

Page 2, line -4: replace “the projection map” by “the bilinear map defined by $(x, a) \rightarrow ax$ ”.

Page 3, line 5: replace two occurrences of ϕ by ψ , as ϕ is already used.

Page 8, line 3: “Let N_0 be a direct factor of N free of finite rank”.

Page 9, Lemma 2.7 (Nakayama’s lemma), Remark 2.8 and Corollary 2.9: replace the local ring A by an arbitrary ring A , and the maximal ideal \mathfrak{m} by the radical $I := \text{Rad}(A)$ of A . This is needed for the proof of the new Proposition 7.1.32.

Page 10, lines 9-10: replace “ideals” by “prime ideals”.

Page 10, line 17: replace “ $n \geq 1$ ” by “ $n \geq 0$ ” because we want the unit to belong to any multiplicative subset.

Page 16, line 4: replace “sum $V_1 + V_2$ ” by “difference $V_1 - V_2$ ”.

Page 19, line 12: $F_j \in \underline{I}$.

Page 21, line -10: $N := x\underline{A}$.

Page 21, line -2: $0 \rightarrow N \rightarrow L \rightarrow M \rightarrow 0$.

Page 22, line -10: $\alpha = \underline{a}(1 - \delta)$.

Page 24, line -5 : replace π_n by π_{n-1} .

Page 25, line 1: Add 0 at the end of the exact sequence.

Page 28, end of the proof of Lemma 1.6(a): replace “Which is impossible” by “Which contradicts the hypothesis on f .”

Page 29, line 16: the assumption “ A_0 noetherian” is not necessary.

Page 29-31, Prop. 1.9 and Lemma 1.18: suppose that A is non-zero.

Page 30, Prop. 1.11: suppose I is a proper ideal.

Page 32, line -12: replace A_p by $A_{\mathfrak{p}}$.

Page 33, line -2: replace $F(V)$ by $\mathcal{F}(V)$.

Page 35, line 1: replace “ $\{U_i\}_i$ ” by “ $\mathcal{U} = \{U_i\}_i$ ”, and “We let” by “let”.

Page 36, lines -14 and -13: $f(\underline{y}) = s_{\underline{y}}$ for every $\underline{y} \in V$.

Pges 41, lines 1-2: replace “these morphisms are isomorphisms” by “the first morphism is surjective”. The original **statement is false**.

Page 41, line 5: the bijection is $\text{Mor}_Y(\mathcal{G}, f_*\mathcal{F}) \simeq \text{Mor}_X(f^{-1}\mathcal{G}, \mathcal{F})$.

Page 43, line 13: replace $V \subset U$ by $V \subseteq U$.

Page 43, line -11: replace “An open subset” by “A proper open subset”.

Page 45, bottom line: replace $\varphi^{-1}(\mathfrak{p})$ by $\varphi^{-1}(\mathfrak{q})$.

Page 51, line -3: rephrase the statement to avoid Overfull.

Page 52, lines 6-7: The equality $V(I) \cap \text{Proj} B = V_+(\mathbb{I}^h)$ is **false**. Use instead the fact that $D(f) \cap \text{Proj} B = \cup_i D_+(f_i)$ if the f_i are the homogeneous components of f . See the new text.

Page 54, line 7: Lemma 3.36 (a).

Page 56, line 10: replace “isolated point.” by “isolated point in X (i.e., $\{x\}$ is an open subset of X).”

Page 72, Lemma 2.5.16: Suppress the hypothesis that A is local. The proof is modified.

Page 73, line -10: replace “degree ≥ 1 ” by “degree 1”.

Page 75, Exercise 5.2: add the statement “If Z is a closed subset of X , then $\text{codim}(Z, X)$ is the minimum of $\dim \mathcal{O}_{X,z}$ for all $z \in Z$ ”.

Page 75, line -4: replace “This is ...” by “One can suppose that \mathfrak{p}_r does not contain any \mathfrak{p}_i , $i < r$.”

Page 77, line -16: replace “discriminant” with “determinant”.

Page 79, line -11: replace “Lemma 2.3.23” by “Proposition 2.3.25”.

Page 82, line 3: replace $B[T_0, \dots, T_n]$ by $B[T_1, \dots, T_n]$.

Page 87, line -2: replace $f^{-1}(Y_i)$ by $f^{-1}(V_i)$.

Page 90, lines 1-5: replace “Let us show ... prime ideal” by “Let \mathfrak{q} be a prime ideal of A_K such that $\mathfrak{q} \cap A = \mathfrak{p}$. Then $\mathfrak{p}A_K \subseteq \mathfrak{q}A_K = \mathfrak{q}$. On the other hand, since $\alpha^q \in A \cap \mathfrak{q} = \mathfrak{p}$ for all $\alpha \in \mathfrak{q}$, we have $\mathfrak{q} \subseteq \sqrt{\mathfrak{p}A_K}$. Hence $\mathfrak{q} = \sqrt{\mathfrak{p}A_K}$ and p is injective.”

Page 95-96, Lemma 2.24 and Corollary 2.27: the proofs are changed. The first part of the original proof of 2.27 is incorrect. See the new text.

Page 99, line 4: add “such that $f^{(p)} \circ F_{X/S} = F_{Y/S} \circ f$ ” at the end of the sentence.

Page 99, line 7: replace $X^{(p)} \rightarrow Y^{(p)}$ by $f^{(p)}$.

Page 109, Remark 3.33, line 4: replace “(3) there exist projective varieties...” by “(3) there exist normal proper varieties...”.

Page 113, line -4: “the radical of the set $\mathfrak{p}(i)$ ”.

Page 122, line -11: in $B/fB \simeq \oplus_{1 \leq i \leq n} B_{\mathfrak{q}_i}/(f)$, replace f by b .

Page 128, line -4: “let (A, \mathfrak{m}) be a Noetherian regular local ring...”.

Page 129, last line of the proof of Proposition 2.11: delete “and”.

Page 131, line 2: replace $(2t, -3s)$ by $(2t, -3s^2)$.

Page 139, Lemma 3.20: replace “ X_y is finite...” with “ X_y is finite, reduced, ...”. In the third line of the proof, replace “ X_y is of dimension 0...” with “ X_y is of dimension 0 and reduced...”.

Page 140-141, proof of Proposition 3.26: move the sentence “As $A \rightarrow B$ is a flat homomorphism... It follows...” right after “ f is étale at x ” because we want to say that $A \rightarrow B$ is injective, which allows us to see A as a subring of B .

Page 142, line -6: add “The set of points $x \in X$ such that f is smooth at x is called the *smooth locus* of f .”

Page 143, line -6: replace m_x/\mathfrak{m}_x^2 by $\mathfrak{m}_x/\mathfrak{m}_x^2$.

Page 145, Exercise 4.3.8(a): weaken the hypothesis on Z by assuming only Z flat at one point. See the new text.

Page 150, line 20: Proposition 3.3.16(e).

Page 151, line 5: replace f^* by $f^\#$.

Page 152, line 14: replace $f_*\mathcal{O}_Y(W)$ by $f_*\mathcal{O}_X(W)$.

Page 153, Proposition 4.11: to clarify the statement, delete “(resp. étale)”, and add “If f is étale, then g is an open immersion.” at the end of the proposition.

Page 154, at the end of the proof of Lemma 4.13: “Exercise 3.2.16”.

Page 155, line 7: replace f_s by $f_s : X_s \rightarrow \mathbb{A}_{k(s)}^d$.

Page 155, line 10 (end of the proof): replace “Corollary 4.13” by “Lemma 4.13”.

Page 155, Exercise 4.4.2(a): replace “principal closed subset” by “non-empty principal closed subset”.

Page 155, Exercise 4.4.2(b): replace “ $V_+(P) \cap \mathbb{P}_V^n$ ” by “ $X \cap V_+(P) \cap \mathbb{P}_V^n$ ”.

Page 162, line -6: the vertical arrows in the commutative diagram are in the opposite sense.

Page 168, lines 20, 22: exchange Lemma 1.25(a) and Lemma 1.25(b).

Page 169, beginning of Remark 1.32: replace with “Let us fix a set of sections s_0, \dots, s_d of $\mathcal{L}(X)$ which generates \mathcal{L} . Then the morphism f as in (b) is unique. Moreover, ...”

Page 170, line -3: replace $h_{ij} \in \mathcal{O}_X(X_j)$ by $h_{ij} \in \mathcal{O}_X(X_{s_j})$.

Page 171, line 2: replace $U_{t_{ij}}$ by $U_{s_{ij}}$.

Page 173, bottom line : add “(called the *Picard group of X*)” after $\text{Pic}(X)$.

Page 181, line -11: replace $(f_i)_i \mapsto f_i|_{U_i \cap U_j}$, by $(f_i)_i \mapsto (f_i|_{U_i \cap U_j} - f_j|_{U_i \cap U_j})_{i,j}$.

Page 184, line -8: replace $C^1(\mathcal{U}, \mathcal{F}'')$ by $\text{Ker}(C^1(\mathcal{U}, \mathcal{F}'') \rightarrow C^2(\mathcal{U}, \mathcal{F}''))$.

Page 185, line 18, Formula (2.5) : replace “ldots” by “...”.

Page 186, at the end of line 16: U_α is an index of $\mathcal{F}|$.

Page 188, bottom line: replace “ $(\text{Ker}\beta/\text{Im}\alpha) \otimes_A \text{Id}_M$ ” by “ $(\text{Ker}\beta/\text{Im}\alpha) \otimes_A M$ ”.

Page 195, line -2: “Exercise 1.16(c)”.

Page 196, line 18: add at the end “(or, as pointed out by C. Ivorra, of a flasque resolution of the sheaf)”.

Page 199, line 14: $H^p(\underline{X}, \mathcal{F}/\alpha^m \mathcal{F})$.

Page 204, Proposition 3.21(c): replace $R^p f_*(\mathcal{F} \otimes_{\mathcal{O}_Y} \mathcal{G})$ by $R^p f_*(\mathcal{F} \otimes_{\mathcal{O}_X} f^* \mathcal{G})$.

Page 205, line 5: “Let \mathcal{I} be a coherent sheaf of ideals”.

Page 205, Example 3.27: replace Lemma 2.1 by Lemma 3.1.

Page 207, line -4 : étale instead of ètale.

Page 212, line 3: the tensor product in on B and not on A .

Page 215, line 5: The first “dim” is \dim_K .

Page 215, end of Remark 1.16: “Proposition 3.2.15”.

Page 219, Exercise 6.1.2: add “in general” after “show that”.

Page 220, Prop. 6.2.2: add a third equivalent property “(iii) X is smooth at x ”. Change the proof of (i) \implies (ii) to (iii) \implies (ii). See the new text.

Page 221, line 9: $\dim_{k(z')} \Omega_{U, z'}^1 \otimes k(z')$.

Page 222, line 9: replace “ $\mathcal{I}(Y)$ ” by “ $\mathcal{O}_Y(Y)$ ”.

Page 222, line 20 (end of the proof) : replace “ $\dim_z Z_s$ ” by “ $\dim_x Z_s$ ”.

Page 222, Prop. 6.2.5: change the proof. See the new text.

Page 222, bottom line: move “where $s = f(x)$ ” to the end of Formula (2.5), and replace $\Omega_{X,x}^1$ by $\Omega_{X/S,x}^1$.

Page 229, Lemma 6.3.10: replace “homomorphism of flat rings” by “flat homomorphism of rings”.

Page 230, lines 3-4: suppose that $X \cap Y'$ is non-empty. The statement on the dimension of X is **false**. replace by : “ $\dim \mathcal{O}_{X,x} = \dim \mathcal{O}_{Y,x} - n$ for all $x \in X$ ” (no finitenesse hypothesis on $\dim Y$).

Page 230, line 20: add “(Exercise 2.5.2)” after “property is local”.

Page 230, line -15: suppress “when $\dim Y'$ is finite”.

Page 231, line 19 (proof of Cor. 3.14): replace “Taking $X = \dots$ in exact sequence (3.7)” with “Applying exact sequence (3.7) to the morphism of Y -schemes $\pi : Y \rightarrow X$ ”.

Page 233, lines 19-20: “ f is a regular immersion”.

Page 234, lines 4 and 5: replace \mathcal{O}_{Z_y} by $\mathcal{O}_{Z_y, x}$.

Page 235, line 10: replace “ F_1, \dots, F_n ” by “ $F_1, \dots, F_n \in \mathcal{O}_Y(V)[T_1, \dots, T_n]$ ”.

Page 235, Exercise 6.3.5(c): replace “ $\mathcal{O}_X(U) \otimes k(y)$ ” with “ $k(y)[T_1, \dots, T_n]$ ”.

Page 235, Exercise 6.3.7: Suppose that $k(x) = k(y)$ and delete the term

$\dim_{k(x)} \Omega_{k(x)/k(y)}^1$ in line -15.

Page 235, line -8: replace “be an l.c.i.” by “be a dominant l.c.i.”

Page 237, lines 11-12: “be respective bases of \underline{L} and $N \dots$ is a basis of \underline{M} ”.

Page 239, line 8: add the sentence “If $f : X \rightarrow Y$ is a regular immersion, then $\omega_{X/Y} = \det(\mathcal{C}_{X/Y})^\vee$.” before “Let us”.

Page 252, lines -17 and -16 : replace “equivalenct” by “equivalent” and “Remark 1.20” by “Proposition 1.32”.

Page 253, end of Lemma 1.2(b): add “and $\mathfrak{p} \cap S = \emptyset$ ”.

Page 254, lines 2, 11: replace $\text{Ann}(x_1A)$ by $\text{Ann}(x_1)$, $\text{Ann}(xA)$ by $\text{Ann}(x)$, and $\text{Ann}(\bar{x}A)$ by $\text{Ann}(\bar{x})$.

Page 254, end of the proof of Corollary 1.5: replace $\text{Ass}(yA) = \mathfrak{p}$ by $\text{Ass}(yA) = \{\mathfrak{p}\}$.

Page 255, Lemma 1.12(a): the **statement is false**. Fortunately, this does not change the rest of the book. See the new text.

Page 256, Proposition 1.5: The proof is rewriting in great part. See the new text.

Page 258, Remark 1.20: replace by the new text.

Page 260, line -6: $E|_X \geq 0$ instead of $E|_X > 0$.

Page 261, Proposition 1.32: give and prove the statement for any quasi-projective scheme over a Noetherian affine scheme. See the new text.

Page 262, line 2: replace “ $f_*\mathcal{K}_x$ ” by “ $f_*\mathcal{K}_X$ ”.

Page 262, at the end of Definition 1.34: add “By construction, f^*D is principal if D is principal. Hence f^* induces a canonical homomorphism $\text{CaCl}(Y) \rightarrow \text{CaCl}(X)$.”

Page 262, at the end of Remark 1.35: replace “ $\mathcal{O}_X(-f^*D)$ ” by “the sheaf of ideals $\mathcal{O}_X(-f^*D)$ ”.

Page 262, Lemma 1.36(a): add “and $[B/\mathfrak{n} : A/\mathfrak{m}] < +\infty$.”

Page 263, Proposition 1.38, line 12: replace $3x$ by x in the displayed formula.

Page 263, line -16: replace “basis of $K(Y)$ over $K(X)$ ” by “basis of $K(X)$ over $K(Y)$ ”.

Page 266, Exercise 1.10(b): the **statement is false in general**. See the new text.

Page 269, Proposition 2.11(b): replace $x \in U$ by $x \in X$.

Page 269, line -6 : replace $\mathcal{O}_{x,x}$ by $\mathcal{O}_{X,x}$.

Page 271, lines 1-2: “Let X be a Noetherian regular integral (hence normal,...)

scheme”.

Page 271, line 13: replace (f_x, U_x) by (U_x, f_x) .

Page 273, line 10: replace $[\Delta_i] = \Gamma_i$ by $[\Delta_i] = f(\Gamma_i)$.

Page 275, line 18: replace $k(x_i)$ by $k(x)$.

Page 275, line -5: add “(note that $\text{mult}_x(D) = 0$ if $x \notin D$)” before “we obtain the lemma”.

Page 276, line 2: replace “finite set, hence closed...” by “finite set. As b_i is a regular element, it does not contain generic points of U_i . Hence $V(b_i)$ is closed...”.

Page 276, line 17: replace the proof of Prop.3.7(b) by “For all $x \in X$, $\text{mult}_x(D)$ does not depend on the base field. So the equality comes from the fact that $[k(x) : k'] = [k : k'][k(x) : k]$.”

Page 276, end of Proposition 3.8: Definition 4.1.19.

Page 278, Remark 7.3.14: replace “equivalence of categories between normal projective curves over k and function fields in one variable over k .” by “equivalence between the category of normal projective curves over k with finite morphisms and the category of function fields in one variable over k with homomorphisms of k -algebras .”

Page 282, line -8: suppress “and Remark 1.20”.

Page 284, line 7: replace “Remark 1.20” par “Proposition 1.32”.

Page 286, lines 14-18 (end of the proof of Lemma 4.3): replace with the new text.

Page 290, line 9: “Then Lemma 1.36(c) can be...”.

Page 290, line -16: “By Proposition 3.8 and Corollary 3.31(a)”.

Page 290, line -14: “we have an injective canonical homomorphism”.

Page 291, Proposition 7.4.21: replace “geometrically reduced” by “smooth over k ”.

Page 291, line -3: “by Exercise 4.3.13(a) and Proposition 3.13”.

Page 301, Exercise 7.4.7(c) : replace by “By considering the degree of $h - \sigma(h)$, show that we have $g \leq 1$. Conclude.”

Page 306, lines 1-4: replace “Using Lemma... (Exercise 1.9).” by “Using Proposition 1.32, we may suppose that $\mathcal{L} \simeq \mathcal{O}_X(D)$ for some Cartier divisor D on X . By Lemma 3.6, we can even suppose D effective.”

Page 313, line 6: replace $p_a(Y) - p_a(X)$ by $p_a(X) - p_a(Y)$.

Page 313, line -7: replace $p_a(X)$ by $p_a(X_{red})$.

Page 318, line -15: add “or (I)” at the end of the line.

Page 319, line -6: replace “ $u = t_2/t_1$ ” by “ $u = y/x \in \text{Frac}(A)$ ”.

Page 319, line -5: replace “Speck[v] with $v = t_1/t_2$ ” by “ $k[v, 1/v]$ with $v = x/y$ ”.

Page 320, line 6: replace “Lemma 1.2(c)” by “Lemma 1.2(e)”.

Page 321, line -6: replace “ $V(\mathcal{I})$ ” by “ $V(\mathcal{I})$ (or \mathcal{I})”.

Page 330, Remark 1.27: replace “integral scheme” by “integral schemes”. And after “everywhere.”, add “Let X be an integral curve over a field k .”

Page 333, bottom line: $\dim \mathcal{O}_{X,x} = \dim \mathcal{O}_{Y,y} \pm 1$.

Page 335, line 3: $y \in \underline{Y}$.

Page 336, line 5: replace SuppM by $\text{Ann}(M)$.

Page 366, lines 1-2: replace “resolve ... (3.11)” with “resolve the possible singularities of X using the sequence (3.11)”.

Page 367, Example 8.3.54: replace “ X_2 ” by “ X'_1 ” (two occurrences) to be coherent with the notation of the sequence (3.11).

Page 367, Figure of Example 8.3.54: \tilde{X}_k meets the elliptic curve transversally.

Page 382, first line in Remark 1.13: $A = \mathcal{O}_{S,s}$.

Page 383, line 5-7: replace “denote the... on S by $D \cdot E$,” by “denote by $D \cdot E$... on S ,”

Page 383, end of Remark 1.17: add “See also Remark 2.13”.

Page 391, Exercise 1.2: replace SuppD_i by SuppD'_i .

Page 395, line 7: “be a birational morphism of ...”.

Page 395, lines 8-9: replace “ $g : X \rightarrow \tilde{Y}$ and $\pi : \tilde{Y} \rightarrow Y$ ” by “ $X \xrightarrow{g} \tilde{Y} \xrightarrow{\pi} Y$ ”.

Page 395, line -9: add “Then $\dim X_y \geq 1$ by Corollary 4.4.3(b)” after “ $y \in f(\mathcal{E})$ ”.

Page 397, line 9, Definition 2.8: suppress integrality hypothesis on X, Y .

Page 397, last line of Remark 2.9: replace “and is proven in the same maner” by “. See also Remark 2.13”.

Page 398, Theorem 2.12(c): replace the letter “ C ” by “ F ” (because C is already used for divisors on X).

Page 398-400, from the beginning of the proof of (b) to the end of page 400: there is a new version because of a gap in the proof of Prop. 2.15.

Page 416, line -4: “Theorem 4.15”.

Page 417, lines -6 : replace “process around ” by “blowing-up process around”

Page 422, line -17: replace $K_{X/S}$ with $K_{Z/S}$ (three occurrences).

Page 427, line 4 and end of Remark in Exercise 3.2: replace “[89]” by “[90]”.

Page 438, lines 2 and 4: switch n and $n + 1$:

$$0 \rightarrow \mathcal{J}^{n+1} \rightarrow \mathcal{J}^n \rightarrow \dots$$

$$0 \rightarrow \mathfrak{m}_y^{n+1} \rightarrow \mathfrak{m}_y^n \rightarrow \dots$$

Page 440, line 11: replace “ $S \setminus U$ ” by “ $S \setminus V$ ”.

Page 457, line 12, Remark 1.11: replace “regular models regular models, with normal crossings” by “regular models, regular models with normal crossings”.

Page 484, line 15: add “By Corollary 9.3.27, the canonical divisor $K_{\mathcal{E}/S}$ is trivial.”

Page 489, at the end of Remark 10.2.3: replace the last sentence “ $\mathcal{E}_s \dots$ ” by “a component of \mathcal{E}_s can be of multiplicity 1 without being geometrically reduced.”

Page 510, proof of Corollary 3.13: delete “ $+2\text{Card}\Gamma_{\text{sing}}$ ” in the display formula.

Page 518, line -10: “a smooth projective curve...”.

Page 521, second line in Lemma 3.40: “discrete valuation ring \mathcal{O}_K ”.

Page 521, line -4: “(Exercise 9.2.9(c))”.

Page 534, line -18: “Let \underline{L} be the compositum”.

Page 535, lines -7 to -2: replace “Let ... hence $\xi \in \widehat{B}$ ” by a new text.

Page 536, lines 2, 6: replace α by α_1 and γ by α_2 .

Page 536, line 5: add “, $\text{Frac}(D) = \text{Frac}(A)$, ” before “and that”.

Page 536, Formulas (4.33), (4.34): remove “ ϵ ” (it is equal to 1).

Page 540, line -3: “Betti number of the graph associated to T ”.

Page 540, line -2: replace “the text preceding Definition 4.1.54” by “Definition 1.46”.

Page 543, Definition 4.27: replace “, and $\mathcal{C} \times_{\text{Spec}\mathcal{O}_L} \overline{\text{Spec}k(s')} \dots$ the *potential stable reduction* of C ” by “. The curve $\mathcal{C} \times_{\text{Spec}\mathcal{O}_L} \overline{\text{Spec}k(s')}$ is called the *potential stable reduction* of C ”.

Page 545, Remark 4.33, lines 21-22 : replace “Proposition 4.30. If the order of G is ...” By “Proposition 4.30 but with group G of order ...”

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