

Index

Symbols

- $A[[T]]$, ring of formal power series with coefficients in A , 18
 A_K , algebra obtained by extension of scalars, 89
 \hat{A} , formal completion of A for the I -adic topology, 18
 $\text{Ann}(M)$, annihilator of a module M , 13
 $\text{Ann } \mathcal{F}$, annihilator of an \mathcal{O}_X -module \mathcal{F} , 173
 $\text{Ass}(M)$, set of associated prime ideals of M , 253
 A^* , set of invertible elements of a ring A , 45
 $a(X)$, Abelian rank of a curve, 315
 $\langle \cdot, \cdot \rangle_s$, symmetric bilinear form on $\text{Div}_s(X)_{\mathbb{R}}$, 385
 $\text{CaCl}(X)$, group of Cartier divisors modulo linear equivalence, 257
 $\text{codim}(Z, X)$, codimension of Z in X , 70
 $\mathcal{C}_{X/Y}$, conormal sheaf of X in Y , 229
 d -uple embedding, 176, 210
 $D(f)$, open subset associated to a function f , 27
 $D_+(f)$, open subset associated to a homogeneous element f , 51
 $D \cdot E$, intersection of a divisor D with a vertical divisor E , 383
 $\text{deg}_k D$, degree of a Cartier divisor D , 275
 $\text{deg}_k \mathcal{L}$, degree of an invertible sheaf, 282
 $\text{degtr}_k K$, transcendence degree of K over k , 74
 $\Delta_{X/Y}$, diagonal morphism, 101
 Δ_W , discriminant of a Weierstrass model, 446
 $\text{depth } M$, depth of a module, 335
 $\text{Der}_A(B, M)$, derivations of B into M , 210
 $\dim A$, dimension of a ring A , 70
 $\dim X$, dimension of a topological space X , 69
 $\dim_x X$, dimension of X at $x \in X$, 69
 $\text{Div}(X)$, group of Cartier divisors, 256
 $\text{Div}_s(X)$, group of divisors with support in X_s , 381
 $\text{Div}_s(X)_{\mathbb{R}}$, real vector space $\text{Div}_s(X) \otimes_{\mathbb{Z}} \mathbb{R}$, 385
 $\text{div}(f)$, principal Cartier divisor associated to a rational function, 256
 $\text{Div}_+(X)$, effective Cartier divisors, 256
 $\text{div}(s)$, Cartier divisor associated to a rational section of an invertible sheaf, 266
 $D|_E$, restriction of a Cartier divisor to a closed subscheme E , 377
 E^2 , self-intersection of a vertical divisor E , 383

- $f \times g$, product of two morphisms, 80
 $f_{S'}$, morphism obtained by base change $S' \rightarrow S$, 81
 $f^*\mathcal{G}$, pull-back of a sheaf of modules, 163
 Φ_E , group of components of the Néron model of E , 497
 $\mathcal{F}(n)$, twist of \mathcal{F} , 166
 $\text{Frac}(A)$, total ring of fractions, 255
 \mathcal{F}_s , pull-back of \mathcal{F} to a fiber, 201
 f^*D , inverse image of a Cartier divisor, 262
 f_*Z , direct image of a cycle, 271
 $\mathcal{F} \otimes_{\mathcal{O}_X} \mathcal{G}$, tensor product of two \mathcal{O}_X -modules, 158
 $\mathcal{F}|_U$, restriction of a sheaf to an open subset U , 34
 \mathcal{F}^\vee , dual of an \mathcal{O}_X -module, 173
 F_X , absolute Frobenius, 94
 $F_{X/S}$, relative Frobenius, 94
 $g(X)$, genus of a smooth projective curve, 280
 G^0 , identity component of an algebraic group G , 496
 \mathbb{G}_a , additive group scheme, 299
 \mathbb{G}_m , multiplicative group scheme, 299
 $\text{gr}_{\mathfrak{m}}(A)$, graded ring associated to an ideal \mathfrak{m} , 135
 $\text{Hom}_{\mathcal{O}_X}(\mathcal{F}, \mathcal{G})$, sheaf of homomorphisms from \mathcal{F} to \mathcal{G} , 172
 $H^p(X, \mathcal{F})$, Čech cohomology group of \mathcal{F} , 182
 $\text{ht}(I)$, height of an ideal I , 70
 \sqrt{I} , radical of an ideal I , 27
 $i_x(D, E)$, intersection number of D and E at x , 377
 $j(E)$, modular invariant of an elliptic curve, 500
 $k(\nu)$, residue field of a valuation ν , 355
 $K(X)$, field of rational functions, 66
 $k(x)$, residue field at a point x , 37
 $K_{X/S}$, canonical divisor on a fibered surface $X \rightarrow S$, 389
 $L(D)$, global sections of $\mathcal{O}_X(D)$, 280
 $l(D)$, dimension of $L(D)$, 280
 $\text{length}_A(M)$, length of an A -module M , 258
 \mathcal{L}^n , n th tensor power of an invertible sheaf, 169
 $M \otimes_A N$, tensor product over A , 2
 M_f , localization of M at f , 10
 $M_{\mathfrak{p}}$, localization of M at a prime ideal \mathfrak{p} , 10
 $M[\alpha]$, the α -torsion elements of M , 198
 $\text{Mor}(X, Y)$, set of morphisms from X to Y , 48
 $\text{Mor}_S(X, Y)$, set of morphisms of S -schemes from X to Y , 81
 $\mu_x(D)$, multiplicity of a hypersurface at a point x , 401
 $\text{mult}_x(D)$, multiplicity of a Cartier divisor at a point x , 260
 $\text{mult}_x(Z)$, multiplicity of a cycle at a point x , 267
 n_G , multiplication by n in a commutative group G , 307
 $\mathcal{N}_{X/Y}$, normal sheaf of X in Y , 229
 \mathcal{O}_K , valuation ring of K , 107
 $\Omega_{X/Y}^r$, differential forms of order r , 238
 $\Omega_{X/Y}^1$ or Ω_X^1 , sheaf of relative differential forms, 216
 $\omega_{X/Y}$, dualizing (or canonical) sheaf, 239
 \mathcal{O}_X , structure sheaf, 37
 $\mathcal{O}_X^{(I)}$, direct sum indexed by I , 158
 $\mathcal{O}_X(n)$, twist of \mathcal{O}_X , 165
 p -adic integers, 18
 $\mathbb{P}(V)$, projective space associated to a vector space V , 54

- $p_a(X)$, arithmetic genus of a curve X , 280
 $p_a(Z)$, arithmetic genus of a vertical divisor Z , 431
 $p_g(X)$, geometric genus, 280
 $\text{Pic}(X)$, Picard group, 173
 $\text{Pic}^0(X)$, group of divisors of degree 0, 300, 307, 430
 $\pi_0(X)$, scheme of connected components of X , 496
 \mathbb{P}_A^n , projective space over a ring A , 50
 \mathbb{P}_S^n , projective space over a scheme S , 82
 $\text{Proj } B$, set of homogeneous prime ideals of a graded algebra, 51
 $\text{Proj } B$, scheme associated to a graded algebra, 53
 $\text{Proj } \mathcal{B}$, scheme associated to a homogeneous sheaf of algebras, 321
 $\text{Rad}(A)$, radical of A , 9
 $\text{Reg}(X)$, set of regular points of X , 131
 $R^p f_* \mathcal{F}$, higher direct image of a sheaf \mathcal{F} , 189
 $\text{Sing}(X)$, set of singular points of X , 131
 $\text{sp}(X)$, underlying topological space of a scheme X , 81
 $\text{Spec } A$, spectrum of A , 26
 $\text{Spec } \mathcal{A}$, spectrum of a quasi-coherent \mathcal{O}_X -algebra, 175
 $\text{Spec } \varphi$, morphism of schemes associated to a ring homomorphism φ , 28
 $s|_V$, restriction of a section s to an open subset V , 34
 $\text{Supp } D$, support of a Cartier divisor, 260
 $\text{Supp } \mathcal{F}$, support of a sheaf \mathcal{F} , 40
 $\text{Supp } M$, support of a module, 336
 $\text{Supp } Z$, support of a cycle, 267
 s_x , germ of a section s , 35
 $T_{f,x}$, tangent map, 126
 $t(X)$, toric rank of a curve, 315
 $u(X)$, unipotent rank of curve, 315
 $V(f)$, principal closed subset associated to f , 27, 75
 $V(I)$, closed subset defined by an ideal I , 26
 $V(\mathcal{J})$, closed subscheme associated to a quasi-coherent sheaf of ideals, 164
 $V_+(I)$, closed subset defined by a homogeneous ideal I , 51
 ν_Γ , valuation associated to a closed irreducible subset Γ of codimension 1, 354
 ν_ξ , valuation associated to a point of codimension 1, 354
 $\overline{\{x\}}$, Zariski closure of $\{x\}$, 64
 $X(K)$, set of points of X with values in a field K , 92
 $X(S)$, set of sections of an S -scheme X , 49
 X_f , open subset of X associated to a function $f \in \mathcal{O}_X(X)$, 44
 $\chi_k(\mathcal{F})$, Euler–Poincaré characteristic of a coherent sheaf \mathcal{F} , 205
 $X_{S'}$, S' -scheme obtained by base change $S' \rightarrow S$, 81
 $X^{(p)}$, twist by the Frobenius, 94
 X_{red} , reduced scheme associated to X , 60
 X_s , open subset defined by a section s of an invertible sheaf on X , 166
 $X \times_S Y$, fibered product of the S -schemes X, Y , 80
 $Z(I)$, set of common zeros of the polynomials contained in I , 31
 $Z(P_1, \dots, P_m)$, set of common zeros of the polynomials P_1, \dots, P_m , 30
 $Z_0(X)$, group of 0-cycles on X , 398
 $Z^1(X)$, group of cycles of codimension 1 on X , 267

A

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