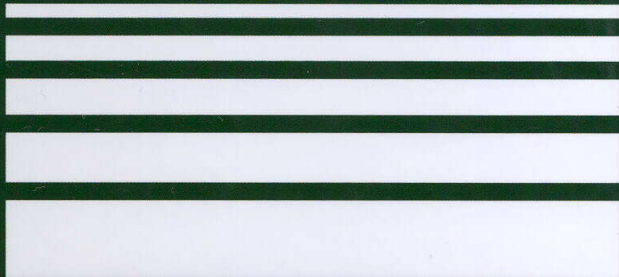


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Progress in Mathematics



Jean Fresnel  
Marius van der Put

# Rigid Analytic Geometry and Its Applications



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