An Immersed Boundary method to study the effect of roughness in supersonic boundary layers

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Laminar-turbulent transition in compressible, high-speed boundary layers is currently not well understood. This is particularly true for cases where transition is influenced or triggered by means of roughness elements. This case is relevant for a number of applications, including the heat-shield of vehicles (re-)entering a planetary atmosphere and the inlet to scramjet combustors for hypersonic cruise vehicles.

Accurate numerical simulations that represent the disturbance amplification are important to identify the physical mechanisms that leads to transition (and eventually turbulence) but are made difficult by the requirement to strictly control numerical diffusion and dispersion. In the present work, solutions to the compressible Navier-Stokes equations are obtained using a sixth-order compact finite-differences together with explicit Runge-Kutta time stepping. The algorithm is effective in representing transition on smooth plates and two-dimensional isolated roughness elements using curvilinear body-fitted grids but is unable to capture three-dimensional roughness. An alternative approach to represent roughness is to use an immersed boundary method in which forcing terms within the domain are introduced to represent the effect of the roughness. We have developed a novel formulation based on a filtered delta-function which allows to achieve high-order in the definition of the source terms thus retaining the accuracy of the compact discretization operators. Figure 1 shows the magnitude of the resulting body-force term for selected locations along the roughness geometry.

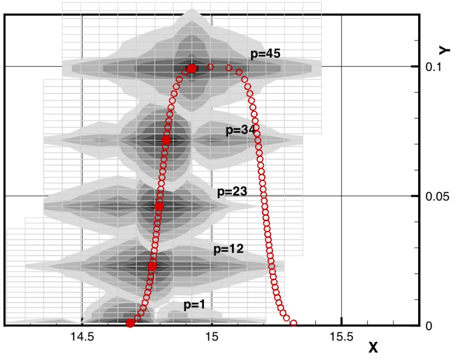

Figure 1: **Absolute value of the body-force term used to impose the immersed boundary for selected grid points along the roughness contour (which is given by circles). The cartesian grid (grey) reflects the grid used in the numerical simulation.**

In this work, we present a numerical investigation of disturbance amplification in a flat-plate boundary layer at $Ma = 4.8$ with two- and three-dimensional roughness elements. The height of the roughness is on the order of, but typically smaller than, the boundary-layer thickness of the unperturbed flow.

The roughness creates weak shocks mostly confined to the spanwise planes which cut the roughness. A small, linear disturbance of fixed frequency is triggered via blowing and suction at the wall close to the inflow boundary. In case of a single two-dimensional roughness (i.e. a spanwise bar), this element considerably alters the instability of the boundary layer (Figure 2). The agreement between results using a body-fitted grid and an immersed boundary method is favorable. This holds true not only with respect to the observed alternation of growth and decay behind the roughness (Figure 2) but also for wall-normal amplitude and phase functions.

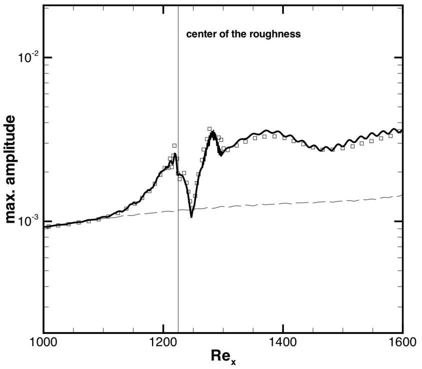

Figure 2: **Streamwise evolution of maximum density $\hat{\rho}_1^{max}$. The 2-D roughness element is centered at x=15 (Re_x=1225) and marked by a line. Symbols: body-fitted grid. Solid lines: immersed-boundary method. Dashed lines: flat plate.**

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