Branch-Cut-and-Price solver for Vehicle Routing Problems

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Contents

Modern Branch-Cut-and-Price for Vehicle Routing

- Bucket graph-based labelling algorithm for the RCSP pricing [Desrosiers et al., 1983] [Righini and Salani, 2006] [Sadykov et al., 2017]
- (Dynamic) partially elementary path (*ng*-path) relaxation [Baldacci et al., 2011] [Roberti and Mingozzi, 2014] [Bulhoes et al., 2018]
- Automatic dual price smoothing stabilization [Wentges, 1997] [Pessoa et al., 2018b]
- Reduced cost fixing of (bucket) arcs in the pricing problem [Ibaraki and Nakamura, 1994] [Irnich et al., 2010] [Sadykov et al., 2017]
- Rounded Capacity Cuts [Laporte and Nobert, 1983] [Lysgaard et al., 2004]
- Limited-Memory Rank-1 Cuts [Jepsen et al., 2008] [Pecin et al., 2017b] [Pecin et al., 2017c] [Pecin et al., 2017a]
- Enumeration of elementary routes [Baldacci et al., 2008] [Contardo and Martinelli, 2014]
- Multi-phase strong branching [Røpke, 2012] [Pecin et al., 2017b]

Motivation

An expert team needs

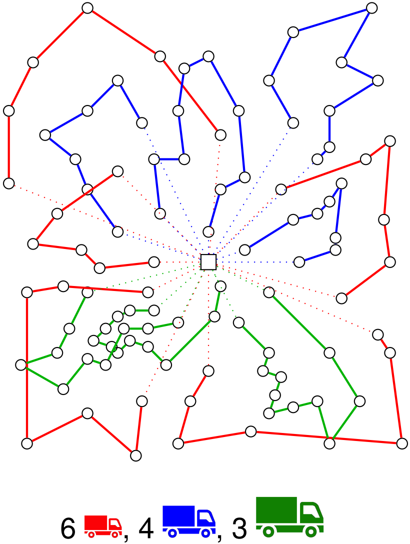
several months of work

to implement a state-of-the-art
Branch-Cut-and-Price algorithm

Our objective

An implementation which may be easily used by researchers

Vehicle Routing Problem



Set V of customers, each $v \in V$ with demand w_v, service time t_v and time window $[l_v, u_v]$.

Set M of vehicle types, each $m \in M$ with a depot $v_{|V|+m}$ with U_m vehicles of capacity W_m, with vectors of travel costs c^m and travel distances d^m.

Variants

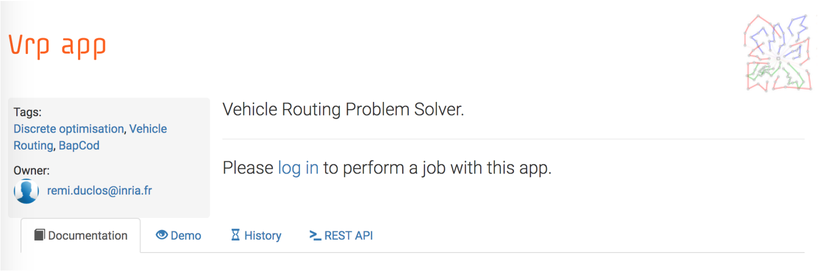
- CVRP (with time windows)
- Distance-constrained VRP
- Heterogeneous VRP
- Multi-depot VRP
- Site-dependent VRP

Contents

Logging in

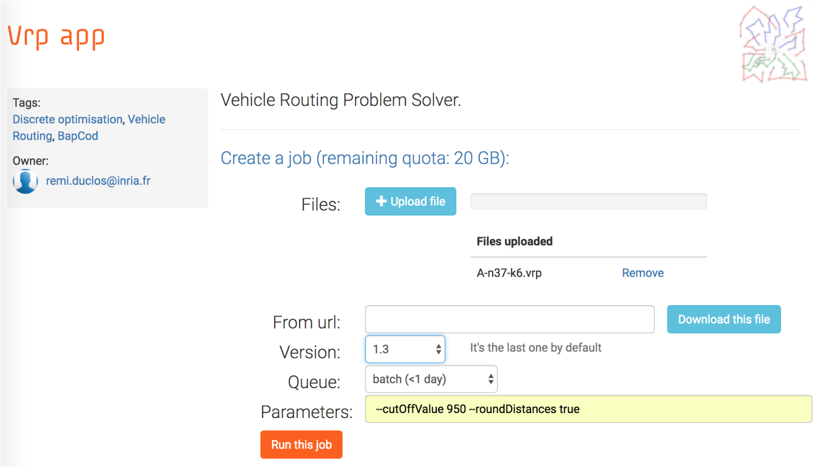
Address

`allgo.inria.fr/app/vehiclerouting`



One needs to sign up (e-mail and password only) and log in.

Running the solver



Parameters

roundDistances — whether to round the distances between clients

Initial upper bound

- By default, jsprit (`github.com/graphhopper/jsprit`) heuristic is used (Ruin-and-Reconstruct Local Search)
- jspritMaxIteration — number of iterations in the heuristic (default is 500),
- cutOffValue — value specified by the user
  if given, jsprit heuristic can be turned off
  (jspritMaxIteration=0)

Supported problems data formats

CVRP/DCVRP	CVRPLIB format (`vrp.atd-lab.inf.puc-rio.br`)
VRPTW	[Solomon, 1987] [Gehring and Homberger, 2002] format (total distance objective)
MDVRP/SDVRP	[Cordeau et al., 1997] format
HVRP	[Taillard, E. D., 1999] format [Duhamel et al., 2011] format (explicit distances) [Pessoa et al., 2018a] format (CVRPLIB)
Other	Let us know!!!

Other problems

Extension to other VRP variants can be considered (contact e-mail is `ruslan.sadykov@inria.fr`)

Output of the solver

Job : 39497

Destroy | Report a problem

App:

VRP App

Version:

1.3

Parameters:

--cutOffValue 950 --roundDistances true --jspritMaxIteration 0

Status:

done

Queue:

batch (<1 day)

Download zip:

allgo_job_39497.zip

Files:

solution_A-n37-k6.txt 636 Bytes

```
Best found solution of value 949 :
Vehicle 1 of type 1 : depot -> 16 -> 35 -> 25 -> 7 -> depot, demand = 99, time = 104
Vehicle 2 of type 1 : depot -> 13 -> 30 -> 15 -> 32 -> 27 -> depot, demand = 100, time
Vehicle 3 of type 1 : depot -> 33 -> 2 -> 28 -> 23 -> 22 -> 12 -> 11 -> 10 -> 4 -> depo
Vehicle 4 of type 1 : depot -> 24 -> 29 -> 36 -> 6 -> 14 -> depot, demand = 88, time =
Vehicle 5 of type 1 : depot -> 18 -> 26 -> 21 -> 9 -> 19 -> 31 -> depot, demand = 97, t
Vehicle 6 of type 1 : depot -> 17 -> 34 -> 1 -> 3 -> 5 -> 8 -> 20 -> depot, demand = 89
Solution is feasible
```

outputSummary_A-n37-k6.txt 101 Bytes

```
Model: A-n37-k6.vrp
bestDualBound: 949
bestIncumbent: 949
bcCountNodeProc: 1

TimeMain= 8.24 seconds
```

A-n37-k6.vrp 808 Bytes

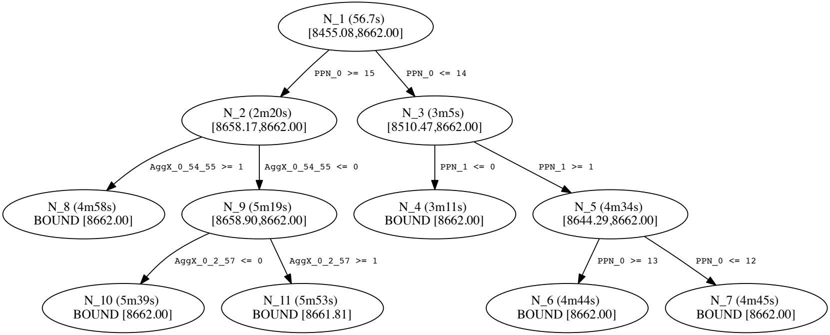
allgo.log 109 Bytes

```
--cutOffValue 950 --roundDistances true --jspritMaxIteration 0
Solution found

====  ALLGO JOB SUCCESS  ====
```

BaPTree_A-n37-k6.pdf 4.24 KB

Example of the branching tree



Remarks

- Academic use only
- Your instance file is kept on the server
- Third party software used:
  - IBM ILOG Cplex — LP and MIP solver (replacement by Cbc is coming!)
  - jsprit (`github.com/graphhopper/jsprit`) — heuristic vehicle routing solver
  - LEMON Graph ibrary (`http://lemon.cs.elte.hu/trac/lemon`)
  - Boost C++ libraries (`www.boost.org`)
  - CVRPSEP — RCC separator [Lysgaard et al., 2004]
- The solver is in beta version, please report us all issues!

Performance

State-of-the-art exact solver for all the problems!

Pessoa, A., Sadykov, R., and Uchoa, E. (2018a).
Enhanced branch-cut-and-price algorithm for heterogeneous fleet vehicle routing problems.
European Journal of Operational Research, 270(2):530–543.

Sadykov, R., Uchoa, E., and Pessoa, A. (2017).
A bucket graph based labeling algorithm with application to vehicle routing.
Cadernos do LOGIS 7, Universidade Federal Fluminense.

Remarks

- CVRP performance is on pair with [Pecin et al., 2017b]
- Performance is very dependent on the initial upper bound

Contents

Set partitioning (master) formulation

- R_m — set of feasible elementary routes for a type m vehicle
- a_v^r — number of times vertex v appear in route r.
- c_r — cost of route r
- Binary variable $\lambda_r = 1$ if and only if route $r \in R_m$ is used by a vehicle of type m

$$\min \sum_{m \in M} \sum_{r \in R_m} c_r \lambda_r$$

$$\begin{aligned} \sum_{m \in M} \sum_{r \in R_m} a_v^r \lambda_r &= 1, \quad \forall v \in V, \\ \sum_{r \in R_m} \lambda_r &\leq U_m, \quad \forall m \in M, \\ \lambda_r &\in \{0, 1\}, \quad \forall r \in R_m, \forall m \in M. \end{aligned}$$

Pricing sub-problem for a vehicle type m

Given dual solution π of the restricted master problem, the pricing problem is

$$\min_{r \in R_m} \bar{c}_r = \sum_{v,v' \in V'_m} \left(c^m_{(v,v')} - \pi_{v'} \right) x^r_{(v,v')}$$

i.e. the elementary shortest path problem with capacity and time resources.

Labels

Each label $L = (v^L, \bar{c}^L, w^L, t^L, \mathcal{V}^L)$ represents a partial route.
It dominates another label L' if

$$v^L = v^{L'},\ \bar{c}^L \leq \bar{c}^{L'},\ w^L \leq w^{L'},\ t^L \leq t^{L'},\ \mathcal{V}^L \subseteq \mathcal{V}^{L'}$$

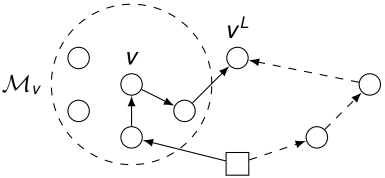
Partial relaxation of the elementarity : *ng*-paths

[Baldacci et al., 2011]

For each vertex $v \in V$, define a memory $\mathcal{M}_v$ of vertices which "remember" v.

If $v^L \notin \mathcal{M}_v$, v is removed from $\mathcal{V}^L$.

Sets $\mathcal{V}^L$ are smaller $\Rightarrow$ stronger domination



Small neighbourhoods (of size ≈8-10) produce a tight relaxation of elementarity constraints for most instances.

Memories $\mathcal{M}_v$ are dynamically augmented [Roberti and Mingozzi, 2014] [Bulhoes et al., 2018]

Baldacci, R., Mingozzi, A., and Roberti, R. (2011).
New route relaxation and pricing strategies for the vehicle routing problem.
Operations Research, 59(5):1269–1283.

Non-robust rank-1 cuts [Jepsen et al., 2008] [Pecin et al., 2017b] [Pecin et al., 2017c] [Pecin et al., 2017a]

Each cut $\eta \in \mathcal{N}$ is obtained by a Chvátal-Gomory rounding of a set $\mathcal{C}_\eta \subseteq V$ of set packing constraints using a vector of multipliers ρ^η $(0 < \rho^\eta_v < 1, v \in \mathcal{C}_\eta)$

$$\sum_{m \in M} \sum_{r \in \mathcal{R}_m} \left\lfloor \sum_{v \in \mathcal{C}_\eta} \rho^\eta_v a^r_v \right\rfloor \lambda_r \leq \left\lfloor \sum_{v \in \mathcal{C}_\eta} \rho^\eta_v \right\rfloor$$

- An active cut $\eta \in \mathcal{N}$ adds one resource in the RCSP pricing
- Limited-memory technique [Pecin et al., 2017b] is critical to reduce the impact on the pricing problem difficulty: for each cut, a memory (on vertices or on arcs) is defined at the separation, making the resource local

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Pecin, D., Pessoa, A., Poggi, M., and Uchoa, E. (2017b).
Improved branch-cut-and-price for capacitated vehicle routing.
Mathematical Programming Computation, 9(1):61–100.

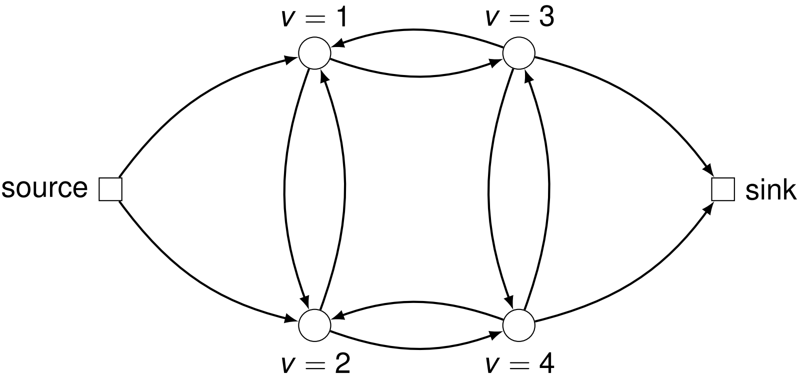
Pricing: structure of RCSPP instances

- A complete directed graph.
- Unrestricted in sign reduced costs on arcs
- Two global (capacity and time) resources with non-negative continuous resource consumption
- Up to $\approx 500 - 1000$ of (more or less) local binary or (small) integer resources
- Many different optimal (or near-optimal) solutions.

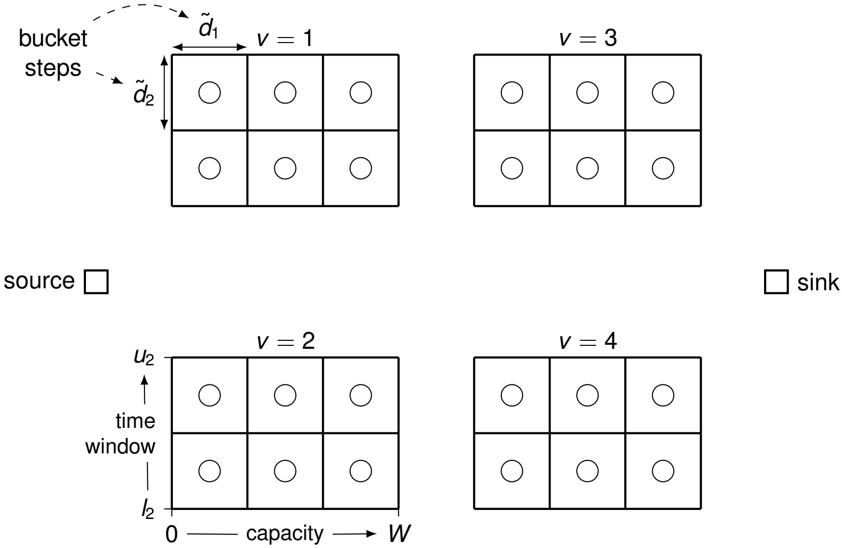
We want to

Find a walk minimizing the total reduced cost respecting the resource constrains, as well as many other (up to 1000) different near-optimal feasible walks

Pricing: original graph



Pricing : the bucket graph [Sadykov et al., 2017]



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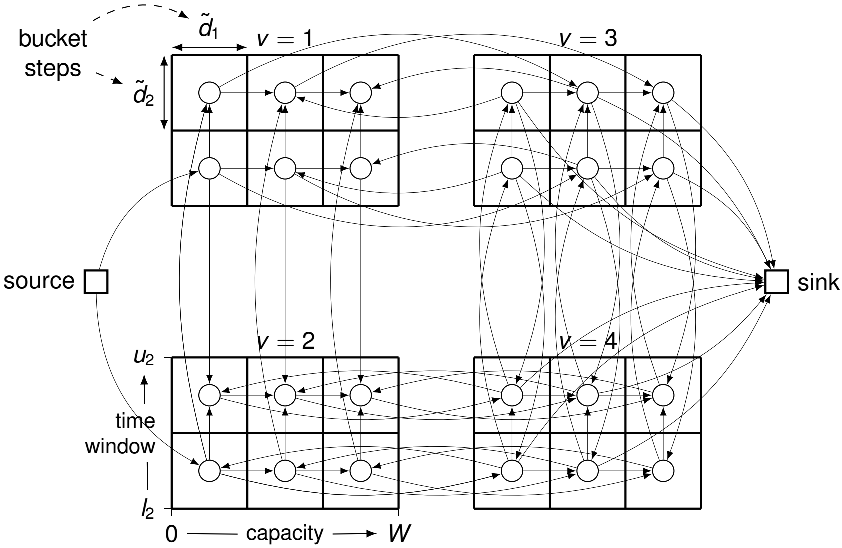
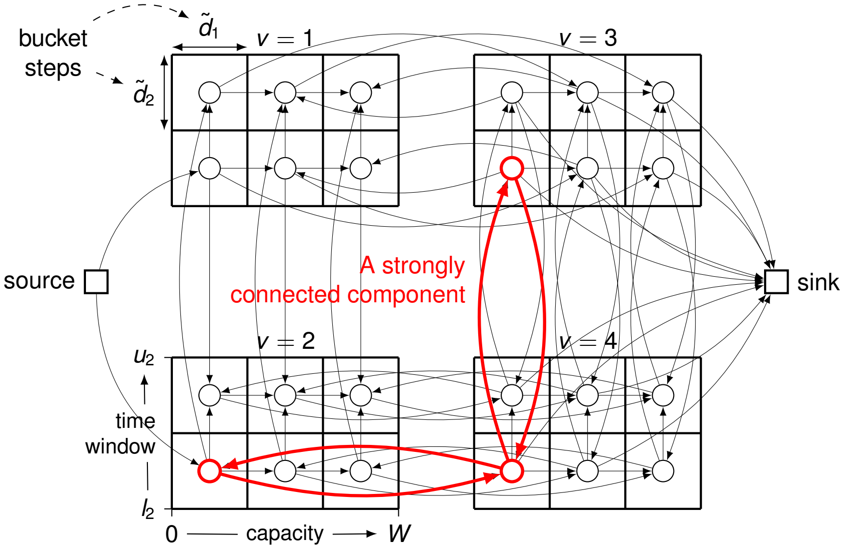


Pricing : the bucket graph [Sadykov et al., 2017]



Bucket graph labeling algorithm [Sadykov et al., 2017]

Features:

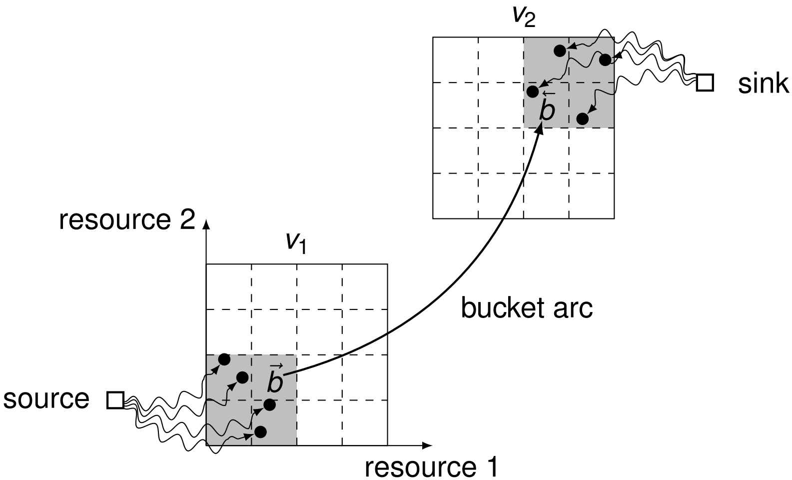
- Bidirectional search + concatenation [Righini and Salani, 2006]
- If the bucket graph is acyclic the algorithm is label setting
- Otherwise, it becomes label correcting. Buckets in the same strongly connected component are treated together.
- The bucket graph structure helps to reduce the number of label dominance checks and speed up concatenation
- Arcs in the bucket graph can be fixed by reduced cost, generalizing the fixing schemes in [Ibaraki and Nakamura, 1994] [Irnich et al., 2010]

Sadykov, R., Uchoa, E., and Pessoa, A. (2017).
A bucket graph based labeling algorithm with application to vehicle routing.
Cadernos do LOGIS 7, Universidade Federal Fluminense.

Fixing of bucket arcs by reduced cost [Sadykov et al., 2017]

A sufficient condition to fix a bucket arc $(\vec{b}, (v_1, v_2), \overleftarrow{b})$

No pair $(\vec{L}, \overleftarrow{L})$ of labels at vertices $v^{\vec{L}} = v_1$, $v^{\overleftarrow{L}} = v_2$, in "gray" buckets producing a path by concatenation along arc (v_1, v_2) with reduced cost smaller than the current primal-dual gap.



Enumeration of elementary paths [Baldacci et al., 2008] [Contardo and Martinelli, 2014]

- After each reduced cost fixing, we try to enumerate into a pool all elementary paths with reduced cost smaller than the current primal-dual gap in each graph G^k
- A labeling algorithm is used for enumeration
- If G^k is enumerated, the pricing can be done by inspection
- If all graphs are enumerated and the total number of paths is "small", the problem can be finished by a MIP solver

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Baldacci, R., Christofides, N., and Mingozzi, A. (2008).
An exact algorithm for the vehicle routing problem based on the set partitioning formulation with additional cuts.
Mathematical Programming, 115:351–385.

Contardo, C. and Martinelli, R. (2014).
A new exact algorithm for the multi-depot vehicle routing problem under capacity and route length constraints.
Discrete Optimization, 12:129 – 146.

Dual price smoothing stabilization [Wentges, 1997]

- $\overline{\pi}$ — current dual solution of the restricted master
- π^* — dual solution giving the best Lagrangian bound so far
- We solve the pricing problem using the dual vector

$$\pi' = (1 - \alpha) \cdot \overline{\pi} + \alpha \cdot \pi^*,$$

  where $\alpha \in [0, 1)$.
- Parameter α is automatically adjusted in each column generation iteration using the sub-gradient of the Lagrangian function at π' [Pessoa et al., 2018b].

Wentges, P. (1997).
Weighted dantzig–wolfe decomposition for linear mixed-integer programming.
International Transactions in Operational Research, 4(2):151–162.

Pessoa, A., Sadykov, R., Uchoa, E., and Vanderbeck, F. (2018b).
Automation and combination of linear-programming based stabilization techniques in column generation.
INFORMS Journal on Computing, 30(2):339–360.

Branching

Strong branching [Røpke, 2012] [Pecin et al., 2017b]

- Multi-strategy
- Branching history (pseudo-costs)
- Multi-phase

Branching strategies

- Number of vehicles
- Assignment of customers to vehicle types
- Participation of arcs in routes

Røpke, S. (2012).
Branching decisions in BCP algorithms for vehicle routing problems.
Presentation in Column Generation 2012.

Pecin, D., Pessoa, A., Poggi, M., and Uchoa, E. (2017b).
Improved branch-cut-and-price for capacitated vehicle routing.
Mathematical Programming Computation, 9(1):61–100.

Questions?

Suggestions?

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Operations Research, 59(5):1269–1283.

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A branch-and-price algorithm for the minimum latency problem.
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Plus court chemin avec contraintes d'horaires.
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Duhamel, C., Lacomme, P., and Prodhon, C. (2011).
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Computers and Operations Research, 38(4):723 – 739.

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Irnich, S., Desaulniers, G., Desrosiers, J., and Hadjar, A. (2010).
Path-reduced costs for eliminating arcs in routing and scheduling.
INFORMS Journal on Computing, 22(2):297–313.

Jepsen, M., Petersen, B., Spoorendonk, S., and Pisinger, D. (2008).
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Operations Research, 56(2):497–511.

Laporte, G. and Nobert, Y. (1983).
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Operations-Research-Spektrum, 5(2):77–85.

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Mathematical Programming, 100(2):423–445.

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New enhancements for the exact solution of the vehicle routing problem with time windows.
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Limited memory rank-1 cuts for vehicle routing problems.
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Roberti, R. and Mingozzi, A. (2014).
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Transportation Science, 48(3):413–424.

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