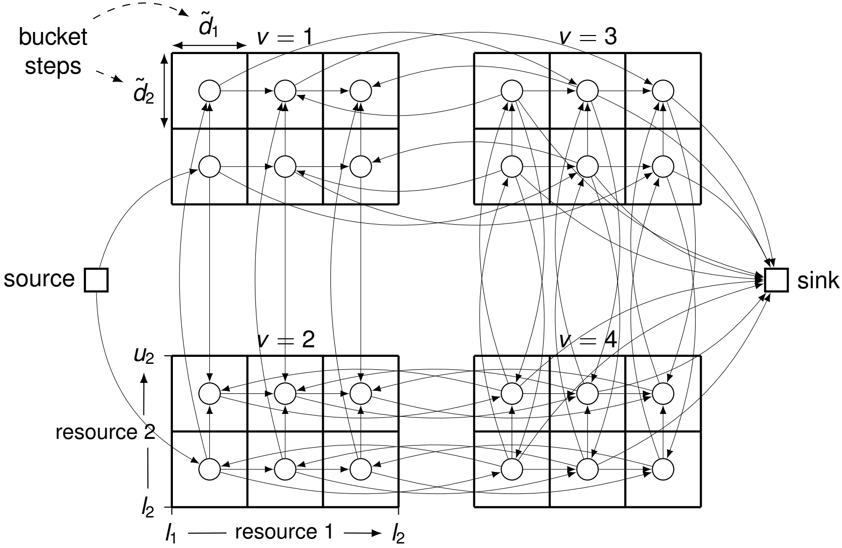

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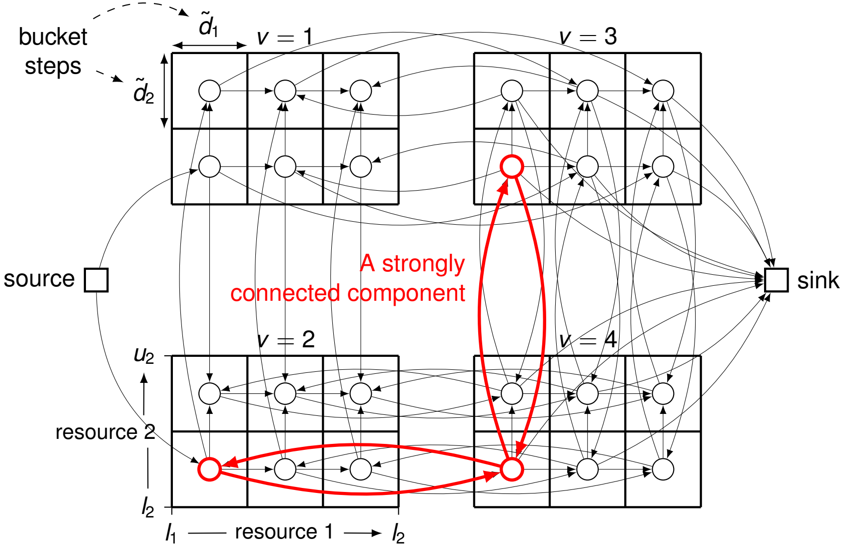
Example with two main resources



Generalized Assignment: computational results

Comparison with on classic instances in class D

$\|K\|$	$\|J\|$	[Posta et al., 2012]	Our BCP
5	100	6s	58s
10	100	55s	1m19s
20	100	1h47m	12m48s
5	200	4s	4m30s
10	200	26m02s	1h40m

Comparison on random instances in class D

$\|K\|$	$\|J\|$	Solved within 3000 seconds	
		[Nauss, 2003]	Our BCP
25	90	0/10	10/10
25	100	0/10	6/10
30	100	0/10	9/10

Gurobi 7.5 can solve only 1 instance in 3 hours.

Bin Packing: computational results

Comparison with the best BCP [Belov and Scheithauer, 2006] (results from [Delorme et al., 2016])

N — number of instances solved within 1 hour
T — average time

Instance class	$\|J\|$	Best BCP		Our BCP	
		N	T	N	T
Falkenauer T	60-501	80/80	$>$24s	80/80	35s
Hard28	160-200	28/28	7s	28/28	54s
ANI200	201	50/50	2m24s	50/50	56s
ANI400	402	1/50	$>$1h	50/50	14m17s

State-of-the-art

Iterative aggregation-disaggregation approach by [Clautiaux et al., 2017]

Vertex Packing: computational results

Comparison with the state-of-the-art on the harderst 2-resources 200-items instances by [Caprara and Toth, 2001]

Algorithm	Class 1		Class 4	
	N	*T*	*N*	*T*
[Brandão and Pedroso, 2016]	10/10	2h07m	0/10	$>$2h
[Hu et al., 2017]	0/10	$>$10m	0/10	$>$10m
[Hessler et al., 2017]	3/10	$>$47m	0/10	$>$1h
Our BCP	10/10	3m53s	10/10	7m40s
	Class 5		Class 9	
[Brandão and Pedroso, 2016]	0/10	$>$2h	0/10	$>$2h
[Hu et al., 2017]	7/10	$>$6m	0/10	$>$10m
[Hessler et al., 2017]	7/10	$>$41m	0/10	$>$1h
Our BCP	7/10	$>$27m	8/10	$>$19m

Team Orienteering: computational results

Standard Team Orienteering problem

Comparison with the state-of-the-art on the most difficult class 4 of classic instances with 100 locations by [Chao et al., 1996]

Algorithm	N	T
[Bianchessi et al., 2018]	39/60	$>$15m
Our BCP	55/60	$>$8m

Capacitated Team Orienteering problem

Comparison with the state-of-the-art on the basic and most difficult instances with 51-200 locations by [Archetti et al., 2009]

Algorithm	N	T
[Archetti et al., 2013]	6/14	$>$35m
Our BCP	13/14	$>$7m

Other computational results

Capacitated Profitable Tour Problem

Comparison with the state-of-the-art on the open instances with 51-200 locations by [Archetti et al., 2009]

Algorithm	N	T
[Bulhoes et al., 2018a]	0/28	$>$1h
Our BCP	24/28	$>$9m

Pickup and Delivery Problem With Time Windows

Comparison with the state-of-the-art on 40 classic instances with 30-75 requests by [Ropke and Cordeau, 2009]

Algorithm	N	T
[Baldacci et al., 2011a]	32/40	$>$13m44s
[Gschwind et al., 2018]	33/40	$>$12m31s
Our BCP	40/40	5m06s

Our BCP is not competitive on instances by [Li and Lim, 2003].

Conclusions

- Our generic BCP algorithm showed good performance approaching or outperforming the state-of-the-art for apparently different problems
- Relatively short developpement time

Issues to address

- Dependence on initial primal bounds in many cases
- Involved parameterisation
- The code needs much more testing before it can be used externally

Perspectives

July 2018 A state-of-the-art web-based solver for some Vehicle Routing problems for ISMP conference

- Capacitated VRP
- VRP with time windows
- Heterogeneous fleet VRP
- Multi-depot VRP
- Site-dependent VRP

$\approx$ 2019 A Julia-JuMP based modelling tool with the precompiled library implementing the modern Branch-Cut-and-Price

References I

Archetti, C., Bianchessi, N., and Speranza, M. (2013).
Optimal solutions for routing problems with profits.
Discrete Applied Mathematics, 161(4):547 – 557.

Archetti, C., Feillet, D., Hertz, A., and Speranza, M. G. (2009).
The capacitated team orienteering and profitable tour problems.
Journal of the Operational Research Society, 60(6):831–842.

Baldacci, R., Bartolini, E., and Mingozzi, A. (2011a).
An exact algorithm for the pickup and delivery problem with time windows.
Operations Research, 59(2):414–426.

Baldacci, R., Christofides, N., and Mingozzi, A. (2008).
An exact algorithm for the vehicle routing problem based on the set partitioning formulation with additional cuts.
Mathematical Programming, 115:351–385.

References II

Baldacci, R., Mingozzi, A., and Roberti, R. (2011b).
New route relaxation and pricing strategies for the vehicle routing problem.
Operations Research, 59(5):1269–1283.

Belov, G. and Scheithauer, G. (2006).
A branch-and-cut-and-price algorithm for one-dimensional stock cutting and two-dimensional two-stage cutting.
European Journal of Operational Research, 171(1):85 – 106.

Bianchessi, N., Mansini, R., and Speranza, M. G. (2018).
A branch-and-cut algorithm for the team orienteering problem.
International Transactions in Operational Research, 25(2):627–635.

Brandão, F. and Pedroso, J. a. P. (2016).
Bin packing and related problems: General arc-flow formulation with graph compression.
Computers & Operations Research, 69:56 – 67.

References III

Bulhoes, T., Hà, M. H., Martinelli, R., and Vidal, T. (2018a).
The vehicle routing problem with service level constraints.
European Journal of Operational Research, 265(2):544 – 558.

Bulhoes, T., Sadykov, R., and Uchoa, E. (2018b).
A branch-and-price algorithm for the minimum latency problem.
Computers & Operations Research, 93:66–78.

Caprara, A. and Toth, P. (2001).
Lower bounds and algorithms for the 2-dimensional vector packing problem.
Discrete Applied Mathematics, 111(3):231 – 262.

Chao, I.-M., Golden, B. L., and Wasil, E. A. (1996).
The team orienteering problem.
European Journal of Operational Research, 88(3):464 – 474.

References IV

Clautiaux, F., Hanafi, S., Macedo, R., Émilie Voge, M., and Alves, C. (2017).
Iterative aggregation and disaggregation algorithm for pseudo-polynomial network flow models with side constraints.
European Journal of Operational Research, 258(2):467 – 477.

Delorme, M., Iori, M., and Martello, S. (2016).
Bin packing and cutting stock problems: Mathematical models and exact algorithms.
European Journal of Operational Research, 255(1):1–20.

Gschwind, T., Irnich, S., Rothenbächer, A.-K., and Tilk, C. (2018).
Bidirectional labeling in column-generation algorithms for pickup-and-delivery problems.
European Journal of Operational Research, 266(2):521 – 530.

Hessler, K., Gschwind, T., and Irnich, S. (2017).
Stabilized branch-and-price algorithms for vector packing problems.
Discussion paper 1713, Johannes Gutenberg University Mainz.

References V

Hu, Q., Zhu, W., Qin, H., and Lim, A. (2017).
A branch-and-price algorithm for the two-dimensional vector packing problem with piecewise linear cost function.
European Journal of Operational Research, 260(1):70 – 80.

Ibaraki, T. and Nakamura, Y. (1994).
A dynamic programming method for single machine scheduling.
European Journal of Operational Research, 76(1):72 – 82.

Irnich, S., Desaulniers, G., Desrosiers, J., and Hadjar, A. (2010).
Path-reduced costs for eliminating arcs in routing and scheduling.
INFORMS Journal on Computing, 22(2):297–313.

Jepsen, M., Petersen, B., Spoorendonk, S., and Pisinger, D. (2008).
Subset-row inequalities applied to the vehicle-routing problem with time windows.
Operations Research, 56(2):497–511.

References VI

Laporte, G. and Nobert, Y. (1983).
A branch and bound algorithm for the capacitated vehicle routing problem.
Operations-Research-Spektrum, 5(2):77–85.

Li, H. and Lim, A. (2003).
A metaheuristic for the pickup and delivery problem with time windows.
International Journal on Artificial Intelligence Tools, 12(02):173–186.

Lysgaard, J., Letchford, A. N., and Eglese, R. W. (2004).
A new branch-and-cut algorithm for the capacitated vehicle routing problem.
Mathematical Programming, 100(2):423–445.

Nauss, R. M. (2003).
Solving the generalized assignment problem: An optimizing and heuristic approach.
INFORMS Journal on Computing, 15(3):249–266.

References VII

Pecin, D., Contardo, C., Desaulniers, G., and Uchoa, E. (2017a).
New enhancements for the exact solution of the vehicle routing problem with time windows.
INFORMS Journal on Computing, 29(3):489–502.

Pecin, D., Pessoa, A., Poggi, M., and Uchoa, E. (2017b).
Improved branch-cut-and-price for capacitated vehicle routing.
Mathematical Programming Computation, 9(1):61–100.

Pecin, D., Pessoa, A., Poggi, M., Uchoa, E., and Santos, H. (2017c).
Limited memory rank-1 cuts for vehicle routing problems.
Operations Research Letters, 45(3):206 – 209.

Pessoa, A., Sadykov, R., Uchoa, E., and Vanderbeck, F. (2017).
Automation and combination of linear-programming based stabilization techniques in column generation.
Technical Report hal-01077984, HAL Inria.

References VIII

Posta, M., Ferland, J. A., and Michelon, P. (2012).
An exact method with variable fixing for solving the generalized assignment problem.
Computational Optimization and Applications, 52:629–644.

Righini, G. and Salani, M. (2006).
Symmetry helps: Bounded bi-directional dynamic programming for the elementary shortest path problem with resource constraints.
Discrete Optimization, 3(3):255 – 273.

Roberti, R. and Mingozzi, A. (2014).
Dynamic ng-path relaxation for the delivery man problem.
Transportation Science, 48(3):413–424.

Ropke, S. and Cordeau, J.-F. (2009).
Branch and cut and price for the pickup and delivery problem with time windows.
Transportation Science, 43(3):267–286.

References IX

Sadykov, R., Uchoa, E., and Pessoa, A. (2017).
A bucket graph based labeling algorithm with application to vehicle routing.
Cadernos do LOGIS 7, Universidade Federal Fluminense.

Sadykov, R., Vanderbeck, F., Pessoa, A., Tahiri, I., and Uchoa, E. (2018).
Primal heuristics for branch-and-price: the assets of diving methods.
INFORMS Journal on Computing, (Forthcoming).

Wentges, P. (1997).
Weighted dantzig–wolfe decomposition for linear mixed-integer programming.
International Transactions in Operational Research, 4(2):151–162.