# Mathematical Modeling in Population Dynamics

August 26-27, 2019.
Bordeaux, France

With the participation of:

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Institut de Mathématiques de Bordeaux
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Closest Tram station: Line B, stop name “Forum”

# PROGRAM

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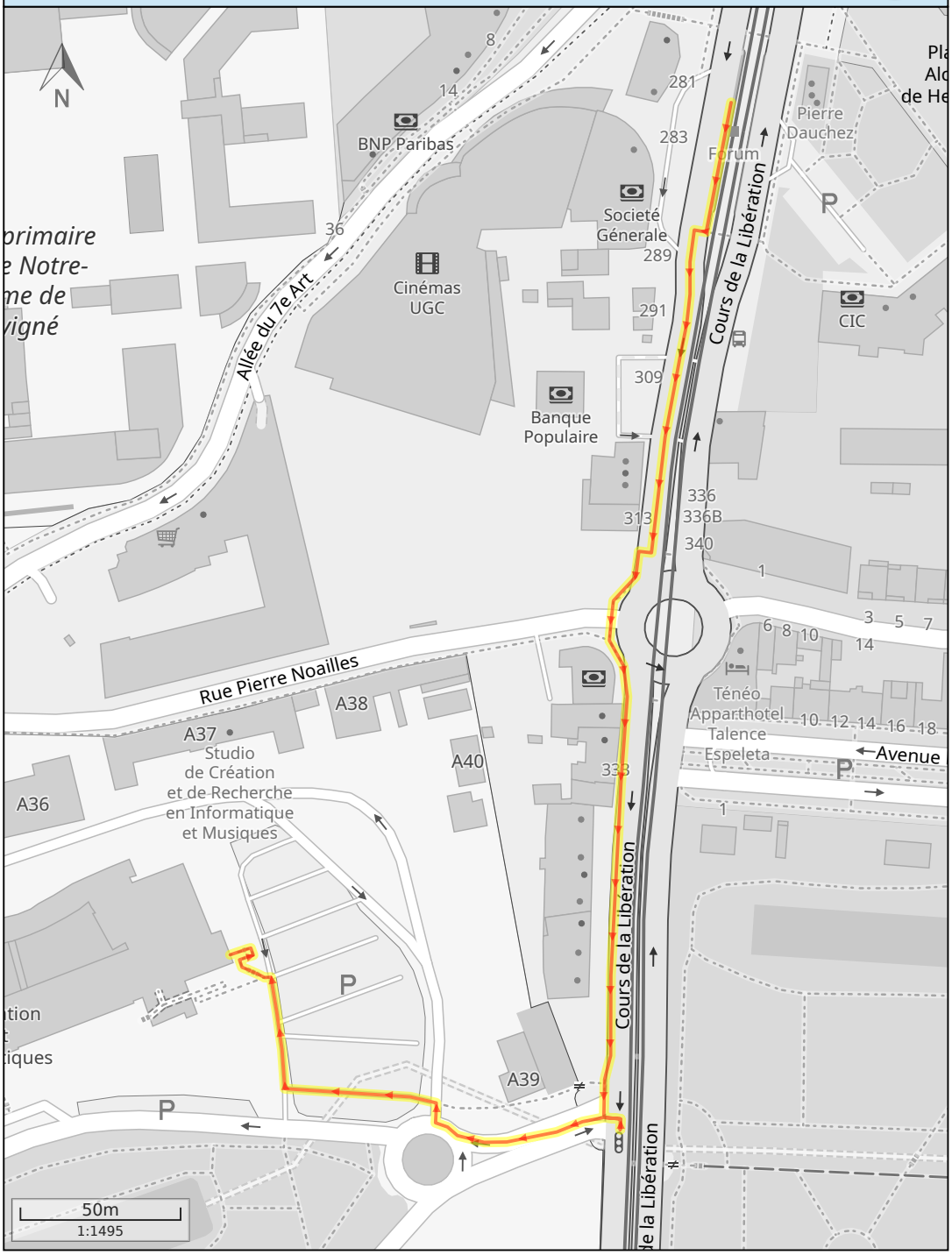
| | **Monday** | **Tuesday** |
|---|---|---|
| 9:00 | Shigui Ruan | Zhi-Cheng Wang |
| 9:45 | Sébastien Motsch | Aziz Alaoui |
| 10:30 | Coffee Break | Coffee Break |
| 11:00 | Óscar Angulo | Frédéric Barraquand |
| 11:45 | Xiaoming Fu | Quentin Richard |
| 12:10 | Lunch | Lunch |
| 14:05 | Coralie Picoche | Lara Abi Rizk |
| 14:30 | Zhihua Liu | Rong Yuan |
| 15:15 | Coffee break | Coffee break |
| 15:45 | Alain Miranville | Arnaud Ducrot |
| 16:30 | Ousmane Seydi | Yu-Yun Chen |
| 17:15 | Free discussions | Sze-Bi Hsu |
| 17:55 | | Closing |
| 19:30 | Conference Banquet | |

# Way from Forum to IMB



# Confirmed speakers (45min)

# Synchronization of Complex Interaction Networks of Reaction-diffusion Systems. Application in Neuroscience

M.A. Aziz-Alaoui

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**Abstract:** Neuroscience consists of the study of the nervous system and especially the brain. The neuron is an electrically excitable cell processing and transmitting information by electrical and chemical signaling, the latter via synapses, specialized connections with other cells. A.L.Hodgkin and A.Huxley proposed the first neuron model to explain the ionic mechanisms underlying the initiation and propagation of action potentials in the squid giant axon. Here, we are interested in the asymptotic behavior of complex networks of reaction-diffusion (PDE) systems of such neuron models. We show the existence of the global attractor and the identical synchronization for the network. We determine analytically, for a given network topology, the onset of such a synchronization. We then present numerical simulations and heuristic laws giving the minimum coupling strength necessary to obtain the synchronization, with respect to the number of nodes and the network topology.

**Keywords:** Complex Systems, Complex Networks, Reaction-diffusion, Synchronisation, Neuroscience.

# Numerical analysis of an age-structured population model with infinite life span

L.M. Abia, O. Angulo, J.C. López-Marcos, M.A. López-Marcos

Dpto Matemática Aplicada, Universidad de Valladolid, Valladolid, Spain

**Abstract:**

Population dynamics modelization entails making different choices. We decided to choose an age-structured population model, which means: individuals are non-homogeneous, they are distinguished by the age and the model is continuous and deterministic. The choice of the age as a physiological parameter to structure a population and describe its dynamics involves the election of the life-span. Models both with finite and unbounded maximum age appear in the literature [2].

The numerical integration of models with unbounded life-span (for instance, in demography) has been integrated with different techniques. It was usually to use a maximum artificial age $A$, an initial condition with compact support on $[0, A]$ and a finite integration time $T$, which allowed to integrate in a finite age interval $[0, A+T]$. However, when the dynamics of the problem is studied (long-time integration), this is unfeasible unless we know properties of the solution [1]. Thus, we look for new numerical techniques to integrate in $[0, \infty)$. This is the aim of the present work.

We propose and analyse a new second order numerical method in this setting. We report numerical experiments to show the convergence properties and the behaviour of the problem to simulate the evolution of Nicholson's blowfies model [3].

## References

[1] O. Angulo, J. C. López-Marcos, M. A. López-Marcos, A numerical integrator for a model with a discontinuous sink term: the dynamics of the sexual phase of monogonont rotifera, Nonlinear Anal. Real World Appl. 6 (2005), 935-954.

[2] M. Iannelli, F. Milner, The Basic Approach to Age-Structured Population Dynamics, Lecture Notes on Mathematical Modelling in the Life Sciences, Springer, Dordrecht, The Netherlands, 2017.

[3] A.J. Nicholson. The self-adjustement of population to change. *Cold Spring Harbor Symp. Quant. Biol.*, 22:153–173, 1957.

# Dynamical behaviour of a stage structured predator-prey model for multispecies demography

Frederic Barraquand and Bachar Tarraf

Institute of Mathematics of Bordeaux (CNRS, U Bordeaux)

**Abstract:** We introduce a stage-structured predator-prey model in discrete-time, that can be fitted to data using the Integrated Population Model framework, which combines different data sources (counts of animals, capture-recapture data producing survival estimates, reproduction data). The species interactions are represented indirectly, through an adult predator reproduction rate that increases with juvenile prey density, and a prey survival rate that decreases with the number of adult predators. The prey and predator components of the model have connections to the single species model of Neubert & Caswell (2000, J Math Biol). To better understand this system, we first study the models without interactions between species, i.e., the prey and predator components. We show that apparently small differences in model structure in the single-species components (parameters changing either with total density or adult density) generate marked structural differences in the sequence of bifurcations observed when increasing fecundities, when compared to Neubert & Caswell (2000, J Math Biol). We then study which type of dynamical behaviour can be exhibited by the full model (when adding interaction between species), for parameters reflecting realistic pairs of predator-prey systems (carnivorous & herbivorous mammals, seed eating rodents & annual plants). We show numerically that the model can exhibit multiple routes to chaos and resonances. Adding stochasticity further increases the potential for cyclic oscillations and modulates the deterministic results on bifurcations.

# General flowering in tropical rain forest of Southeast Asia: the causes and consequences on the forests

Yu-Yun Chen

National Dong Hwa University

**Abstract:** Hundreds of forest species exhibit synchronized flowering and fruiting at long and irregular intervals, called the general flowering (GF), is a unique phenomenon spreads over a large area of hyper-diverse lowland tropical rain forests in Southeast Asia. This phenomenon contradicts with predictions from competition models that communities of high reproductive synchrony should experience strong competitive exclusion and result in low biodiversity. Species joining GF events may enjoy disproportional benefits such as enhanced pollination efficiency, seed survival, and seed dispersal by the sudden increase of resource for mutualists. Regardless of evolutionary path, signals from the environments are important triggers for synchronization among individuals and species. Previous studies identified climatic cues, such as cool temperature and drought, that trigger GF events. As climatic fluctuation increases, frequencies and intensity of environmental cues for GF may change, thus altering demographic benefits gained by species of GF. This presentation introduces theories of GF, proximate cues for the community-wide masting phenomenon, and discusses on challenges of forest regeneration face the SE Asian forests under the new climate regime.

# Large time behaviour for a logistic equation with nonlinear and nonlocal diffusion and periodic boundary conditions

Arnaud Ducrot

LMAH, Université Le Havre Normandie

**Abstract:** In this talk we discuss the large time behaviour of a logistic type equation with a nonlinear and nonlocal diffusion term, that corresponds to a nonlocal Darcy law of motion with a smooth kernel. The model we consider is endowed with periodic boundary conditions. Under some conditions on the kernel function, we investigate the large time behaviour of the solutions. A general multi-dimensional – weak – convergence result is obtained by coupling energy functional arguments with Young measure topology, in order to overcome the possible lack of asymptotic compactness of the solutions for a sufficiently strong topology. This result is improved in the one-dimensional setting for which we are able to show that, for a large class of initial data, the solutions converge for the strong topology of $L^1$.

# A reaction-diffusion system modeling intraguild predation and internal storage

Sze-Bi Hsu

National Tsing-Hua University

**Abstract:** The theory of intraguild predation were established by Holt and Polis in 1989. Namba (2005,Ecology) found chaos in his numerical simulations for a simple Lotka-Volterra model with intraguild predations.. Hsu, Ruan and Yang (JMAA2015) analyzed the LV model and find the chaos is created by Hopf Bifurcation and then as parameter increases the routes of period doubling leads to chaos. However the intraguild predation phenomena was never found in laboratory before. Huisman et (Oikos 2014) is the first one to do experiments on intraguild predation and construct a model of intraguild predation of Droop type in the chemostat. In this talk we continue our study on Huisman's ODE model (TJM 2019) to a PDE of Droop type with intraguild predation. We apply our research results on PDE of Droop type (JDE 2010) to construct a reaction-diffusion system of Droop type with intraguild predation in an unstirred chemostat. We analyze a nonlinear eigenvalue problem using a generalized Krein-Rutman Theorem discovered by Mallet Paret and Nussbaum (2010). We prove the existence of principle eigenvalue and its relation to solution behavior for the extinction and coexistence of competing species. The conclusion is that intraguild predation will promote the coexistence.

# Bogdanov-Takens bifurcation for age structured models

Zhihua Liu

Beijing Normal University, Beijing, China

**Abstract:** The main purpose of this talk is to present our works recently on Bogdanov-Takens bifurcation in age structured models. We derive an easily feasible method for the determination of Bogdanov-Takens singularity in age structured models and show that some age structured models undergo the Bogdanov-Takens bifurcation. The analysis is based on the normal form theory and the center manifold theory for semilinear equations with non-dense domain combined with integrated semigroup theory.

# The Cahn-Hilliard equation with a proliferation term

Alain Miranville

University of Poitiers, France

**Abstract:** Our aim in this talk is to discuss (well-posedness, blow-up, global existence and asymptotic behavior) a Cahn-Hilliard type model with applications in wound healing and tumor growth.

# Tumor growth: from agent-based model to free-boundary problem

Sebastien Motsch

Arizona State University

**Abstract:** In this talk, we investigate the large time behavior of a agent based model modeling tumor growth. This microscopic model combines short-range repulsion and cell division. We derive the associated macroscopic dynamics leading to a porous media type equation. In order to capture the long-time behavior of the microscopic model, we have to modify the porous media in order to include a density threshold for the repulsion. The main difficulty is then to investigate the limit as the repulsion between cells becomes singular (modeling non-overlapping constraint). We show formally that such asymptotic limit leads to a free-boundary problem (Hele-Shaw type). Numerical results confirm the relevance of such limit.

# On a Network Model of Two Competitors with Applications to the Invasion and Competition of *Aedes Albopictus* and *Aedes Aegypti* Mosquitoes in the United States

Shigui Ruan

Department of Mathematics, University of Miami, Coral Gables, FL 33146, USA

**Abstract:** Based on the invasion of the *Aedes albopictus* mosquitoes and the competition between *Ae. albopictus* and *Ae. aegypti* mosquitoes in the United States, we study a two-species competition model in a network, that is with discrete Laplacian diffusion. In the case of strong-weak competition where the invasive competitor is stronger than the local one, it is shown that the invasive species (*Ae. albopictus*) wins over the local species (*Ae. aegypti*) and the solutions converge uniformly to the semi-positive equilibrium such that the invasive species survives while the local species becomes extinct, and vice versa. In the case of weak-weak competition, the solutions converge uniformly to the positive equilibrium so that both invasive and local species coexist. By using numerical simulations, we apply the two-species competition model in a network to explain the invasion and competition of *Ae. Albopictus* and *Ae. Aegypti* mosquitoes in the United States. We also show that discrete Laplacian diffusion induces different spreading speeds in different invasive directions. (Based on a joint paper with Canrong Tian)

## Persistence of a normally hyperbolic manifold for a system of non densely defined Cauchy problems

Ousmane Seydi

École Polytechnique de Thiès, Sénégal

**Abstract:** In this presentation we will consider a system of non densely defined Cauchy problems and we investigate the persistence of normally hyperbolic manifolds. The notion of exponential dichotomy is used to characterize the normal hyperbolicity and a generalized Lyapunov-Perron approach is used in order to prove our main result. The result presented in this talk extend the previous results on the center manifold by allowing a nonlinear dynamic in the unperturbed central part of the system. We consider two examples to illustrate our results. The first example is a parabolic equation coupled with an ODE that can be considered as an interaction between an antimicrobial and bacteria while the second one is a Ross-Macdonald epidemic model with age of infection. In both examples we were able to reduce the infinite dimensional system to an ordinary differential equation

## Monotonicity and Global Dynamics of a nonlocal two-species phytoplankton model

Zhicheng Wang

Lanzhou University

**Abstract:** We investigate a nonlocal reaction-diffusion-advection system modeling the population dynamics of two competing phytoplankton species in a eutrophic environment, where nutrients are in abundance and the species are limited by light only for their metabolism. We first demonstrate that the system does not preserve the competitive order in the pointwise sense. Then we introduce a special cone $\mathcal{K}$ involving the cumulative distributions of the population densities, and a generalized notion of super- and subsolutions, where the differential inequalities hold in the sense of the cone $\mathcal{K}$. A comparison principle is then established for such super- and subsolutions, which implies the monotonicity of the semiflow with respect to the cone $\mathcal{K}$. As application, we study the global dynamics of the single species system and the competition system. The latter has implications for the evolution of movement for phytoplankton species. This is a joint work with Dr. Danhua Jiang, Prof. King-Yeung Lam, and Prof. Yuan Lou.

## Almost periodic solutions and its module

Rong YUAN

Beijing Normal University, Beijing 100875, P. R. China

**Abstract:** In this talk, I would like to introduce almost periodic functions, Stepanovs almost periodic functions, piecewise continuous almost periodic functions, and its module. Some relationship and results will be shown.

# Short communications (20min)

# Asymptotic behavior of a nonlocal advection system with two populations

Xiaoming Fu

IMB, The University of Bordeaux

**Abstract:** In this talk, we consider a nonlocal advection model for two populations on a bounded domain. The first part of the talk is devoted to the existence and uniqueness of solutions and the associated semi-flow properties. Next, by proving segregation property, we construct an energy functional to investigate the asymptotic behavior of the solution. By using Young measure, we get a description of the asymptotic behavior of the solution. In the last section, we will talk about numerical simulations and some perspectives.

# How self-regulation, the storage effect and their interaction contribute to coexistence in stochastic and seasonal environments

C. Picoche[*,1] & F. Barraquand[1,2]

1-Integrative and Theoretical Ecology, University of Bordeaux
2-Institute of Mathematics of Bordeaux, CNRS

**Abstract:** Explaining coexistence in species-rich communities of primary producers remains a challenge for ecologists because of the likely competition for shared resources. Following Hutchinson's seminal suggestion, many theoreticians have tried to create diversity through a fluctuating environment, which impairs or slows down competitive exclusion. There are now several fluctuating-environment models allowing coexistence, but they often produce only a dozen of coexisting species at best. Here, we investigate how to create even richer communities in fluctuating environments, using an empirically parameterized model. Building on the forced Lotka-Volterra model of Scranton and Vasseur (2016) inspired by phytoplankton communities, we have investigated the effect of two coexistence mechanisms, namely the storage effect and higher intra- than interspecific competition strengths (i.e., strong self-regulation). We tuned the competition ratio based on empirical analyses, in which self-regulation usually dominates interspecific interactions. Although a strong self-regulation maintained more species (50%) than the storage effect (25%), we show that none of the two coexistence mechanisms considered could, by itself, ensure the coexistence of all species present at the beginning of our simulations. Realistic seasonal environments only aggravated that picture, as they decreased persistence relative to a random environment. Our results suggest that combining different mechanisms for biodiversity maintenance into community models might be more fruitful than trying to find which mechanism explains best the observed diversity levels. We additionally highlight that while trait-biomass distributions provide some clues regarding coexistence mechanisms, they cannot indicate by themselves which coexistence mechanisms are at play.

# Asymptotic behavior of age-structured and delayed Lotka-Volterra models

Quentin RICHARD

UMR 5251 Institut de Mathématiques de Bordeaux, Université de Bordeaux, France.

**Abstract:** In this talk, we will investigate the asymptotic behavior of the solutions of an age-structured Lotka-Volterra model [1]. We enlighten the existence of two thresholds. Depending on their value, we prove that the solutions can go extinct or explode in infinite time. We also show some numerical simulations suggesting the possibility for the solution to converge either to a periodic function or to a nontrivial equilibrium. Finally, we consider a specific choice of functional parameters that allows us to formulate a delayed problem [2]. Using Lyapunov functional, we show that the attractive set is reduced either to the nontrivial equilibrium or to a periodic solution. We characterize the existence of such periodic solution and we prove the asymptotic stability of the coexistence equilibrium in some basin of attraction.

## References

[1] A. Perasso and Q. Richard, *Implication of age-structure on the dynamics of Lotka-Volterra equations* 32, (2019), 91–120.

[2] A. Perasso and Q. Richard, *Asymptotic behavior of age-structured and delayed Lotka-Volterra models*, submitted.

# Travelling wave solutions for a non-local evolutionary-epidemic system

Lara Abi Rizk

Institut de Mathématiques de Bordeaux

**Abstract:** In this talk we study the existence of a travelling wave solutions for a spatially distributed system of equations modelling the evolutionary epidemiology of plant-pathogen interaction, we prove that the wave solutions connects two determined stationary states and have a rather simple structure, provided some parameters condition expressed using the principle eigenvalue of some integral operator.
An important contribution of this work is to overcome a difficulty on the lack of positivity due to the sign-changing of the higher eigenvectors. This analysis allows us to reduce the infinite dimensional travelling wave profile system of equations to a 4-dim ODE system .