Branch-cut-and-price algorithms for the vehicle routing problem with backhauls

Eduardo Queiroga[1] Yuri Frota[1] **Ruslan Sadykov**[2]
Anand Subramanian[3] Eduardo Uchoa[1] Thibaut Vidal[4]

[1] Univ. Federal Fluminense Brazil
[2] Inria Bordeaux, France
[3] Univ. Federal da Paraíba Brazil
[4] PUC-Rio Brazil

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Motivation 1

Perspectives in the survey by [Koç and Laporte, 2018]:

- *"Our belief that further studies should focus on developing effective and powerful exact methods, such as branch-and-cut-and-price, to solve all available standard VRPB instances to optimality."*
- *"No exact algorithm has yet been proposed for the time windows extension of the VRPB. This type of effective algorithms could be applied to the VRPB with time windows."*

Koç, Ç. and Laporte, G. (2018).
Vehicle routing with backhauls: Review and research perspectives.
Computers and Operations Research, 91:79 – 91.

Motivation 2

Generic Branch-Cut-and-Price (BCP) state-of-the-art solver for Vehicle Routing Problems (VRPs) [Pessoa et al., 2019].

`vrpsolver.math.u-bordeaux.fr`

Non-trivial modelling allows us to solve VRPB more efficiently using the solver.

Pessoa, A., Sadykov, R., Uchoa, E., and Vanderbeck, F. (2019).
A generic exact solver for vehicle routing and related problems.
In Lodi, A. and Nagarajan, V., editors, Integer Programming and Combinatorial Optimization, volume 11480 of Lecture Notes in Computer Science, pages 354–369, Springer International Publishing.

Plan of the talk

Generic Model for Vehicle Routing Problems

Models for the Vehicle Routing Problem with Backhauls

Results and Conclusions

Generic Model: Graphs for Resource Constrained Shortest Path (RCSP) generation

Define a set R of resource indices, partitioned into main resources R^M and secondary resources R^N

Define directed graphs $G^k = (V^k, A^k)$, $k \in K$:

- Special vertices v^k_{source}, v^k_{sink}
- Arc consumption $q_{a,r} \in \mathbb{R}_+$, $a \in A^k$, $r \in R$
  - cycles with zero main resource consumption should not exist
  - secondary resources may be of a special type that allow negative consumption
- Accumulated resource consumption intervals $[l_{a,r}, u_{a,r}]$, $a \in A^k$, $r \in R$
  - May also be defined on vertices ($[l_{v,r}, u_{v,r}]$, $v \in V^k$, $r \in R$)

Let $V = \cup_{k \in K} V^k$ and $A = \cup_{k \in K} A^k$

Generic Model: Graphs for RCSPs generation (2)

Resource Constrained Path (disposable resources)

A path $p = (v^k_{\text{source}} = v_0, a_1, v_1, \ldots, a_{n-1}, v_{n-1}, a_n, v_n = v^k_{\text{sink}})$ over G^k is resource constrained iff for every $r \in R$, the accumulated resource consumption $t_{j,r}$ at visit j, $0 \le j \le n$, does not exceed $u_{a_j,r}$, where

$$t_{j,r} = \begin{cases} 0, & j = 0, \\ \max\{l_{a_j,r}, t_{j-1,r} + q_{a_j,r}\}, & j > 0 \end{cases}$$

Generic Model: Variables and Mappings

Define continuous and/or integer variables:

1. Mapped x variables
   - Each variable x_j, $1 \leq j \leq n_1$, is mapped into a non-empty set $M(j) \subseteq A$.
   - The inverse mapping of arc a is $M^{-1}(a) = \{j | a \in M(j)\}$.
2. Additional (non-mapped) y variables
3. Bounds $[\bar{L}^k, \bar{U}^k]$ for the number of paths in P^k in a feasible solution.

Define also

- For each $k \in K$, P^k is the set of all resource constrained paths in G^k
- $P = \cup_{k \in K} P^k$
- λ_p = how many times path $p \in P$ is used in the solution.
- h_a^p = how many times arc a is used in path p

Generic Model: Formulation

$$
\begin{aligned}
\text{Min} \quad & \sum_{j=1}^{n_1} c_j x_j + \sum_{s=1}^{n_2} f_s y_s && && \text{(1a)}\\
\text{S.t.} \quad & \sum_{j=1}^{n_1} \alpha_{ij} x_j + \sum_{s=1}^{n_2} \beta_{is} y_s \geq d_i, && i = 1, \ldots, m, && \text{(1b)}\\
& x_j = \sum_{k \in K} \sum_{p \in P^k} \left(\sum_{a \in M(j)} h_a^p \right) \lambda_p, && j = 1 \ldots, n_1, && \text{(1c)}\\
& \bar{L}^k \leq \sum_{p \in P^k} \lambda_p \leq \bar{U}^k, && k \in K, && \text{(1d)}\\
& \lambda_p \in \mathbb{Z}_+, && p \in P, && \text{(1e)}\\
& x_j \in \mathbb{N},\ y_s \in \mathbb{N} && j = 1, \ldots, n_1,\ s = 1, \ldots, n_2. &&
\end{aligned}
$$

(1b) may be separated on demand through callback routines

Generic Model: Collection of Packing Sets

Define a collection $\mathcal{S}$ of mutually disjoint **packing sets**, each one being a subset of A, such that the constraints:

$$\sum_{a \in S} \sum_{p \in P} h_a^p \lambda_p \leq 1, \quad S \in \mathcal{S}, \tag{2}$$

are satisfied by at least one optimal solution (x^*, y^*, λ^*) of Formulation (1).

- The definition of a proper $\mathcal{S}$ is part of the modeling
- Packing sets can be defined on vertices (each one is a subset of V)

Packing sets generalize customers in the classical CVRP

Why we need Packing Sets?

Knowledge about packing sets allows the solver to use state-of-the-art techniques in a generalized form:

- *ng*-paths [Baldacci et al., 2011]
  - Distance matrix for packing sets is expected from the user to obtain initial *ng*-neighbourhoods
- Limited Memory Rank-1 Cuts [Pecin et al., 2017]
- Elementary path enumeration [Baldacci et al., 2008]
  - Additional condition to use enumeration:
    Two partial paths ending in the same vertex and mapped to different columns in (1b) *should correspond to different collection of packing sets*

Rounded Capacity Cuts (RCC) Separators

Interface for separating RCCs [Laporte and Nobert, 1983].
CVRPSEP code [Lysgaard, 2003] is used by the solver

Each separator is characterized by a triple

- sub-collection $\mathcal{S}' \subseteq \mathcal{S}$ of packing sets,
- a demand d_S for each $S \in \mathcal{S}'$,
- capacity Q.

Conditions to use

- Collection of packing sets is defined on vertices.
- For every $S \subseteq \mathcal{S}'$, $\sum_{v \in S} \sum_{p \in P} h_v^p \lambda_p = 1$.
- For every path $p \in P$, $\sum_{S \in \mathcal{S}'} d_S \cdot \sum_{v \in S} h_v^p \leq Q$.

Vehicle Routing Problem with Backhauls

Data

- Depot 0
- Set $L = \{1, \dots, n\}$ of linehaul vertices
- Set $B = \{n+1, \dots, n+m\}$ of backhaul vertices
- Graph $G = (V, A)$, $V = \{0\} \cup L \cup B$, $A = \{(i, j) : i, j \in V, i \neq j\}$.
- Travelling cost c_a, $a \in A$.
- Demands d_i, $i \in L \cup B$.
- K homogeneous vehicles of capacity Q.

Feasible solution

K vehicle routes

- start and finish at the depot
- serve at least one linehaul customer
- serve backhaul customers strictly after linehaul ones
- total demand of linehaul customers $\leq Q$
- total demand of backhaul customers $\leq Q$

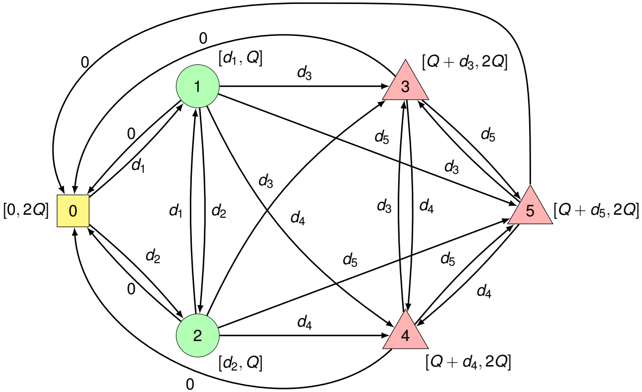
Objective

Minimize the total travelling cost

Standard model with one graph

- Graph $G^1 = (V, A^1)$, $v^1_{\text{source}} = v^1_{\text{sink}} = 0$,
  $A^1 = A \setminus \{(0,j) : j \in B\} \setminus \{(j,i) : j \in B, i \in L\}$,
- One main capacity resource :

$$q_{(i,j)} = \begin{cases} d_j, & j \in L \cup B, \\ 0, & j = 0. \end{cases} \qquad [l_v, u_v] = \begin{cases} [d_v, Q], & v \in L, \\ [Q + d_v, 2Q], & v \in B, \\ [0, 2Q], & j = 0. \end{cases}$$



Standard model with one graph: formulation

- Variable x_a is mapped to arc a, $a \in A^1$.
- $[\bar{L}^1, \bar{U}^1] = [K, K]$
- Formulation:

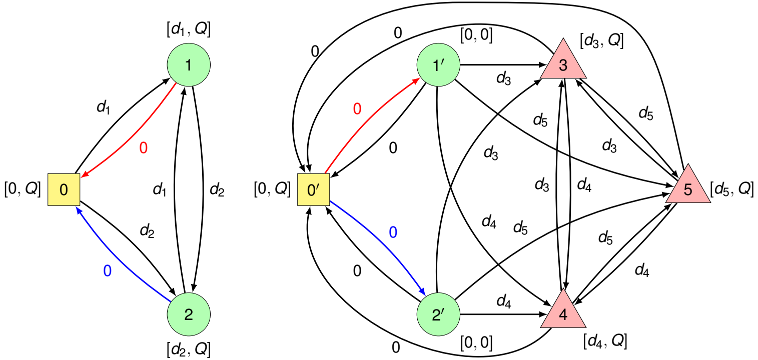
$$
\begin{aligned}
\text{Min} \quad & \sum_{a \in A^1} c_a \cdot x_a \\
\text{S.t.} \quad & \sum_{(i,j) \in A^1} x_a = 1, \quad \forall j \in L \cup B, \\
& x_a \in \{0, 1\}, \quad \forall a \in A^1.
\end{aligned}
$$

- Packing sets are defined on vertices: $\mathcal{S} = \mathcal{S}_L \cup \mathcal{S}_B$, $\mathcal{S}_L = \big\{\{v\} : v \in L\big\}$, $\mathcal{S}_B = \big\{\{v\} : v \in B\big\}$.
- Distance matrix is based on travelling costs
- First RCC separator: $(\mathcal{S}_L, \{d_v\}_{v \in L}, Q)$
- Second RCC separator: $(\mathcal{S}_B, \{d_v\}_{v \in B}, Q)$
- Branching on variables x

New model with two graphs

- Graph $G^1 = (V^1, A^1)$, $V^1 = \{0\} \cup L$, $A^1 = V^1 \times V^1$
- Graph $G^2 = (V^2, A^2)$, $V^2 = \{0'\} \cup L' \cup B$, $A^2 = \{(0,j) : j \in L'\} \bigcup \{(i,j) : i \in L', j \in \{0\} \cup B\} \bigcup \{(i,j) : i \in B, j \in \{0\} \cup B\}$.
- One main capacity resource :

$$q_{(i,j)} = \begin{cases} d_j, & j \in L \cup B, \\ 0, & j \in \{0\} \cup L'. \end{cases} \qquad [l_v, u_v] = \begin{cases} [d_v, Q], & v \in L \cup B, \\ [0,0], & v \in L', \\ [0,Q], & j \in \{0, 0'\}. \end{cases}$$



New model with two graphs: formulation

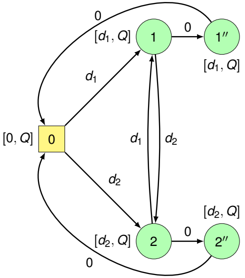
- Variable x_a is mapped to arc a, $a \in \{(i,j) : j \in L \cup B \cup \{0'\}\}$
- Variable z_j is mapped to arc $(j,0) \in A^1$, $j \in L$
- Variable w_j is mapped to arc $(0',j) \in A^2$, $j \in L'$
- $[\bar{L}^1, \bar{U}^1] = [\bar{L}^2, \bar{U}^2] = [K, K]$
- Formulation:

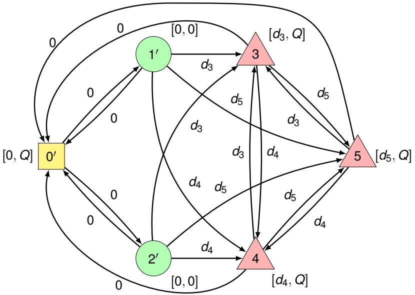
$$
\begin{aligned}
\text{Min} \quad & \sum_{a \in A^1} c_a \cdot x_a \\
\text{S.t.} \quad & \sum_{(i,j) \in A^1} x_a = 1, \quad \forall j \in L, \\
& \sum_{(i,j) \in A^2} x_a = 1, \quad \forall j \in B, \\
& z_j = w_j, \quad \forall j \in L, \\
& x_a \in \{0,1\}, \quad \forall a \in A^1 \cup A^2, \\
& z_j, w_j \in \{0,1\}, \quad \forall j \in L.
\end{aligned}
$$

- Branching on variables x and y

New model with two graphs: packing sets

We modify graph G^1 by duplicating vertices in L to L'':





- Packing sets are defined on vertices: $\mathcal{S} = \mathcal{S}_L \cup \mathcal{S}_{L'} \cup \mathcal{S}_{L''} \cup \mathcal{S}_B$.
- Packing sets in $\mathcal{S}_{L'} \cup \mathcal{S}_{L''}$ are artificial and serve to satisfy condition to use enumeration
- Distance to an artificial packing set is ∞
- Same two RCC separators as for the first model

Instances and exact approaches in the literature

Instances

- [Goetschalckx and J.-B., 1989]: 25–200 customers
- [Toth and Vigo, 1997]: 21–100 customers
- [Uchoa et al., 2017] (modified CVRP): 101–1001 customers

Exact approaches in the literature

TV [Toth and Vigo, 1997]: Lagrangian relaxation + branch-and-bound

MGB [Mingozzi et al., 1999]: Heuristic solution of the dual of an LP relaxation of route-based formulation + enumeration of routes with small reduced cost

GES [Granada-Echeverri and Santa, 2019]: MIP formulation

Computational results

Comparison with the literature

Instance set	Solved/Tried instances			
	TV	MGB	GES	Ours
[Goetschalckx and J.-B., 1989]	29/34	30/47	47/62	68/68
[Toth and Vigo, 1997]	23/30	24/33	28/33	33/33

Comparison of two models

Instances	Size	One graph		Two graphs	
		Time (s)	Nodes	Time (s)	Nodes
Classic	25–200	284	2.0	106	1.9
New	101–167	4814	28	1120	12

Large instances with 172–1001 customers

- 77 from 255 instances are solved to optimality in 60 hours
- The largest solved instance has 655 customers
- The smallest unsolved instance has 190 customers

Conclusions

- Wishes of [Koç and Laporte, 2018] are fulfilled:
  - All classic instances of the VRPB are solved to optimality
  - All literature instances of the VRPBTW and HFFVRPB are also solved to optimality
    (straightforward modifications of the model)
- Many open instances for future works
- Model with two graphs is experimentally much better
- The code of the second model (≈130 lines of Julia code) ~~is~~ will be available as a VRPSolver demo at

  vrpsolver.math.u-bordeaux.fr

- Demos for several other VRP variants are available
  (CVRP, VRPTW, HFVRP, PDPTW, TOP, CARP, 2E-CVRP)

Thank you!
Questions?

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