IMPERIAL

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Online

Conditioned Stochastic Stability of Invariant Measures on Repellers

THERMOGAMAS

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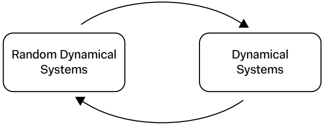
Joint work with Jeroen S.W. Lamb (Imperial) and Matheus M. Castro (UNSW).

From Random to Deterministic Dynamical Systems



Adding noise to a system can make it easier to deal with – randomness smoothens dynamics.

Stochastic Stability:

Problem: Invariant sets may support infinitely many ergodic invariant measures.

Strategy: Only those robust under small random perturbation are statistically relevant.

Ya. G. Sinaĭ, "Gibbs Measures in Ergodic Theory," Uspehi Mat. Nauk, vol. 27, no. 4(166), pp. 21–64, 1972.
L.-S. Young, "Stochastic Stability of Hyperbolic Attractors," ETDS, vol. 6, no. 2, pp. 311–319, 1986.
Y. Kifer, Random Perturbations of Dynamical Systems, vol. 16, Progress in Probability and Statistics. Boston, MA: Birkhäuser, 1988.
V. Baladi and M. Viana, "Strong Stochastic Stability and Rate of Mixing for Unimodal Maps," Ann. Sci. École Norm. Sup. (4), vol. 29, no. 4, pp. 483–517, 1996.
J. F. Alves, V. Araújo, and C. H. Vásquez. "Stochastic stability of non- uniformly hyperbolic diffeomorphisms." Stoch. Dyn. 7.3, pp. 299–333, 2013.
W. Shen and S. van Strien, "On Stochastic Stability of Expanding Circle Maps with Neutral Fixed Points," Dynamical Systems, vol. 28, no. 3, pp. 423–452, 2013.

Stochastic Stability for Attractors - An Example

The Hénon map

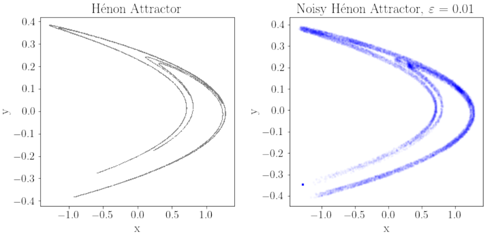
$$\begin{cases} x_{n+1} &= 1 - a x_n^2 + y_n \ + \omega_1 \\ y_{n+1} &= b x_n \ + \omega_2, \end{cases} \qquad \text{with } a = 1.4, b = 0.3, \quad \omega_1, \omega_2 \sim \mathrm{Unif}(-\varepsilon, \varepsilon),$$

has a very complicated attractor.

Noise smoothens and mixes the dynamics.[1]

The unique stationary measure μ_ε highlights the random system's statistical properties.

Stochastic Stability: Show that μ_ε converges to a unique invariant measure as $\varepsilon \to 0$.
$\Rightarrow$ The limiting measure is the statistically relevant one for the deterministic map.



IMPORTANT: This "works" partly because the stationary measure sits around the deterministic attractor.

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[1] M. Benedicks and M. Viana. "Random perturbations and statistical properties of Hénon-like maps". *Ann. Inst. H. Poincaré C Anal. Non Linéaire* 23.5 (2006), pp. 713–752

Could this work for repelling invariant sets?

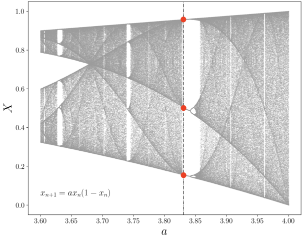
They may also support infinitely many (ergodic) invariant measures.

Random trajectories near repellers eventually go away, so...

There is no stationary measure near a repelling invariant set!

But... some invariant measures on repellers are “relevant”.

Identifying Relevant Measures on Repellers - Numerics for the Logistic Map

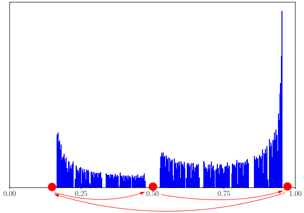


The logistic map

$$x_{n+1} = 3.83 \cdot x_n(1 - x_n)$$

has a three-periodic attractor.

There is also a repelling invariant Cantor set.



Trajectories that remain close to the Cantor set for a long time are distributed according to a particular measure.[2]

GOAL: We want to identify this "natural", seemingly statistically relevant, measure.

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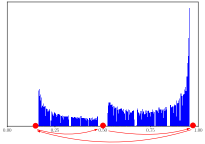
[2] H. Kantz and P. Grassberger. "Repellers, semi-attractors, and long-lived chaotic transients". *Phys. D* 17.1 (1985), pp. 75–86

Identifying Relevant Measures on Repellers - Previous work



1. Several sampling techniques in the transient chaos literature try to approximate these "natural measures":[3]
    - Ensemble method[4], PIM-triple method[5], etc.
2. Open Dynamical Systems provide a strategy to find "relevant" statistics of the deterministic transient.
    - Absolutely continuous conditional invariant measures (accim).[6]
    - Lyapunov exponents, escape rates, etc. for maps with holes.[7]
3. Thermodynamic formalism on repelling invariant sets:[8]
    - The relevant invariant measure satisfies a variational principle, i.e. maximises the topological pressure.

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[3] Y.-C. Lai and T. Tél. *Transient chaos. Complex dynamics on finite-time scales*. Vol. 173. Springer, New York, 2011, pp. xiv+497

[4] H. Kantz and P. Grassberger. "Repellers, semi-attractors, and long-lived chaotic transients". *Phys. D* 17.1 (1985), pp. 75–86

[5] H. E. Nusse and J. A. Yorke. "A procedure for finding numerical trajectories on chaotic saddles". *Phys. D* 36.1-2 (1989), pp. 137–156

[6] G. Pianigiani and J. A. Yorke. "Expanding maps on sets which are almost invariant. Decay and chaos". *Trans. Amer. Math. Soc.* 252 (1979), pp. 351–366

[7] M. F. Demers, P. Wright, and L.-S. Young. "Entropy, Lyapunov exponents and escape rates in open systems". *ETDS* 32.4 (2012), pp. 1270–1301

[8] D. Ruelle. *Thermodynamic formalism*. Vol. 5. Encyclopedia of Mathematics and its Applications. Addison-Wesley Pub. Co., MA, 1978, pp. xix+183

Identifying Relevant Measures on Repellers - Previous work

Are these measures “stochastically stable”?

Again,

Random trajectories near repellers eventually go away, so...

There is no stationary measure near the repelling invariant set!

New approach proposed: Employ Conditioned Random Dynamical Systems and consider quasi-ergodic measures instead of stationary measures.

1. Several sampling techniques in the transient chaos literature try to approximate these “n[illegible] me[illegible]
   - Ensemble method[4], PIM-triple method[5], etc.
2. Open Dynamical Systems provide a strategy to find “relevant” statistics of the deterministic transient.
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   - Lyapunov e[illegible][7]
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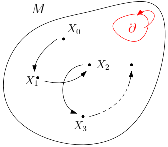
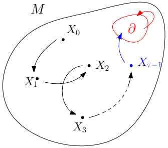
Conditioned Random Dynamics and Quasi-ergodic Measures





Consider an absorbing Markov process X on M, absorbed at ∂ (cemetery state), so that:

- if $X_n \in \partial$ then $X_{n+1} \in \partial$, for all $n \in \mathbb{N}$, and
- set $\tau := \min\{n > 0 : X_n \in \partial\}$.

We employ conditioned random dynamics to study conditioned Birkhoff averages:[9] for an observable $\varphi : M \setminus \partial \to \mathbb{R}$, we analyse if

$$\lim_{n\to\infty} \mathbb{E}_x\left[\frac{1}{n}\sum_{i=0}^{n-1} \varphi \circ X_i \,\middle|\, \tau > n\right] = \int \varphi \, \mathrm{d}\nu, \quad \nu\text{-a.s. on } x \in M \setminus \partial. \tag{1}$$

Definition 1

We say that ν is a quasi-ergodic measure of the absorbing process X if (4) holds.

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[9] M. M. Castro et al. "Existence and uniqueness of quasi-stationary and quasi-ergodic measures for absorbing Markov chains: a Banach lattice approach". *Stochastic Process. Appl.* 173 (2024), Paper No. 104364, 19.

Conditioned Stochastic Stability

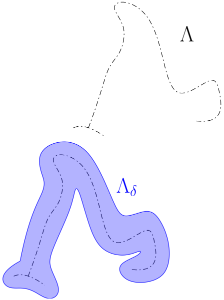
For a map $T : M \to M$, consider the process $X^{\varepsilon}, \varepsilon > 0$, generated by ε-small random perturbations of T:

$$X_0^{\varepsilon} = x \in M, \quad X_n^{\varepsilon}(\omega) \coloneqq T_{\omega}^n(x) \coloneqq T_{\omega_{n-1}} \circ \cdots \circ T_{\omega_0}(x), \quad \mathrm{dist}_{\mathcal{C}^2}(T_{\omega_i}, T) \leq C\|\omega_i\|, \quad \omega_i \in [-\varepsilon, \varepsilon],$$

satisfying some mild regularity assumptions.

Suppose that T has an invariant repelling set $\Lambda \subset M$.

- (local) Set $\partial \coloneqq M \setminus \Lambda_{\delta}$, with $\Lambda \subset \Lambda_{\delta}$ a small δ-neighbourhood, $\delta > 0$.



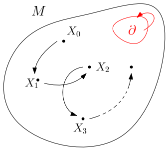
Strategy:

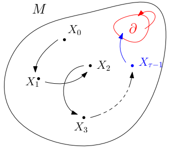
1. Show that X_{ε}^{ϕ} has a unique quasi-ergodic measure ν_{ε}^{ϕ} on Λ_{δ}.
2. Show that $\nu_{\varepsilon}^{\phi} \xrightarrow{\mathrm{w}^*} \nu^{\phi} \in \mathrm{Erg}(T, \Lambda)$, as $\varepsilon \to 0$.
3. Characterise the measure ν^{ϕ}.

Definition 2

The T-invariant measure ν^{ϕ} on Λ is (local) conditioned stochastically stable if $\nu_{\varepsilon}^{\phi} \xrightarrow{\mathrm{w}^*} \nu^{\phi}$ as $\varepsilon \to 0$.

Conditioned Random Dynamics and Quasi-ergodic Measures with Potentials





Consider an absorbing Markov process X on M, absorbed at ∂ (cemetery state), so that:

- if $X_n \in \partial$ then $X_{n+1} \in \partial$, for all $n \in \mathbb{N}$, and
- set $\tau := \min\{n > 0 : X_nX_n^{\phi} \in \partial\}$.
- fix $\phi : M \setminus \partial \to (-\infty, 0]$ and set: $X_{n+1}^{\phi} := \begin{cases} X_{n+1}, & \text{with prob. } e^{\phi(X_n)} \\ \partial, & \text{with prob. } 1 - e^{\phi(X_n)}. \end{cases}$

We employ conditioned random dynamics to study conditioned Birkhoff averages:[10] for an observable $\varphi : M \setminus \partial \to \mathbb{R}$, we analyse if

$$\lim_{n\to\infty} \mathbb{E}_x\mathbb{E}_x^{\phi}\left[\frac{1}{n}\sum_{i=0}^{n-1}\varphi\circ X_iX_i^{\phi}\,\middle|\,\tau>n\right] = \int \varphi \,\mathrm{d}\nu\nu^{\phi}, \quad \nu\nu^{\phi}\text{-a.s. on } x\in M\setminus\partial. \tag{2}$$

Definition 3

We say that $\nu\nu^{\phi}$ is a quasi-ergodic measure of the absorbing process XX^{ϕ} if (4) holds.

We employ conditioned random dynamics to study conditioned Birkhoff averages:[11] for an observable

Conditioned Stochastic Stability with Potentials

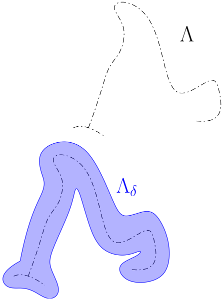
For a map $T : M \to M$, consider the process $X^\varepsilon, \varepsilon > 0$, generated by ε-small random perturbations of T:

$$X_0^\varepsilon = x \in M, \quad X_n^\varepsilon(\omega) := T_\omega^n(x) := T_{\omega_{n-1}} \circ \cdots \circ T_{\omega_0}(x), \quad \mathrm{dist}_{\mathcal{C}^2}(T_{\omega_i}, T) \leq C\|\omega_i\|, \quad \omega_i \in [-\varepsilon, \varepsilon],$$

satisfying some mild regularity assumptions.

Suppose that T has an invariant repelling set $\Lambda \subset M$.

- (local) Set $\partial := M \setminus \Lambda_\delta$, with $\Lambda \subset \Lambda_\delta$ a small δ-neighbourhood, $\delta > 0$.
- Fix $\phi : \Lambda_\delta \to (-\infty, 0]$ and consider X_ε^ϕ.



Strategy:

1. Show that X_ε^ϕ has a unique quasi-ergodic measure ν_ε^ϕ on Λ_δ.
2. Show that $\nu_\varepsilon^\phi \xrightarrow{\mathrm{w}^*} \nu^\phi \in \mathrm{Erg}(T, \Lambda)$, as $\varepsilon \to 0$.
3. Characterise the measure ν^ϕ.

Definition 6

The T-invariant measure ν^ϕ on Λ is (local) conditioned stochastically stable if $\nu_\varepsilon^\phi \xrightarrow{\mathrm{w}^*} \nu^\phi$ as $\varepsilon \to 0$.

Challenges to Overcome

Existence and Uniqueness of quasi-ergodic measures, $\varepsilon > 0$.

$$\mathbb{E}_x^{\phi}\left[\frac{1}{n}\sum_{i=0}^{n-1}\varphi\circ X_i^{\varepsilon}\,\middle|\,\tau > n\right] := \frac{1}{\mathbb{E}_x\left[e^{\sum_{i=0}^{n-1}\phi\circ X_i}\mathbb{1}_{M\setminus\partial}(X_n)\right]}\mathbb{E}_x\left[e^{\sum_{i=0}^{n-1}\phi\circ X_i}\mathbb{1}_{M\setminus\partial}(X_n)\frac{1}{n}\sum_{i=0}^{n-1}\varphi\circ X_i\right]$$

$$\xrightarrow[?]{n\to\infty}\int\varphi\,\mathrm{d}\nu_{\varepsilon}^{\phi}$$

Conditioned Stochastic Stability: understanding ν_{ε}^{ϕ} in the limit $\varepsilon \to 0$.

Main Results

Establishing (local) Conditioned Stochastic Stability

$T: M \to M$, $\Lambda \subset M$ is T-inv, $\quad \partial = M \setminus \Lambda_\delta, \quad \phi : \Lambda_\delta \to \mathbb{R}$, Hölder

X_ε^ϕ, generated by e^ϕ-weighted ε-small random perturbation of T

ν_ε^ϕ is a quasi-ergodic measure of X_ε^ϕ on Λ_δ

Conditioned Stochastic Stability

For a map $T : M \to M$, consider the process $X^\varepsilon, \varepsilon > 0$, generated by ε-small random perturbations of T:

$$X_0^\varepsilon = x \in M, \quad X_n^\varepsilon(\omega) := T_\omega^n(x) := T_{\omega_{n-1}} \circ \cdots \circ T_{\omega_0}(x), \quad d_{C^2}(T_{\omega_i}, T) \le C\|\omega_i\|, \quad \omega_i \in [-\varepsilon, \varepsilon],$$

satisfying some mild regularity assumptions.

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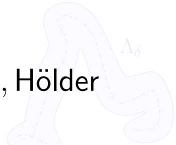

Strategy:

1. Show that X_ε^ϕ has a unique quasi-ergodic measure ν_ε^ϕ on Λ_δ.
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3. Characterise the measure ν^ϕ.

Definition 2

The T-invariant measure ν^ϕ on Λ is (local) conditioned stochastically stable if $\nu_\varepsilon^\phi \xrightarrow{w^*} \nu^\phi$ as $\varepsilon \to 0$.

Understanding $\varepsilon \to 0$. Setup

Spoiler: in the limit $\varepsilon \to 0$, the quasi-ergodic measure ν_ε^ϕ converges to the equilibrium state associated with the potential $\psi = \phi - \log|\det DT|_{E^u}|$.

Definition 7 (Equilibrium state)

Consider a T-invariant set $\Lambda \subset M$ and let $\psi : \Lambda \to \mathbb{R}$. The T-invariant measure ν on Λ is an equilibrium state if its metric pressure is equal to the topological pressure $P_{\mathrm{top}}(T, \psi, \Lambda)$ of the system on Λ, i.e.

$$h_\nu(T) + \int \psi \, \mathrm{d}\nu = \sup_{\mu \in \mathrm{Erg}(T,\Lambda)} \left(h_\mu(T) + \int \psi \, \mathrm{d}\mu \right) =: P_{\mathrm{top}}(T, \psi, \Lambda),$$

where $h_\mu(T, \Lambda)$ is the Kolmogorov-Sinai (metric) entropy.

Disclaimer: we do *not* build a thermodynamic formalism, hence work under conditions that guarantee equilibrium states exist.

The repeller: $\Lambda = \cap_{m \in \mathbb{Z}} T^m(\overline{V})$ for $V \supset \Lambda$ an open set is uniformly expanding (or hyp.) and transitive.

Theorem 1 (Ruelle, T uniformly expanding)

Let T be uniformly expanding on Λ. For every α-Hölder potential $\psi : \Lambda \to \mathbb{R}$, $\alpha > 0$, consider

$$\mathcal{L} : C^0(\Lambda) \to C^0(\Lambda), \quad f \mapsto \sum e^{\psi(y)} f(y)$$

Understanding $\varepsilon \to 0$. Main (local) result

Stochastic setting: Quasi-ergodic measures are often obtained from the dominant eigenfunction of

$$\mathcal{P}_\varepsilon : f \mapsto e^{\phi(X_0^\varepsilon)}\mathbb{E}_x[f \circ X_1^\varepsilon \cdot \mathbb{1}_{\Lambda_\delta} \circ X_1^\varepsilon] \qquad \text{(assume strong Feller)}$$

and its dual.

Theorem 2 (Local, T uniformly expanding, Theorem 5.1.6)

Let $\delta > 0$ be small enough and consider the weight $e^\phi : \Lambda_\delta \to \mathbb{R}$ on Λ. Then for $\varepsilon > 0$ sufficiently small:

(1) X_ε^ϕ admits a unique (quasi-stationary) measure μ_ε on Λ_δ such that $\mathcal{P}_\varepsilon^* \mu_\varepsilon = \lambda_\varepsilon \mu_\varepsilon$.

(2) There is a unique $g_\varepsilon \in \mathcal{C}^0(\Lambda_\delta, \rho)$ such that $\mathcal{P}_\varepsilon g_\varepsilon = \lambda_\varepsilon g_\varepsilon$.

(3) The measure $\nu_\varepsilon^\phi(\mathrm{d}x) := g_\varepsilon(x)\mu_\varepsilon(x)$ is the unique quasi-ergodic measure of X_ε^ϕ on $\{g_\varepsilon > 0\}$.

(4) $\nu_\varepsilon^\phi(\mathrm{d}x) \to \nu_0^\phi(\mathrm{d}x) = \nu^\psi$ in the weak-* topology as $\varepsilon \to 0$, the unique equilibrium state for the potential $\psi = \phi - \log|\det \mathrm{d}T|$ on Λ.

Uniformly Expanding Repellers vs Hyperbolic Sets

	Uniformly Expanding $T_{\vert\Lambda} : \Lambda \to \Lambda$	**Hyperbolic Sets** $T_{\vert\Lambda} : \Lambda \to \Lambda$
Function spaces	$\mathcal{P}_\varepsilon : L^\infty \to L^\infty$ $\mathcal{L}_\varepsilon : L^1 \to L^1$	Anisotropic Banach spaces[13] $B^{t,s}, t > 0, s < 0$
$\exists\, \nu_\varepsilon^\phi$, Potentials	No significant changes	Important differences
$\exists!\, \nu_\varepsilon^\phi$ s.t. $\Lambda \subset \operatorname{supp} \nu_\varepsilon^\phi$	Follows from uniform expansion	Need more control + perturbation args.[14]
$\varepsilon \to 0$	Standard bounds, no perturbation args.	Compact embedding for $B^{t,s}$ required + perturbation args.[15]

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[15] V. Baladi. *Dynamical zeta functions and dynamical determinants for hyperbolic maps. A functional approach.* Vol. 68. Results in Mathematics and Related Areas. 3rd Series. A Series of Modern Surveys in Mathematics. Springer, Cham, 2018, pp. xv+291

[15] P.-D. Liu. "Random perturbations of Axiom A basic sets". *J. Statist. Phys.* 90.1-2 (1998), pp. 467–490

[15] G. Keller and C. Liverani. "Stability of the spectrum for transfer operators". *Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4)* 28.1 (1999), pp. 141–152

Global Conditioned Stochastic Stability

To compute the QEM we approximate the right and left maximal eigenfunctions of a (substochastic) transition matrix from Λ_δ to Λ_δ.

$$\mathcal{P}_\varepsilon : L^\infty(\Lambda_\delta) \to L^\infty(\Lambda_\delta), \quad f \mapsto e^{\phi(X_0^\varepsilon)}\mathbb{E}_x[f \circ X_1^\varepsilon \cdot \mathbb{1}_{\Lambda_\delta} \circ X_1^\varepsilon]$$

Where is Λ?

Uniformly Expanding Repellers vs Hyperbolic Sets

	Uniformly Expanding	Hyperbolic Sets
Function spaces	$\mathcal{P}_\varepsilon : L^\infty \to L^\infty$ $\mathcal{L}_\varepsilon : L^1 \to L^1$	Anisotropic Banach spaces[10] $B^{t,s}, t > 0, s < 0$
ν_ε, Potentials	No significant changes	Important differences Section 6.4
$\exists!\ \nu_\varepsilon^\phi$ s.t. $\Lambda \subseteq \text{supp } \nu_\varepsilon^\phi$	Follows from uniform expansion	Need more control Section 6.5.1
$\varepsilon \to 0$	Standard bounds no perturbation args.	Compact embedding for $B^{t,s}$ required + perturbation args.[11]

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[11] V. Baladi. *Dynamical zeta functions and dynamical determinants for hyperbolic maps. A functional approach.* Vol. 68. Results in Mathematics and Related Areas. 3rd Series. A Series of Modern Surveys in Mathematics. Springer, Cham, 2018, pp. xv+291

[11] G. Keller and C. Liverani. "Stability of the spectrum for transfer operators". *Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4)* 28.1 (1999), pp. 141–152

Global Conditioned Stochastic Stability

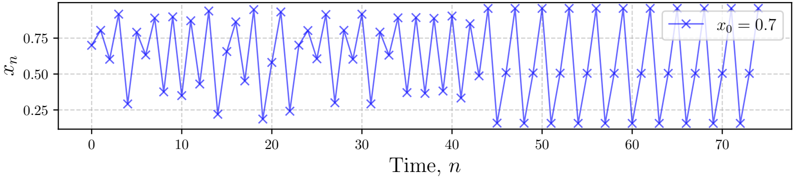

Figure 1: Noisy iterates of the logistic map $X_{n+1} = 3.83 \cdot X_n(1 - X_n) + \omega_n$, with $\omega_n \sim \text{Unif}(-\varepsilon, \varepsilon)$, $\varepsilon = 0.00012$.

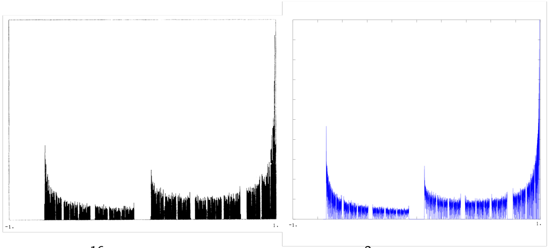

Figure 2: (left) Natural measure[16] of the system $x_{n+1} = 1 - 1.75488x_n^2$, and (right) limiting quasi-ergodic measure.

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[16] H. Kantz and P. Grassberger. "Repellers, semi-attractors, and long-lived chaotic transients". *Phys. D* 17.1 (1985), pp. 75–86, Fig. 4.

Global Conditioned Stochastic Stability

Set ∂ to be a trapping region, e.g. containing an attractor $\mathcal{A}$.

Consider the global transfer operator

$$\mathcal{P}_\varepsilon : L^\infty(M \setminus \partial) \to L^\infty(M \setminus \partial), \quad f \mapsto e^{\phi(X_0^\varepsilon)} \mathbb{E}_x[f \circ X_1^\varepsilon \cdot \mathbb{1}_{M \setminus \partial} \circ X_1^\varepsilon],$$

which we can effectively approximate from data.

Challenges:

1. There may be many repelling invariant sets in $M \setminus \partial$.
2. Control the interaction between repellers:
    - We build an ordering based on the topological pressure of each repeller, i.e. the escape rate, and dynamical filtrations.[17]
3. Leverage the local results to obtain global conditioned stochastic stability.

Results:

1. Existence of global quasi-ergodic measures depending on the initial condition.
2. Conditioned stochastic stability for the equilibrium states of maximal topological pressure.

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[17] D. E. Norton. "The fundamental theorem of dynamical systems". *Comment. Math. Univ. Carolin.* 36.3 (1995), pp. 585–597

Open Questions and Future Directions

Conceptual:

1. Can this strategy be adapted to non-uniformly hyperbolic systems?
2. Can we induce quasi-ergodic measures or construct them in other ways?
3. In learning dynamics from data, how do these objects play a role?

Technical:

1. Remove the assumption of existence of equilibrium states.
2. Adapt to flows perturbed by SDEs with potentials.
3. Compare Quenched and Annealed conditioned RDS with their thermodynamic formalism counterpart.

Key Take-aways

Conceptual:

1. A new strategy is needed to talk about stochastic stability for repelling sets.
2. Conditioned Random Dynamics provides a natural object: quasi-ergodic measures.
3. Quasi-ergodic measures can be easily approximated numerically.

Technical:

1. Existence and uniqueness of QEMs shown with good properties, e.g. $\Lambda \subset \nu_\varepsilon^\phi$.
2. As $\varepsilon \to 0$ we recover equilibirum states on deterministic invariant repellers.
3. Results established for uniformly expanding repellers and hyperbolic sets for diffeomorphisms.
4. Extension to global conditioned stochastic stability.

Thank you for your time and attention!