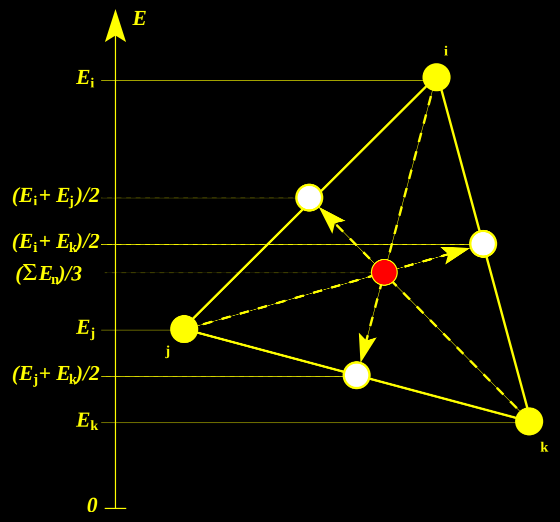
Shear flow compositions on the Galerkin piano

A unified theory for instabilities, strange attractors, statistical mechanics, and attractor control

is currently emerging



Bernd R. Noack & friends

Berlin Institute of Technology *& many other places*

Acknowledgements: DFG, CNRS, ERCOFTAC, NSF, USAFOSR, ...

Workshop: Industrial Applications of Low Order Models based on POD
Bordeaux, 2008-03-31..04-02

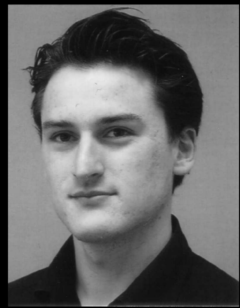
Bernd's friends

(co-author subset for FTT ordered by distance from my office)

at the Berlin Institute of Technology

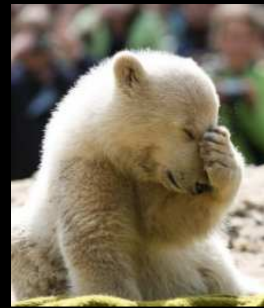
Michael Schlegel

Mark Luchtenburg



Mark Pastoor

Rudibert King



and elsewhere

Gerd Mutschke

Marek Morzyński

Pierre Comte

Gilead Tadmor

Boye Ahlborn

Overview

1. Introduction

— *low-order Galerkin modeling*

2. Control of laminar shear flow

— *low-order modeling of weakly nonlinear dynamics*

3. Control of turbulent shear flow

— *low-order modeling of strongly nonlinear dynamics*

4. Instabilities, turbulence and control

— *an emerging unifying theory*

5. Concluding remarks and outlook

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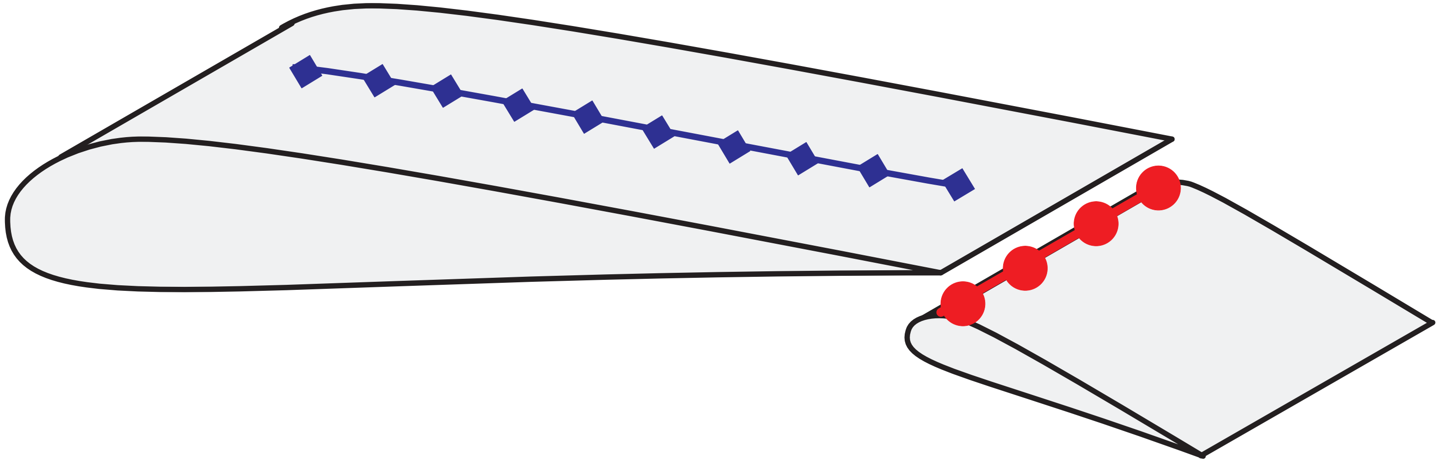
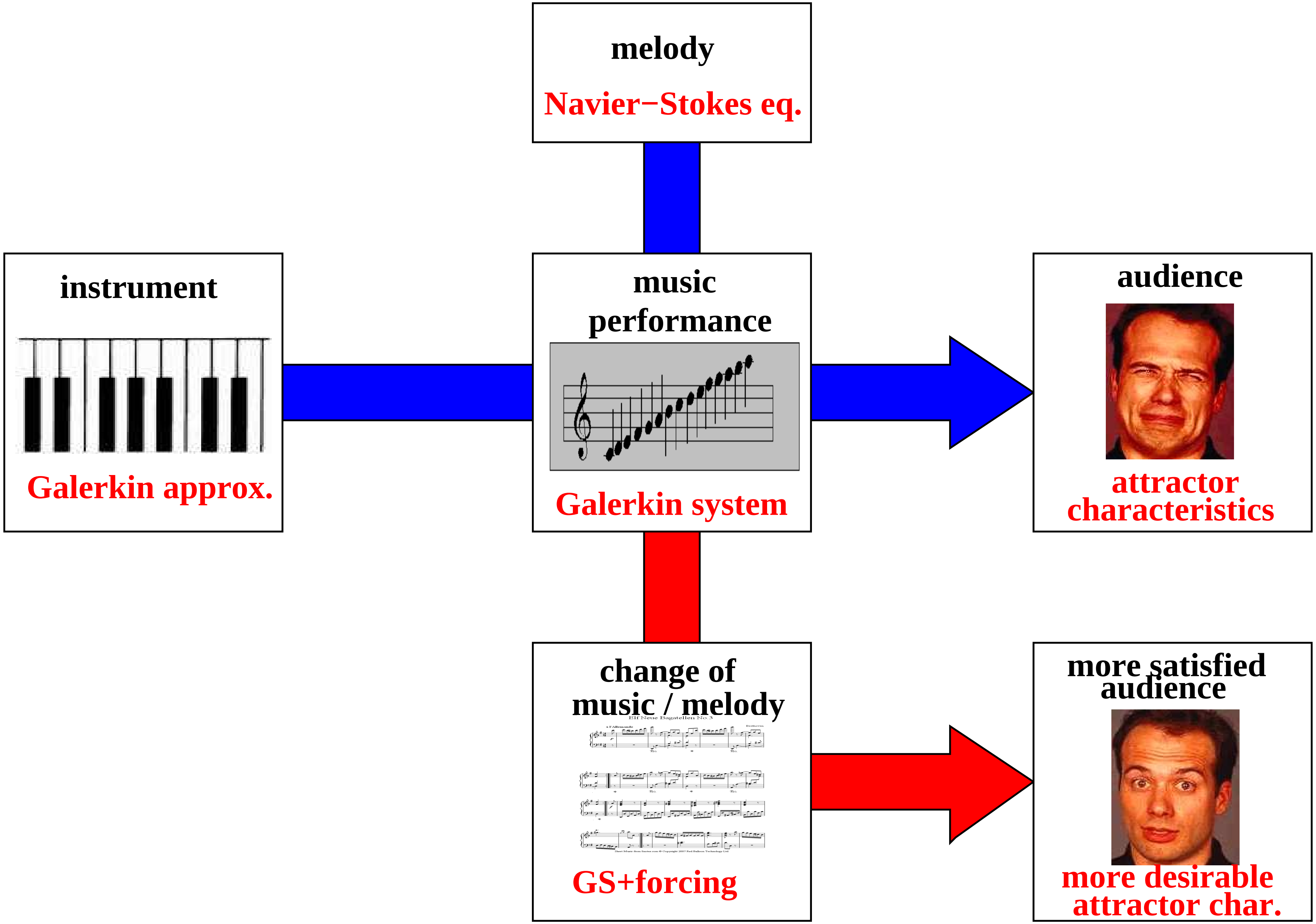
Low-order modeling for flow control



- Myriads of **actuation**- and **sensor**-opportunities:
  - • kind, • location, • amplitude and frequency range,
  - • control design
- No time for myriads of high-fidelity simulations.
- **Complementary low-dimensional models needed for exploration, optimization and control design.**

Low-order Galerkin modelling – piano analogy



'Traditional' Galerkin method

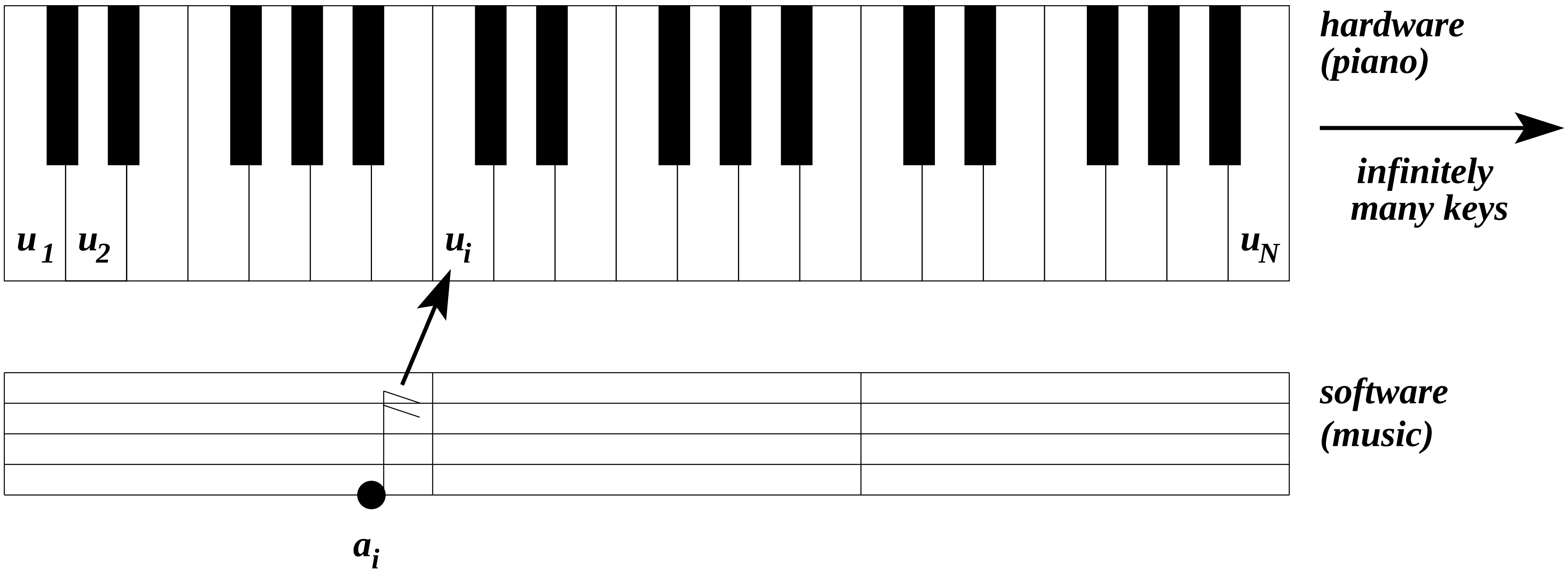
— *Fletcher 1984 Computational Galerkin Methods, Springer* —

Galerkin method

$$\mathbf{u}(\mathbf{x},\mathbf{t}) \quad \rightarrow \quad \partial_t \mathbf{u} = \frac{1}{Re}\triangle \mathbf{u} - \nabla(\mathbf{u}\mathbf{u}) - \nabla p$$

$$\downarrow \qquad\qquad \downarrow$$

$$\mathbf{u}^{[N]} = \sum_{i=0}^{N} a_i(t)\mathbf{u}_i(\mathbf{x}) \quad \rightarrow \quad \frac{da_i}{dt} = c_i + \sum_{j=1}^{N} l_{ij}\, a_j + \sum_{j,k=1}^{N} q_{ijk}\, a_j\, a_k$$



Low-order Galerkin modelling

Noack, Cordier, King, Morzyński, Siegel, Tadmor (2009+) Springer

Kinematics	Dynamics	Control
$\mathbf{u} = \sum_{i=0}^{N} a_i \, \mathbf{u}_i$	$\dot{a}_i = f_i(\mathbf{a})$	$\dot{a}_i = f_i(\mathbf{a}, \mathbf{b})$, $\mathbf{b} = \mathbf{h}(\mathbf{a})$
• basic modes • choice of Hilbert space • POD • other empirical modes [Kunisch, Ravindran] • stability eigenmodes • mathematical modes • actuation modes • control-theory modes [Rowley] • ...	• Galerkin projection • pressure model [Bergmann] • subgrid turbulence • actuation effect [Weller] • robustness [Mathelin,Willcox] • parameter identification • structure identification • ... • system reduction [Antoulas] • intertial manifolds • modal balance equations	Observer • LSE • Kalman filter • Volterra series • dynamic observer [Lombardi,Rempfer] Controller • linear control • optimal control [Cordier] • full-info, MIMO, ...
This talk:	**• attractor closure**	**• nonlinear control**

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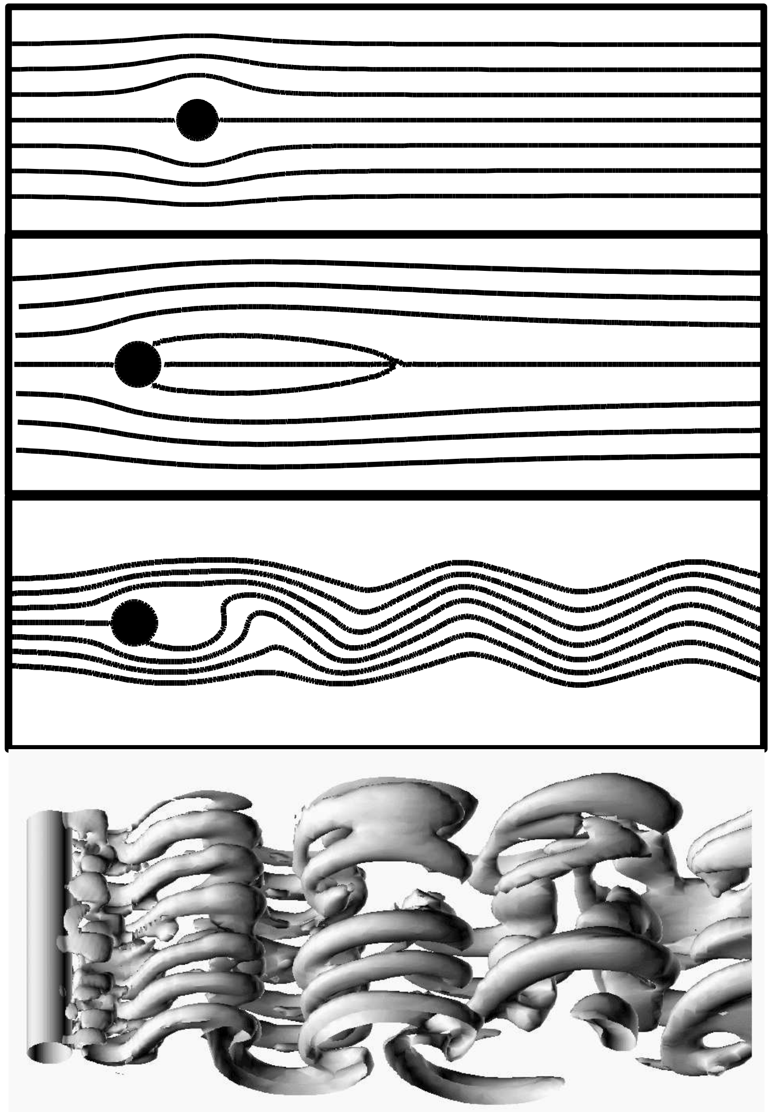
Phenomenogram of cylinder wake

Reynolds number $Re = \frac{UD}{\nu}$



$Re < 4$ — 2D steady flow without vortex pair

$Re < 47$ — 2D steady flow *with* vortex pair

$Re < 180$ — 2D vortex shedding

$180 < Re$ — 2D vortex shedding superimposed by 3D modes / fluctuations

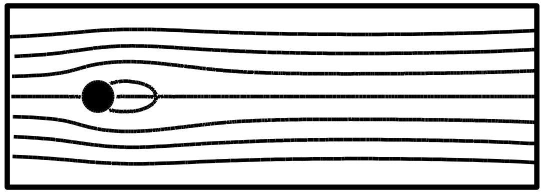
POD Galerkin model

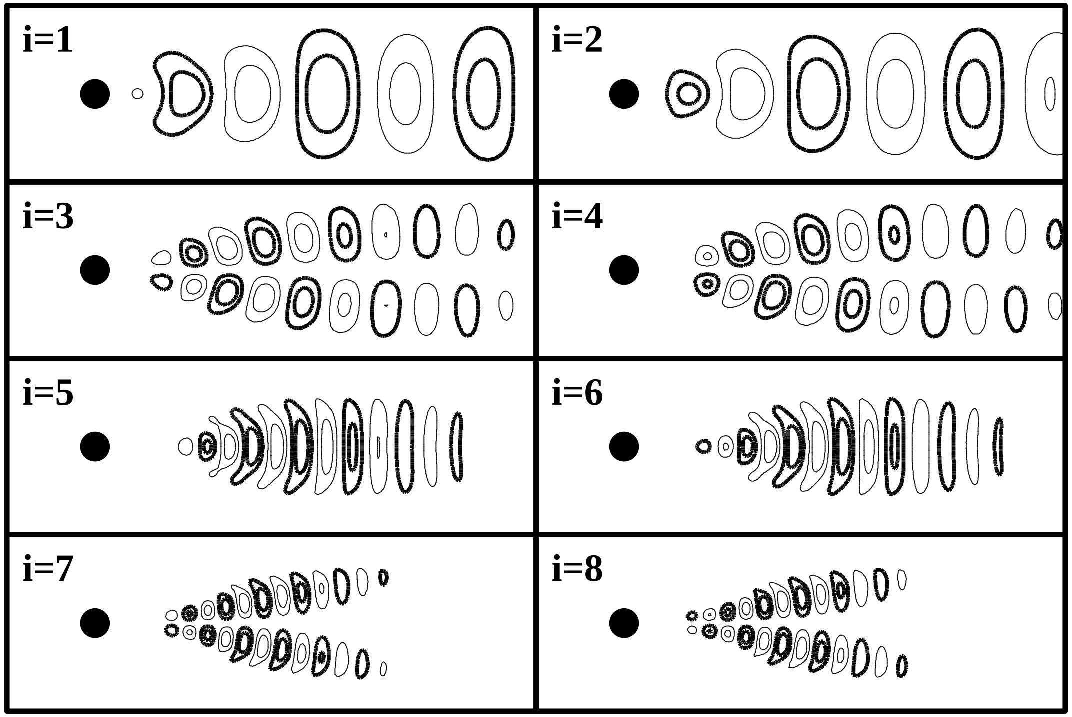
Noack, Afanasiev, Morzyński, Tadmor & Thiele (2003) JFM

POD at $Re = 100$

Deane et al (1991) PF

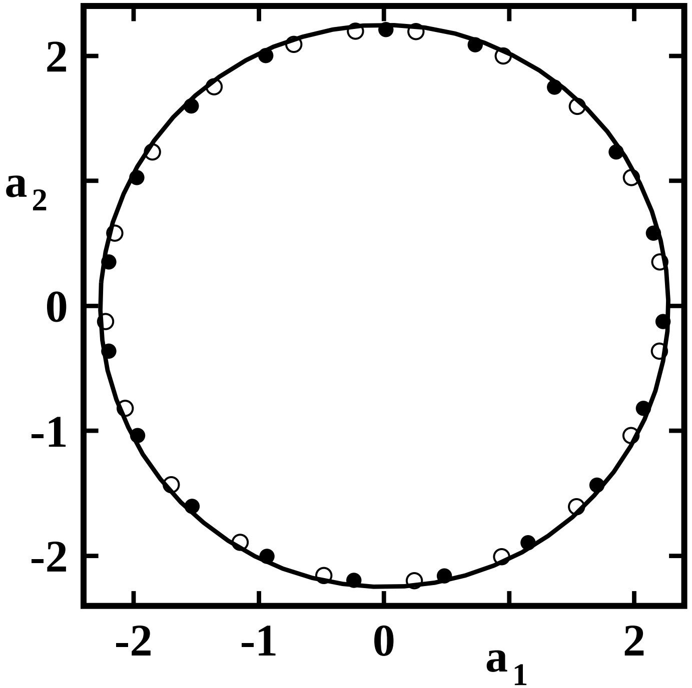
$$\mathbf{u} = \sum_{i=0}^{8} a_i \mathbf{u}_i$$





Galerkin solution

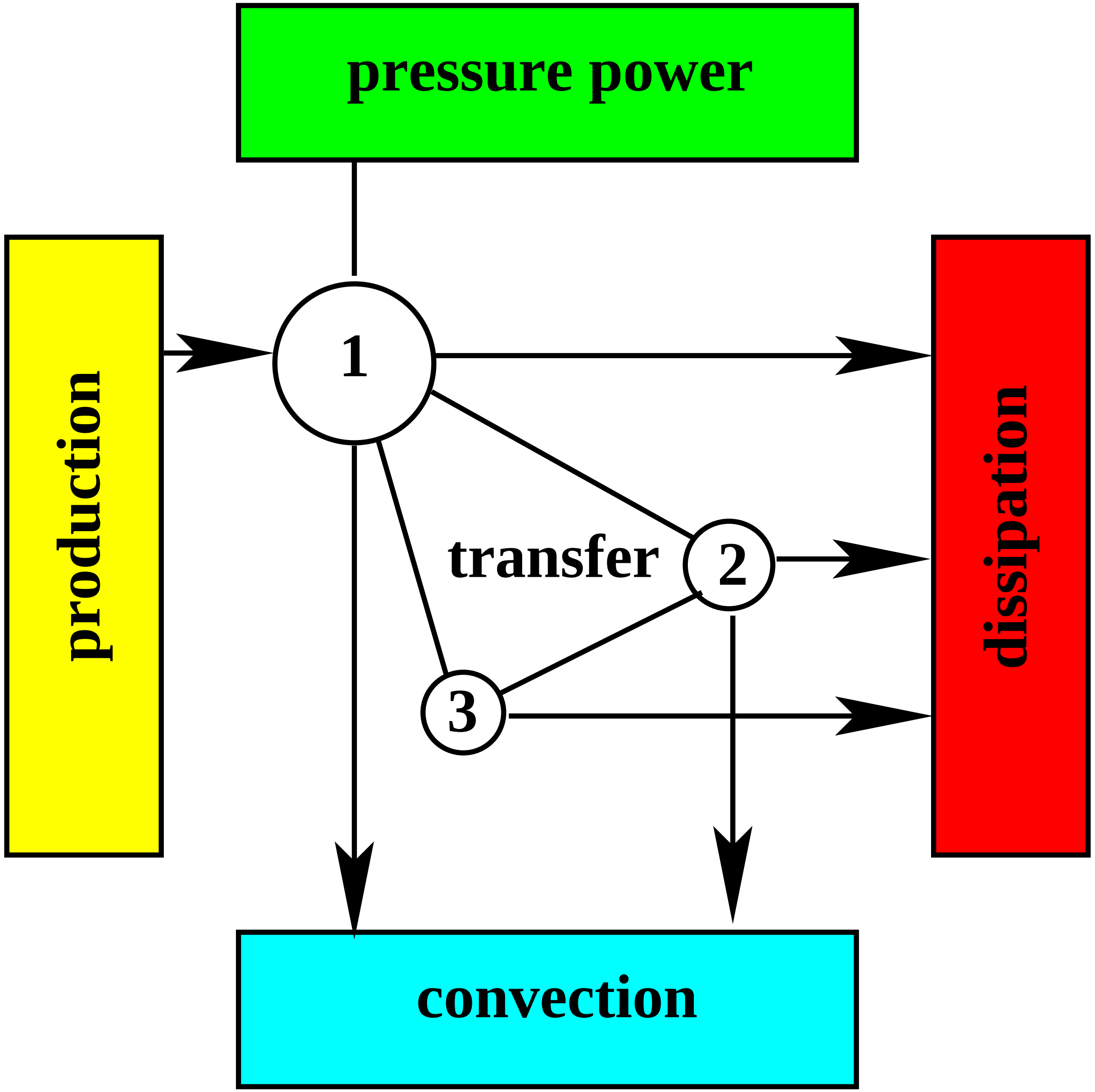
$$\frac{da_i}{dt} = \nu \sum_j l_{ij} a_j + \sum_{j,k} q_{ij} a_j a_k$$



- 8-dim. POD model reproduces DNS.

Modal enery flow analysis

— *Noack, Papas & Monkewitz (2005) JFM* —



$$
\begin{array}{ccc}
P & = & \sum P_i \\
+ & & + \\
D & = & \sum D_i \\
+ & & + \\
C & = & \sum C_i \\
+ & & + \\
T & = & \sum T_i \\
+ & & + \\
F & = & \sum F_i \\
= & & = \\
0 & = & 0
\end{array}
$$

Modal fluid dynamics

— Noack, Papas & Monkewitz (2005) JFM —

In a nutshell:

Galerkin approximation . $\mathbf{u} = \mathbf{u}_0 + \mathbf{u}', \quad \mathbf{u}_0 := \overline{\mathbf{u}}, \quad \mathbf{u}' := \sum_{i=1}^{N} a_i \mathbf{u}_i$

Navier-Stokes Eq. . . . . $\mathcal{R}(\mathbf{u}) = 0$

Galerkin system . . . . $\left(\mathbf{u}_i, \mathcal{R}(\mathbf{u}^{[N]})\right)_\Omega = 0$

Modal energy flow balance $\left(a_i \mathbf{u}_i, \mathcal{R}(\mathbf{u}^{[N]})\right)_\Omega = 0$

Global energy flow balance $\left(\mathbf{u}', \mathcal{R}(\mathbf{u}^{[N]})\right)_\Omega = 0$

$$\overline{F} = \frac{1}{T} \int_0^T dt \; F$$

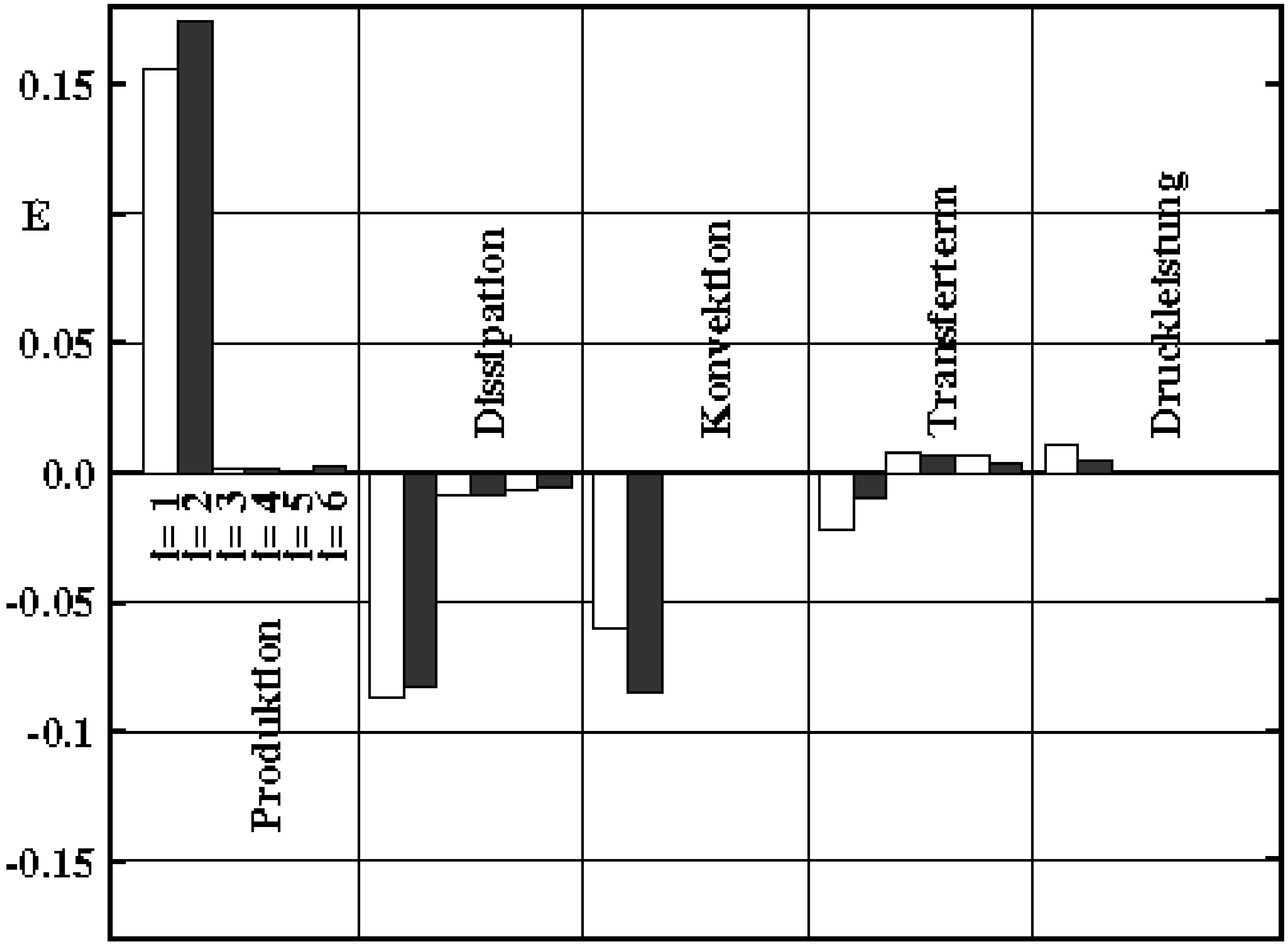
$$(\mathbf{u}, \mathbf{v})_\Omega := \int_\Omega dV \; \mathbf{u} \cdot \mathbf{v}$$

Im some detail:

NSE	NSE II	GS	modal E	
$\partial_t \mathbf{u} =$	$\partial_t \mathbf{u}' =$	$da_i/dt =$	$\frac{d}{dt}\overline{a_i^2}/2 =$	$d\, K_i /dt =$
$-\nabla \cdot \mathbf{u}\mathbf{u}$	$-\nabla \cdot \mathbf{u}_0\mathbf{u}_0$	$+q_{i00}$		
	$-\nabla \cdot \mathbf{u}'\mathbf{u}_0$	$+\sum_{j=1}^{N} q_{ij0} a_j$	$+2 q_{ii0}\, K_i$	$+\mathcal{P}_i$
	$-\nabla \cdot \mathbf{u}_0\mathbf{u}'$	$+\sum_{j=1}^{N} q_{i0j} a_j$	$+2 q_{i0i}\, K_i$	$+\mathcal{C}_i$
	$-\nabla \cdot \mathbf{u}'\mathbf{u}'$	$+\sum_{j,k=1}^{N} q_{ijk} a_j a_k$	$+\sum_{j,k=1}^{N} q_{ijk} \overline{a_i a_j a_k}$	$+\mathcal{T}_i$
$+\nu \triangle \mathbf{u}$	$+\nu \triangle \mathbf{u}_0$	$+\nu l_{i0}$		
	$+\nu \triangle \mathbf{u}'$	$+\nu \sum_{j=1}^{N} l_{ij} a_j$	$+2\nu l_{ii}\, K_i$	$+\mathcal{D}_i$
$-\nabla p$	$-\nabla p$	$+\sum_{j,k=1}^{N} q_{ijk}^{\pi} a_j a_k$	$+\sum_{j,k=1}^{N} q_{ijk}^{\pi} \overline{a_i a_j a_k}$	$+\mathcal{F}_i$

Modal energy flow analysis of cylinder wake

— Noack 2006 —



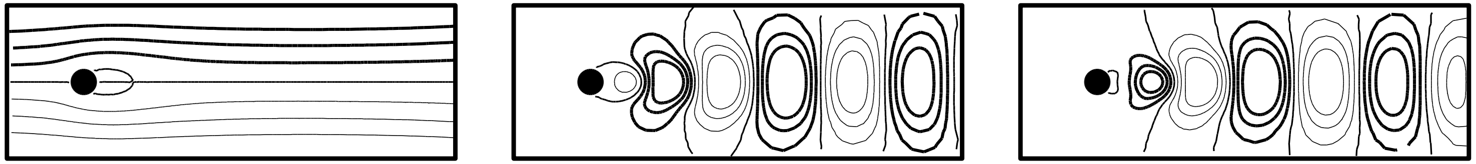
Semi-spectral characterization of e-flow cascade

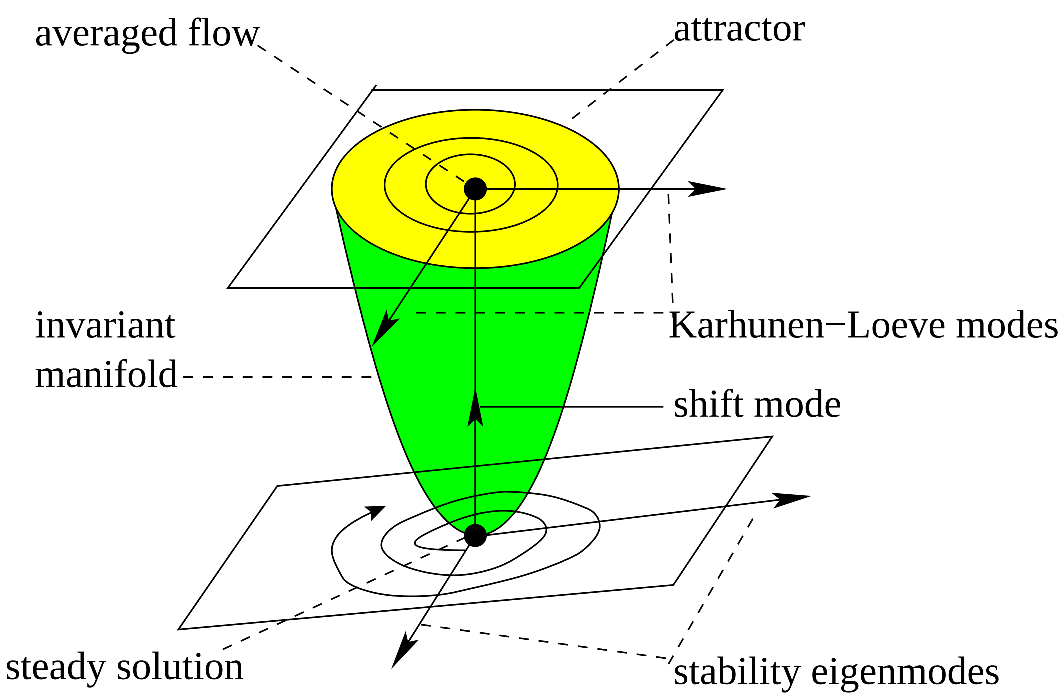
Transient dynamics of wake

Noack, Afanasiev, Morzyński, Tadmor & Thiele (2003) JFM —

Operating condition II

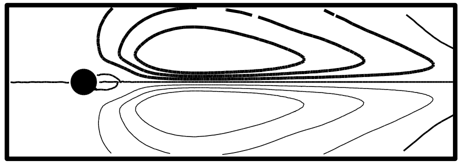
on attractor





↑

transient on paraboloid



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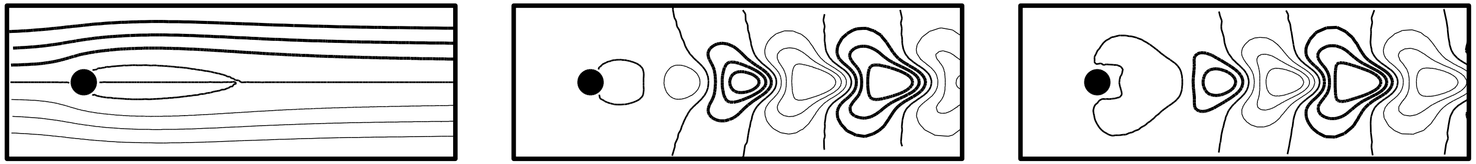
Operating condition I

near fixed point



Zielinska & Wesfreid (1995) PF

Continuous mode interpolation

— *Morzyński, Stankiewicz, Noack, King, Thiele & Tadmor (2006) AFC* —

mode 1 mode 2

$\mathbf{u}_{1,2}^{\kappa=1}$

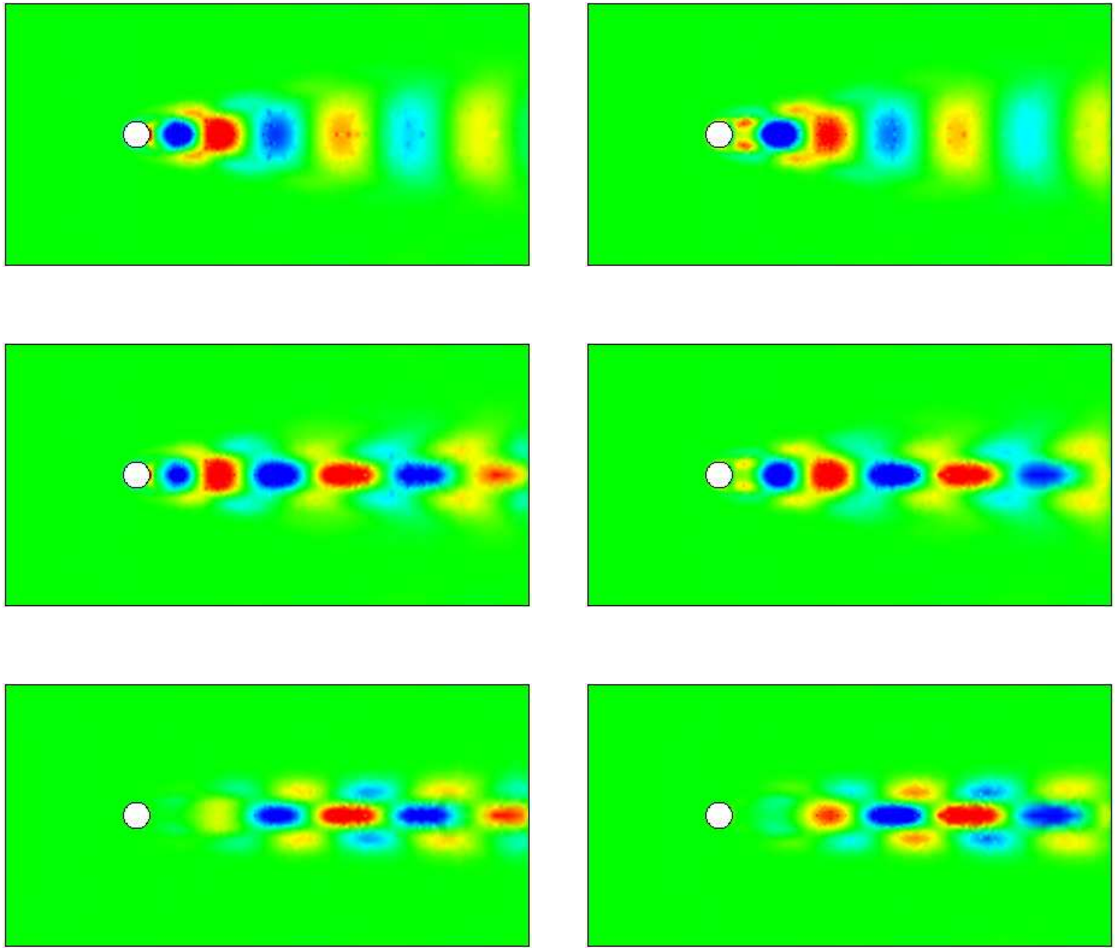
first POD modes

$\mathbf{u}_{1,2}^{\kappa=0.5}$

interpolated modes

$\mathbf{u}_{1,2}^{\kappa=0}$

stability eigenmode



- **Mode interpolation resolves intermediate states.**

Generalized mean-field model

— Morzyński, Stankiewicz, Noack, Thiele & Tadmor (2006) AIAA —

3-dim. Galerkin approx.:

$\mathbf{u} = \mathbf{u}_B + \mathbf{u}'$

$\mathbf{u}_B = \mathbf{u}_s + a_\Delta \mathbf{u}_\Delta$

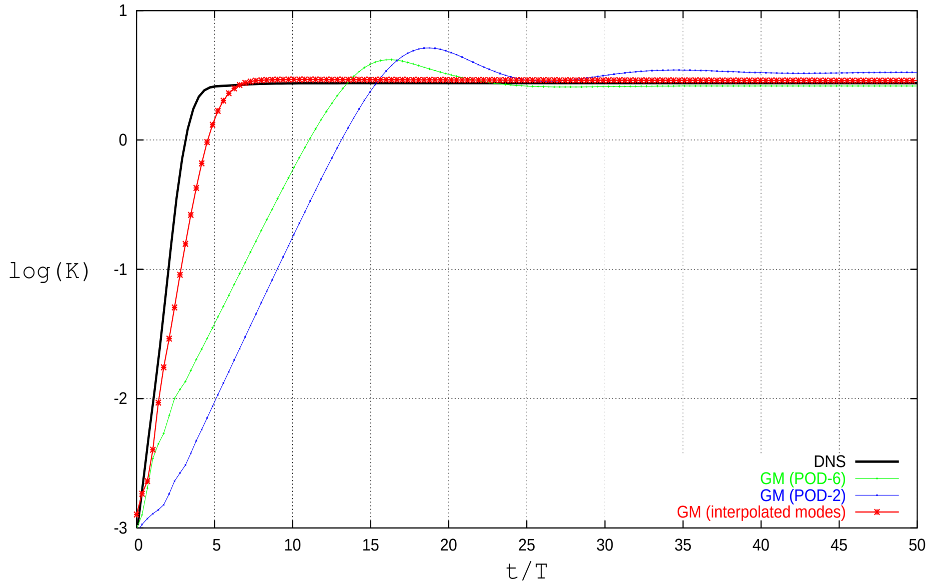
$\mathbf{u}' = a_1^\kappa \mathbf{u}_1^\kappa + a_2^\kappa \mathbf{u}_2^\kappa$

Galerkin system

$da_1^\kappa/dt = \sigma a_1^\kappa - \omega a_2^\kappa$

$da_2^\kappa/dt = \sigma a_2^\kappa + \omega a_1^\kappa$

Fluctuation energy for transient



$$da_\Delta/dt = \sigma_D a_\Delta + c\left((a_1^\kappa)^2 + (a_2^\kappa)^2\right)$$

$$\sigma = \sigma_1 - \beta a_\Delta,\ \omega = \omega_1 + \gamma a_\Delta,\ \kappa = a_\Delta / a_\Delta^\infty$$

- **Generalized 3-dim. model $\sim$ 10% error.**

Wake stabilization as benchmark problem

— ≡ *Lehmann, Luchtenburg, Noack, King, Morzyński & Tadmor (2005) CDC-ECC* —

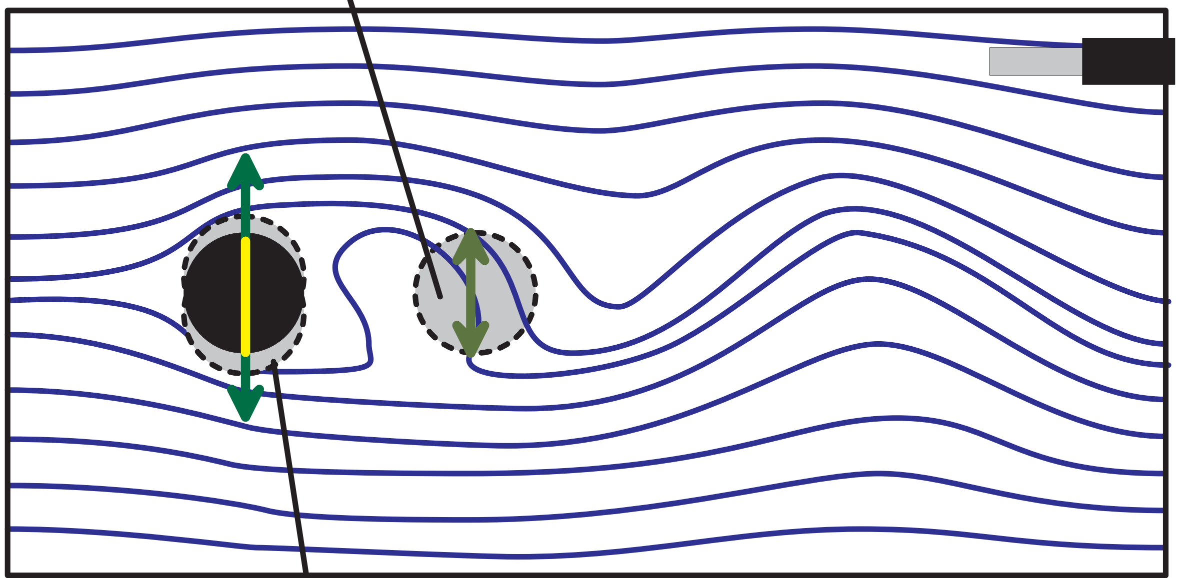
Control problem: stabilize wake at $Re = 100$

≡ *Protas & Szewzyk 2002 PF*

≡ *Protas & Wesfreid 2003 Mecanique*

≡ *Siegel et al. 2003 AIAA*

≡ *Bergmann, Cordier & Brancher (2005) PF*



- **Actuation:** (A) volume force, (B) cylinder oscillation
- **Sensing with hot-wire:** $S(t) = u(6.5D, 2D, t)$

Strategy of GM-based SISO control

— ≡ *Lehmann, Luchtenburg, Noack, King, Morzyński & Tadmor (2005) CDC-ECC* —

(1) Galerkin model

$$\mathbf{u} = \sum a_i \mathbf{u}_i$$

$\frac{d\mathbf{a}}{dt} = \mathbf{f}(\mathbf{a}, b)$, b: control

(2) Energy-based control

Let $K = \frac{1}{2} \sum a_i^2$.

Determine $b = b(\mathbf{a})$

so that $dK/dt = \sigma K, \quad \sigma < 0$

(3) Dynamic observer

Plant: $\mathbf{u} \Rightarrow S(t)$

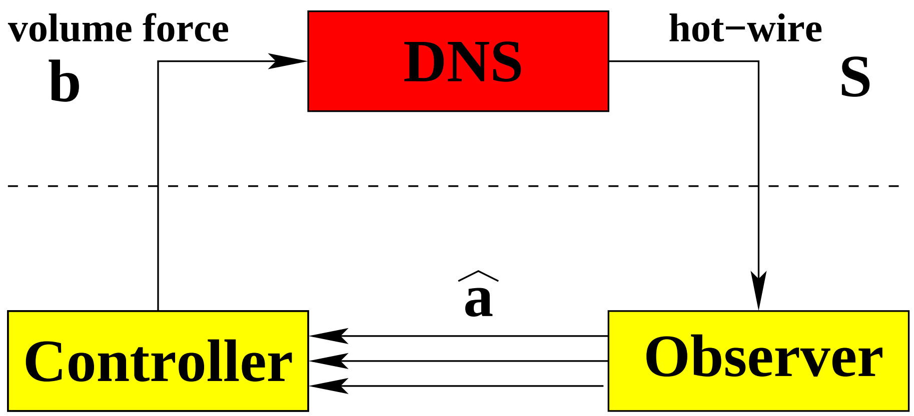
Observer: $\hat{\mathbf{a}} \Rightarrow \hat{\mathbf{u}} \Rightarrow \widehat{S}$

$$\frac{d\hat{\mathbf{a}}}{dt} = \mathbf{f}(\hat{\mathbf{a}}, b) + L\,(\widehat{S} - S)$$

(4) DNS with GM-based SISO control

Control law

$$b(S) = b(\hat{\mathbf{a}}(S))$$

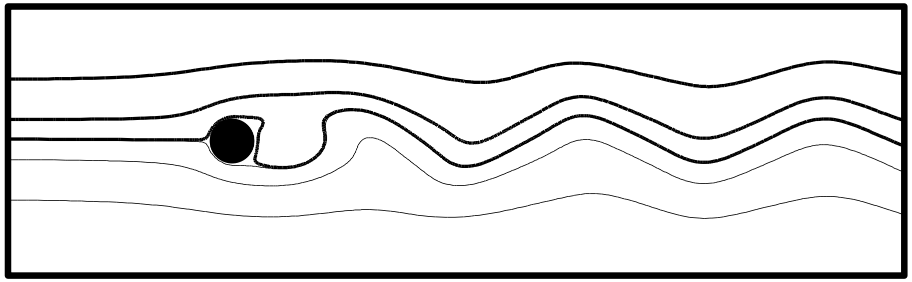


SISO wake stabilization in simulation

— ≡ *Lehmann, Luchtenburg, Noack, King, Morzyński & Tadmor (2005) CDC-ECC* —

Natural flow

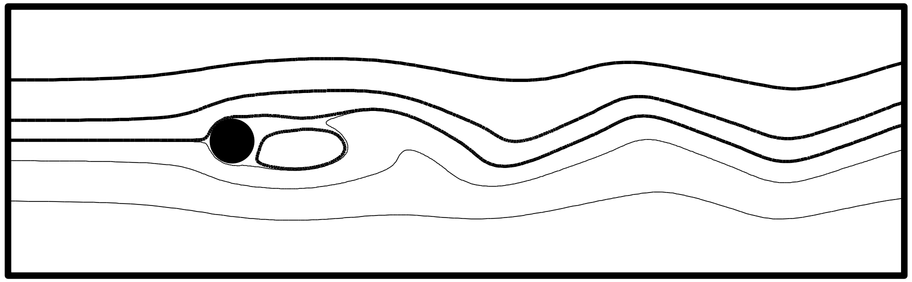
100% TKE ....



GM-based control

with standard POD model

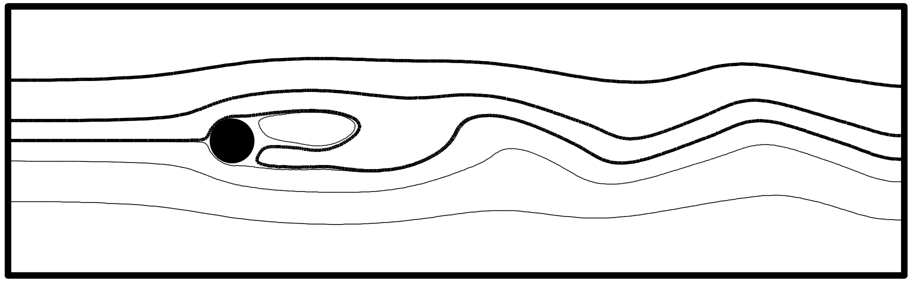
$(N = 3)$ 80% TKE .....



GM-based control II

with hybrid model

$(N = 3)$ 25% TKE .....



- **Better models improve control!**

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   — *low-order modeling of weakly nonlinear dynamics*

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   — *an emerging unifying theory*

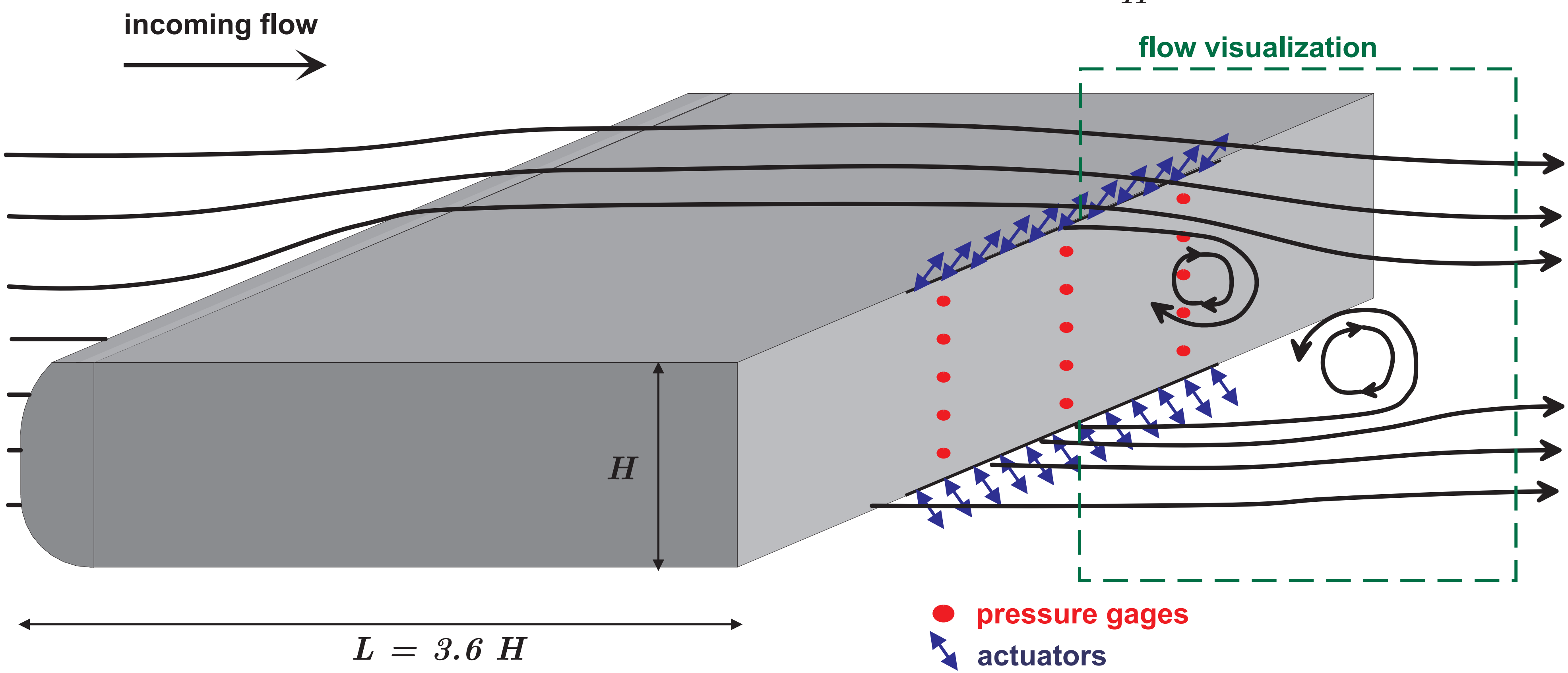
5. **Concluding remarks and outlook**

Phase control applied to the D-shaped body

— Pastoor, Henning, Noack, King & Tadmor (2008) JFM (accepted) —

Experimental setup

$Re_H = 20\,000 \ldots 60\,000$

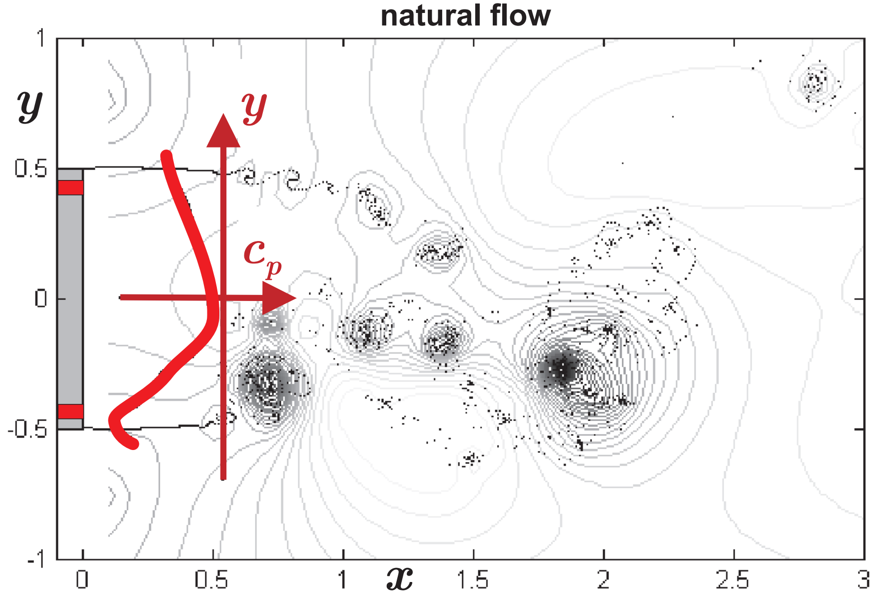


Phase control applied to the D-shaped body

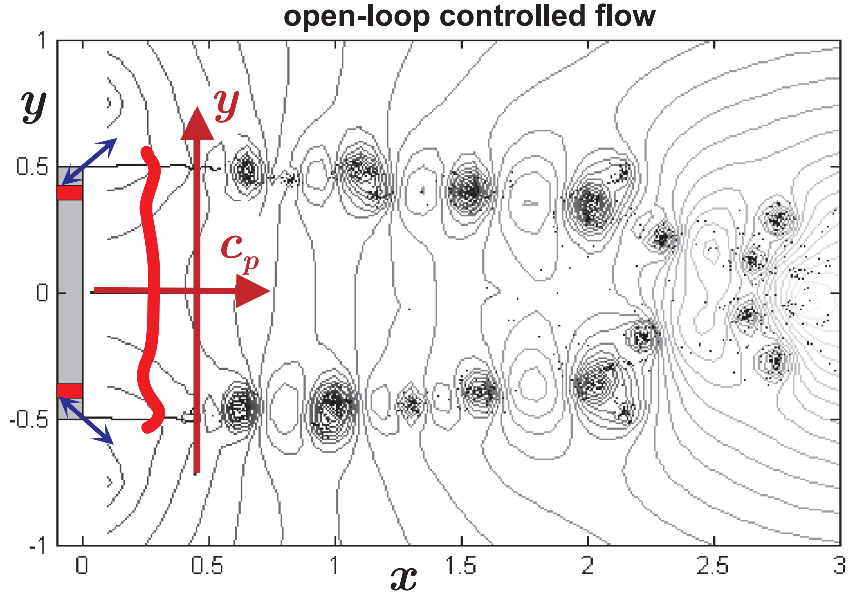
— Pastoor, Henning, Noack, King & Tadmor (2008) JFM (accepted) —

Vortex model with $\sim$ 1000 vortices.

Natural flow



Open-loop forcing

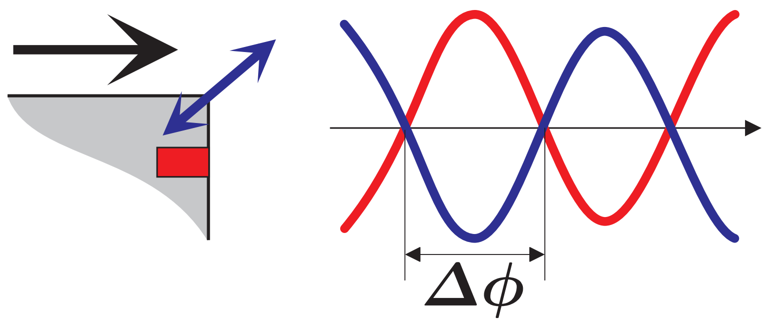


Closed-loop forcing

Phase angle

$$\Delta\Phi_{c_\mu,c_p} = \angle(c_\mu, c_p)$$

optimal: $\Delta\Phi_{c_\mu,c_p} = \pi$



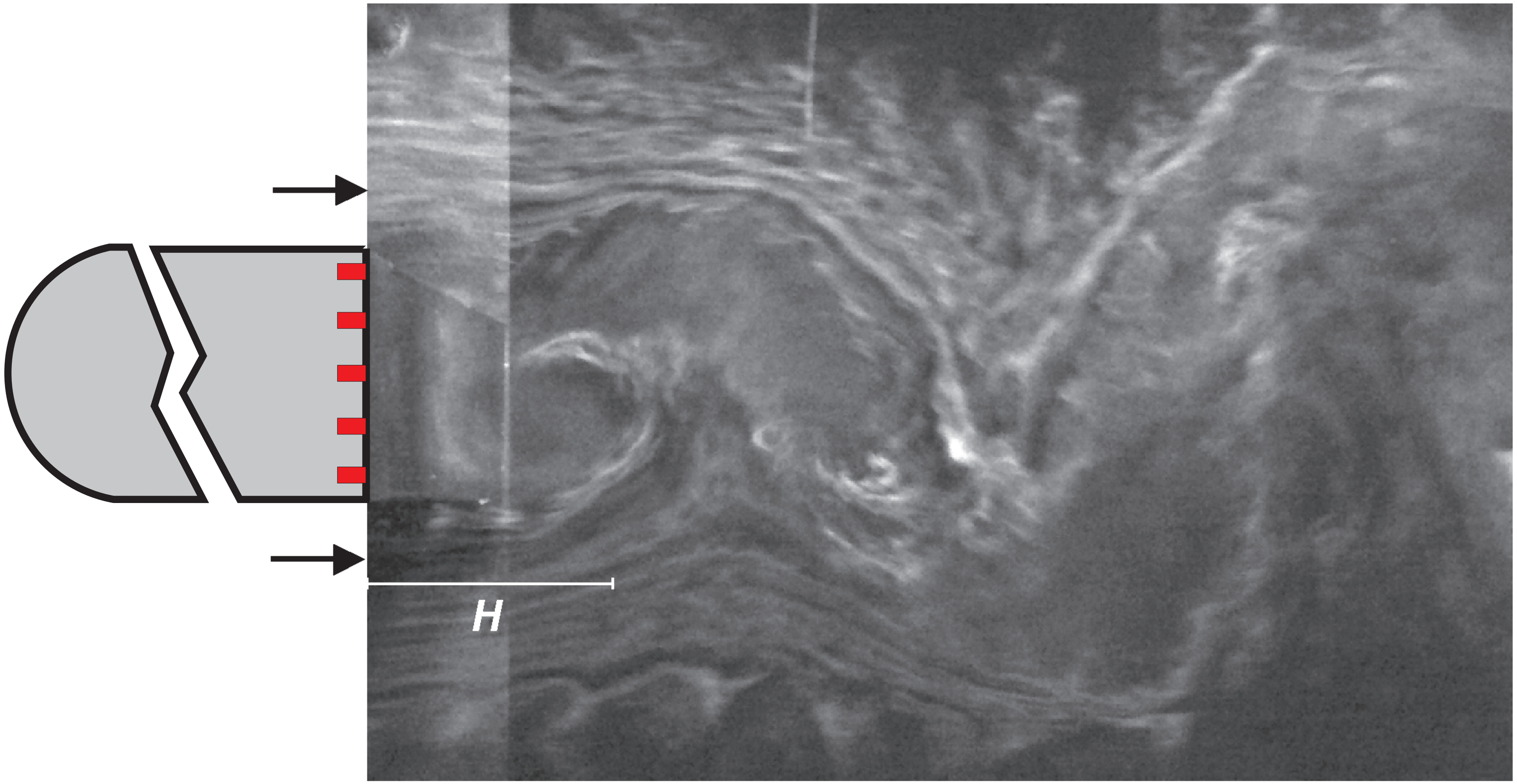
Phase control applied to the D-shaped body

— Pastoor, Henning, Noack, King & Tadmor (2008) JFM (accepted) —

Natural flow

smoke visualization

$Re_H = 40\,000$



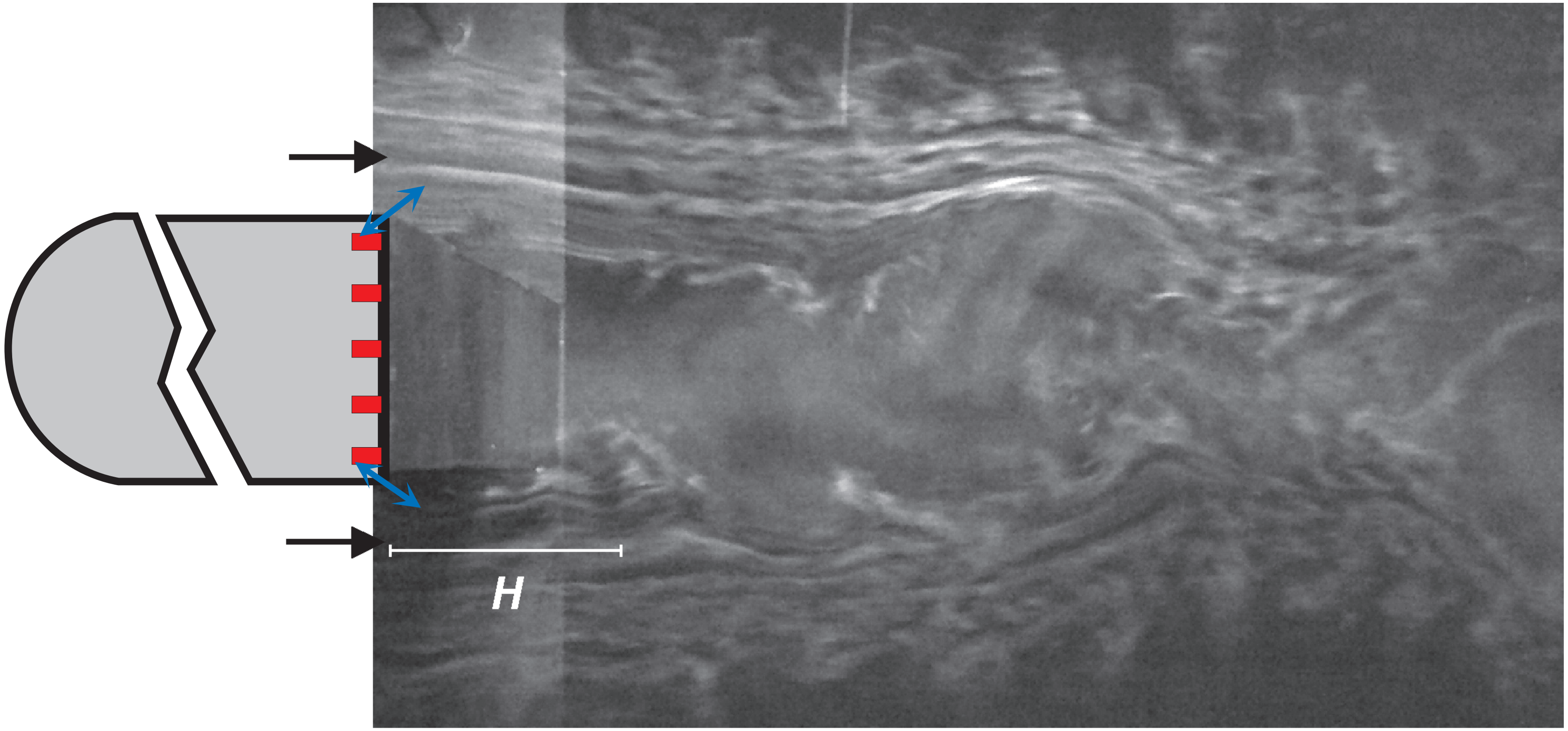
$St_{wake} = 0.20 \quad c_{D,0} = 1.2 \quad \bar{c}_{P,0} = -0.5$

Phase control applied to the D-shaped body

— ≡ *Pastoor, Henning, Noack, King & Tadmor (2008) JFM (accepted)* —

Open-loop controlled flow . . . . . *suggested by low-order model*

$Re_H = 40\,000$



$c_\mu = 0.015$ $St_A = 0.126$ $c_D/c_{D,0} = 0.85$ $\overline{c_D}/|\bar{c}_{P,0}| = -0.6$

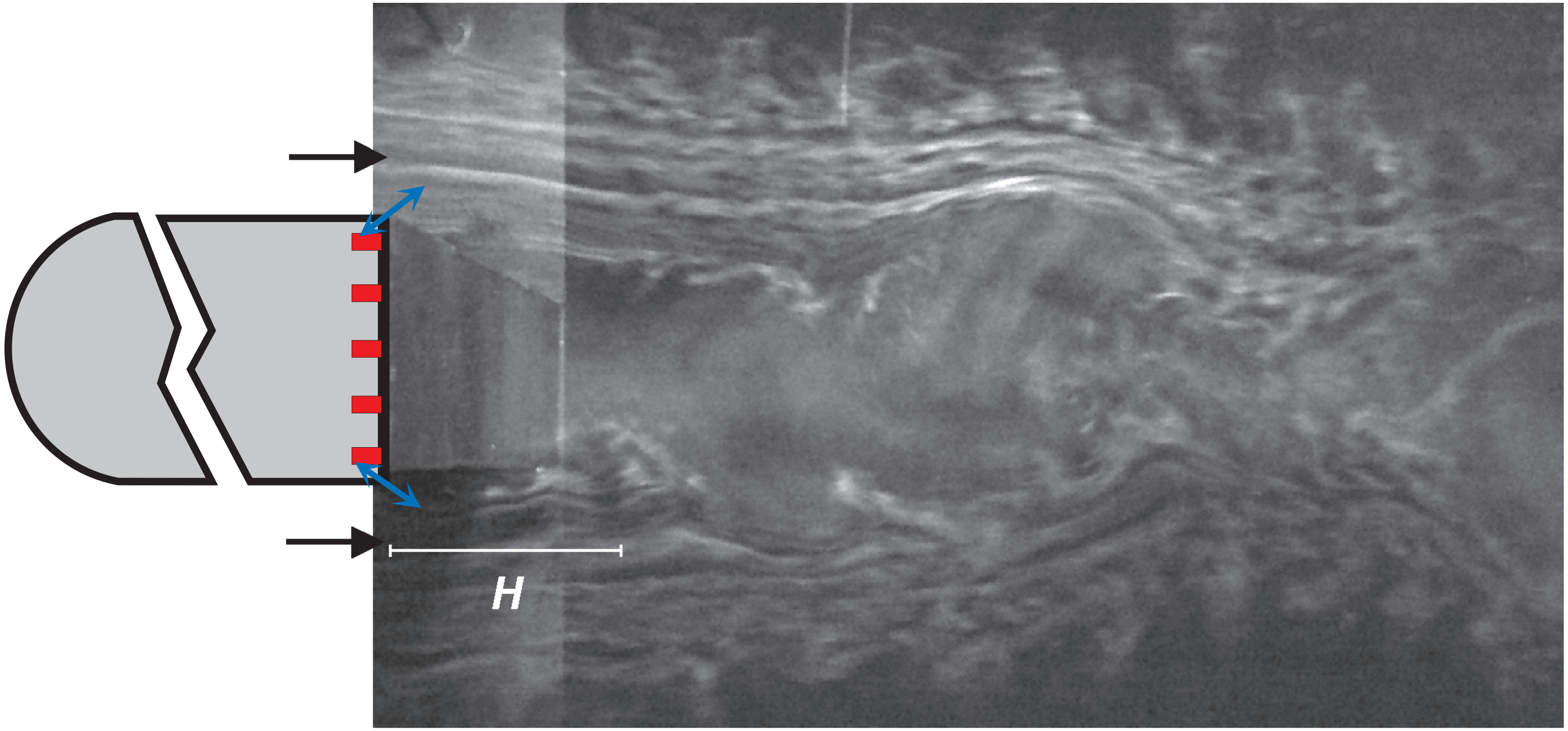
- **40% increase in base pressure.**
- **20% decrease in drag.**

Phase control applied to the D-shaped body

— Pastoor, Henning, Noack, King & Tadmor (2008) JFM (accepted) —

Closed-loop controlled flow . . . . *derived from low-order model*

$Re_H = 40\,000$



$c_\mu = 0.015 \quad St_A = 0.17 \quad c_D/c_{D,0} = 0.85 \quad \overline{c_D}/|\overline{c}_{P,0}| = -0.6$

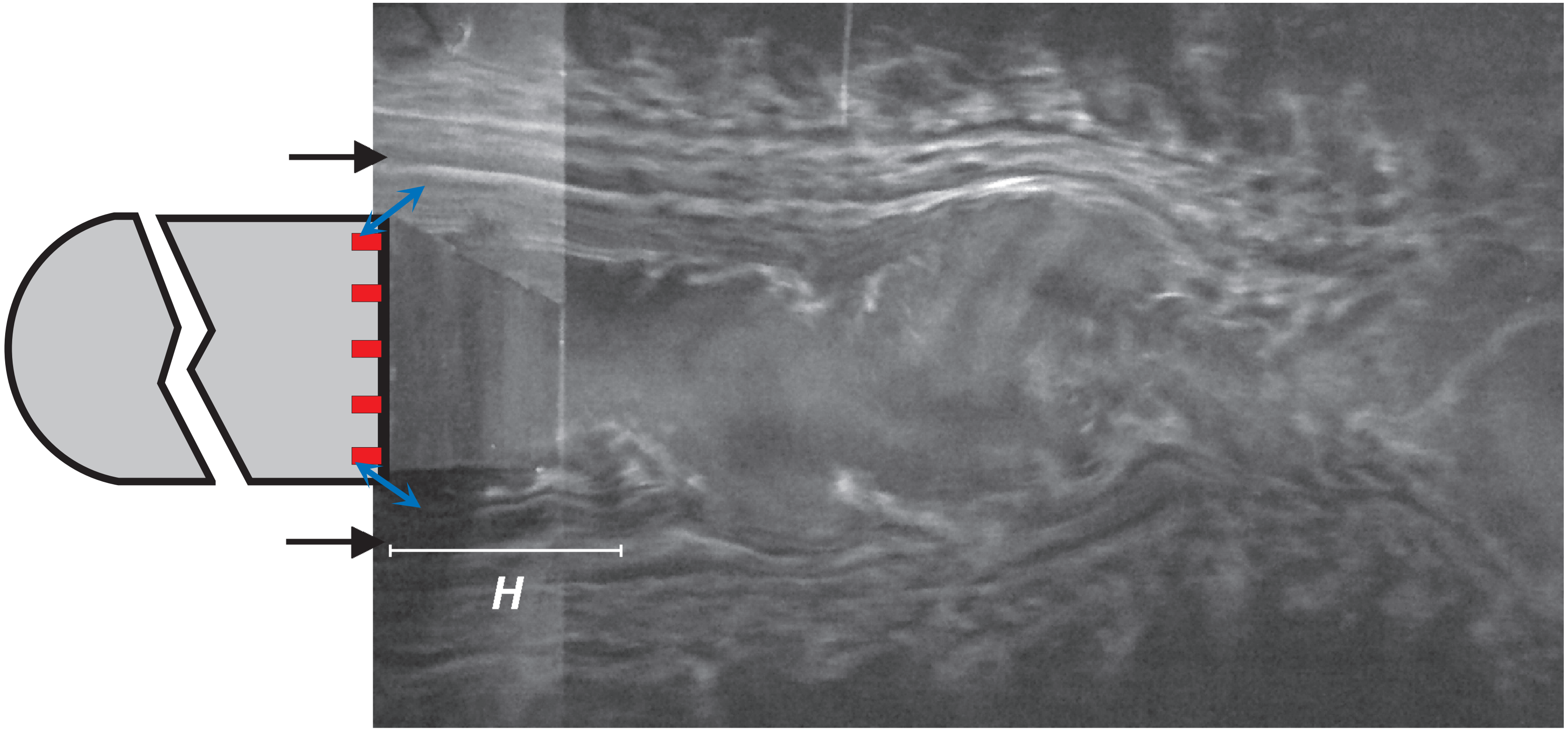
- Same drag reduction.
- **But with 40% less actuation energy.**

Phase control applied to the D-shaped body

— ≡ *Pastoor, Henning, Noack, King & Tadmor (2008) JFM (accepted)* —

Closed-loop controlled flow . . . . derived from low-order model

$Re_H = 40\,000$

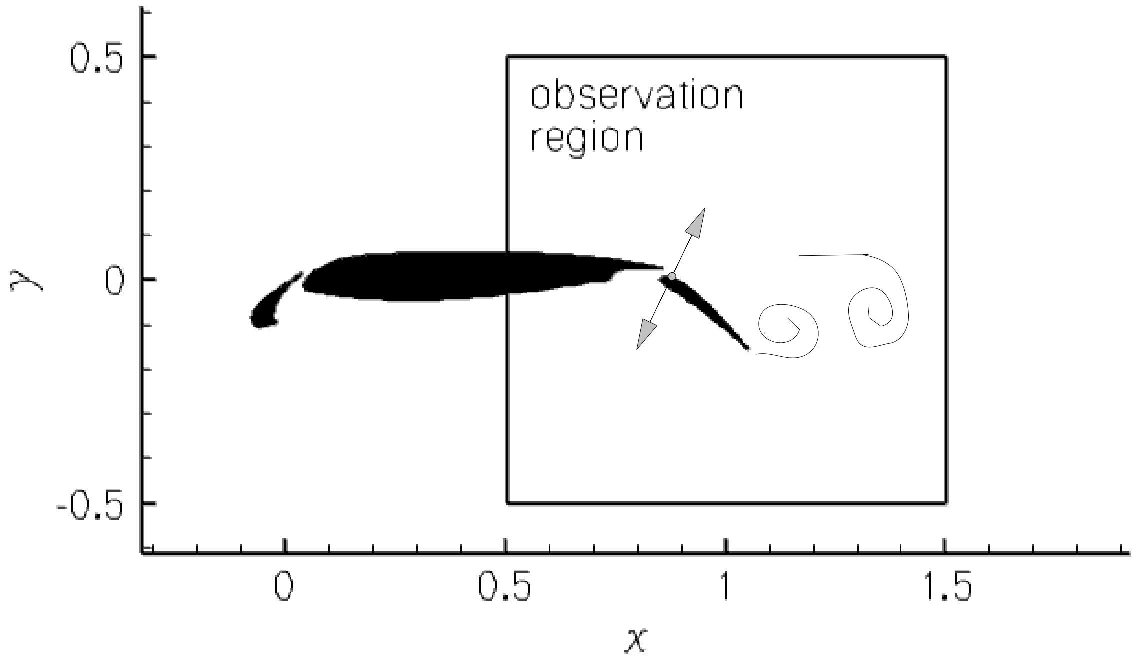


$c_\mu = 0.015$ $St_A = 0.126$ $c_D/c_{D,0} = 0.85$ $\overline{c_D}/|\bar{c}_{P,0}| = -0.6$

- Same drag reduction with 40% less actuation energy.
- **But only one (!) actuator.**

Sketch of the high-lift configuration

— *Luchtenburg, Günther, Noack, King & Tadmor (2007) JFM preprint* —



Configuration: $Re = U_\infty\, c/\nu = 10^6$, angle of attack 6°

Actuation: acoustic actuator at the upper side of the trailing flap

Simulation: 2D URANS, LLR k-ω model, structured grid, ∼90000 cells

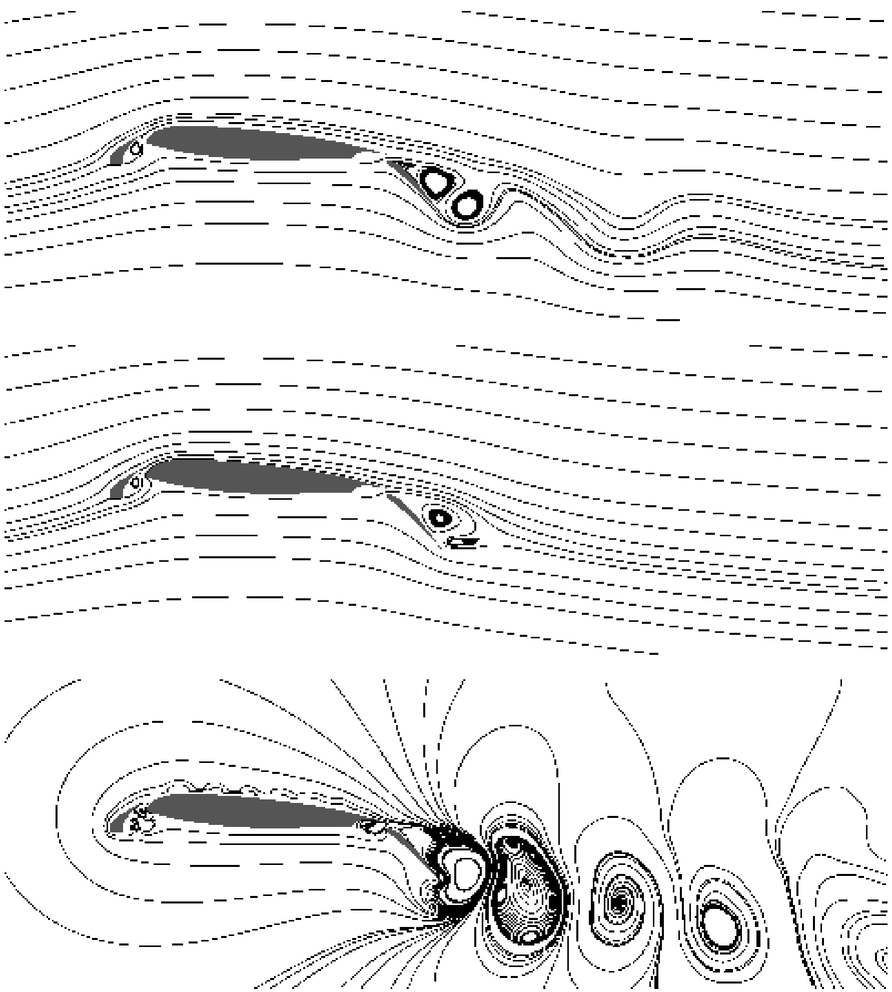
Modeling: Least-order Galerkin model for observation region explaining the control mechanism

URANS simulation of high-lift configuration

— Luchtenburg, Günther, Noack, King & Tadmor (2007) JFM preprint —

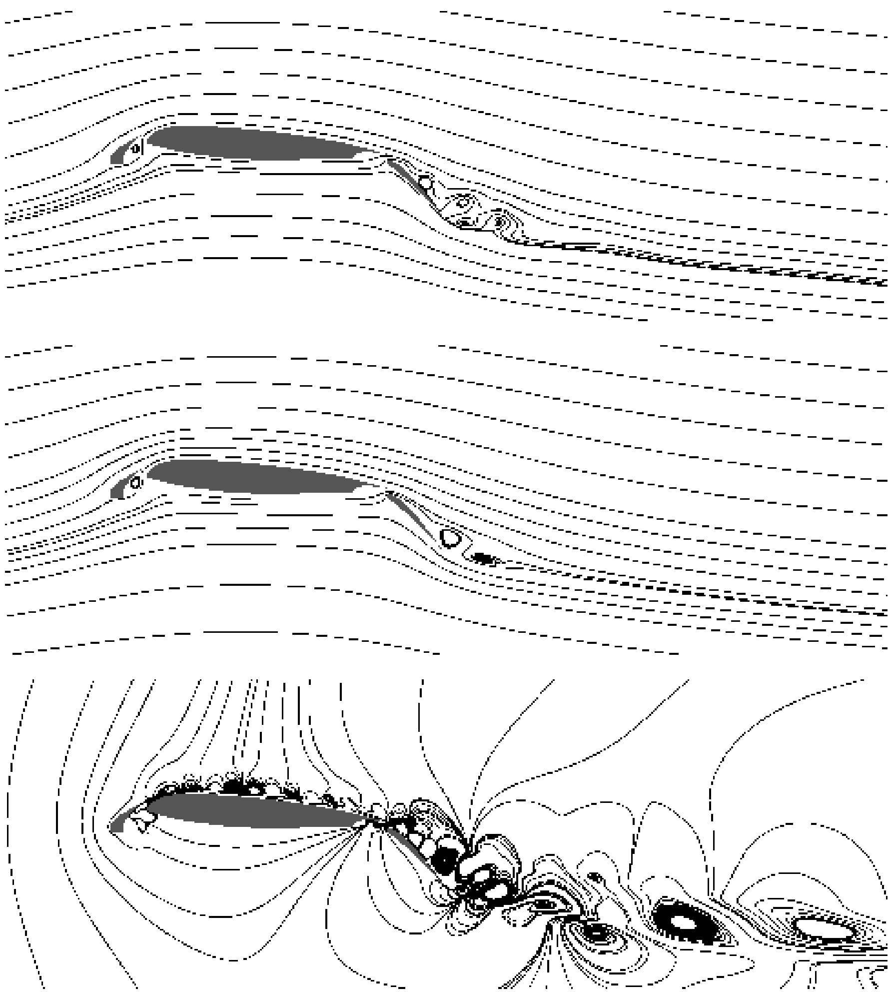
natural flow

$$St^n_{\mathsf{fl}} = f^n c_{\mathsf{fl}}/U_\infty = 0.32$$



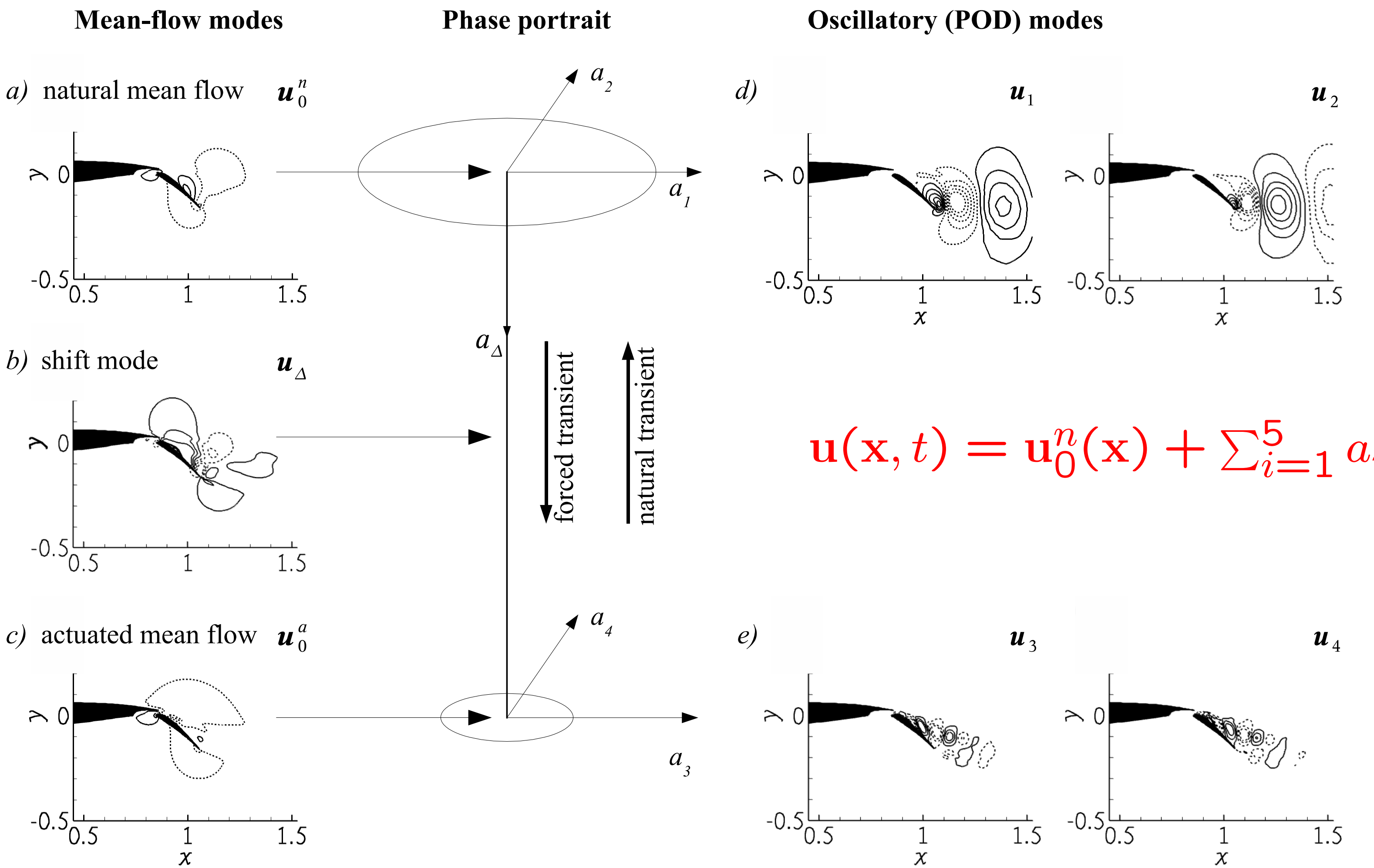
actuated flow

$$St^a_{\mathsf{fl}} = f^a c_{\mathsf{fl}}/U_\infty = 0.6$$



Generalized mean field model

— Luchtenburg, Günther, Noack, King & Tadmor (2007) JFM preprint —



$$\mathbf{u}(\mathbf{x},t) = \mathbf{u}_0^n(\mathbf{x}) + \sum_{i=1}^{5} a_i(t)\mathbf{u}_i(\mathbf{x})$$

$$\frac{da_i}{dt} = c_i + \sum_{j=1}^{5} c_{ij}a_j + \sum_{j,k=1}^{5} c_{ijk}a_ja_k + g_i^1 b + g_1^2 \frac{db}{dt}$$

Generalized mean field model

— *Luchtenburg, Günther, Noack, King & Tadmor (2007) JFM preprint* —

Dynamical system structure:

$$\frac{d}{dt}\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \begin{bmatrix} \tilde{\sigma}^n & -\tilde{\omega}^n & 0 & 0 \\ \tilde{\omega}^n & \tilde{\sigma}^n & 0 & 0 \\ 0 & 0 & \tilde{\sigma}^a & -\tilde{\omega}^a \\ 0 & 0 & \tilde{\omega}^a & \tilde{\sigma}^a \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \kappa & -\lambda \\ 0 & 0 & \lambda & \kappa \end{bmatrix} \mathbf{b}.$$

with state-dependent coefficients

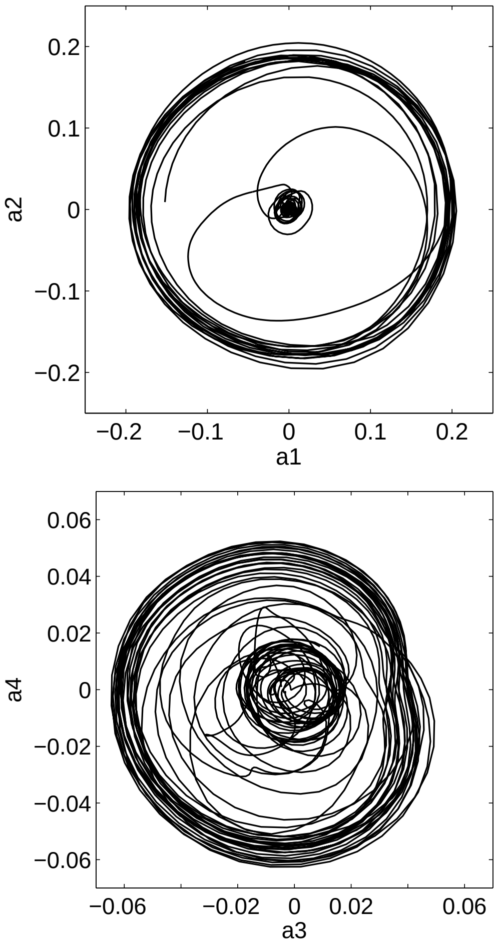
$$\begin{aligned} \tilde{\sigma}^n &= \sigma^n - \sigma^{n,n}\,(A^n)^2 - \sigma^{n,a}\,(A^a)^2, \\ \tilde{\omega}^n &= \omega^n + \omega^{n,n}\,(A^n)^2 + \omega^{n,a}\,(A^a)^2, \\ \tilde{\sigma}^a &= \sigma^a - \sigma^{a,n}\,(A^n)^2 - \sigma^{a,a}\,(A^a)^2, \\ \tilde{\omega}^a &= \omega^a + \omega^{a,n}\,(A^n)^2 + \omega^{a,a}\,(A^a)^2, \\ a_5 &= c + c^n\,(A^n)^2 + c^a\,(A^a)^2, \end{aligned}$$

with $A^n = \sqrt{a_1^2 + a_2^2}$, $A^a = \sqrt{a_3^2 + a_4^2}$ and $\mathbf{b} = (b, \dot{b}/\tilde{\omega}^a)$

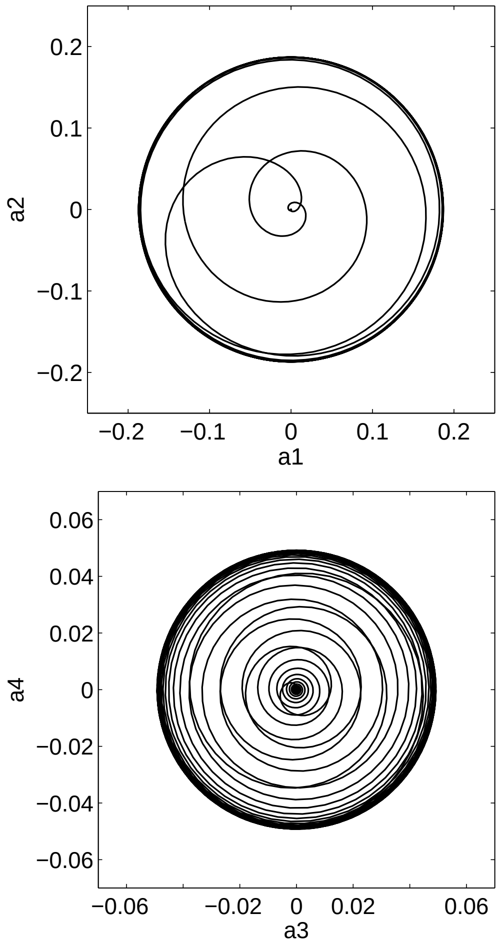
Phase portraits of transient flow: URANS vs. generalized mean-field model

— Luchtenburg, Günther, Noack, King & Tadmor (2007) JFM preprint —

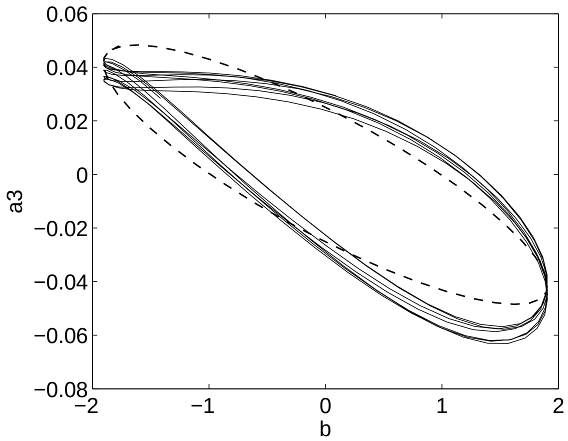
URANS



model

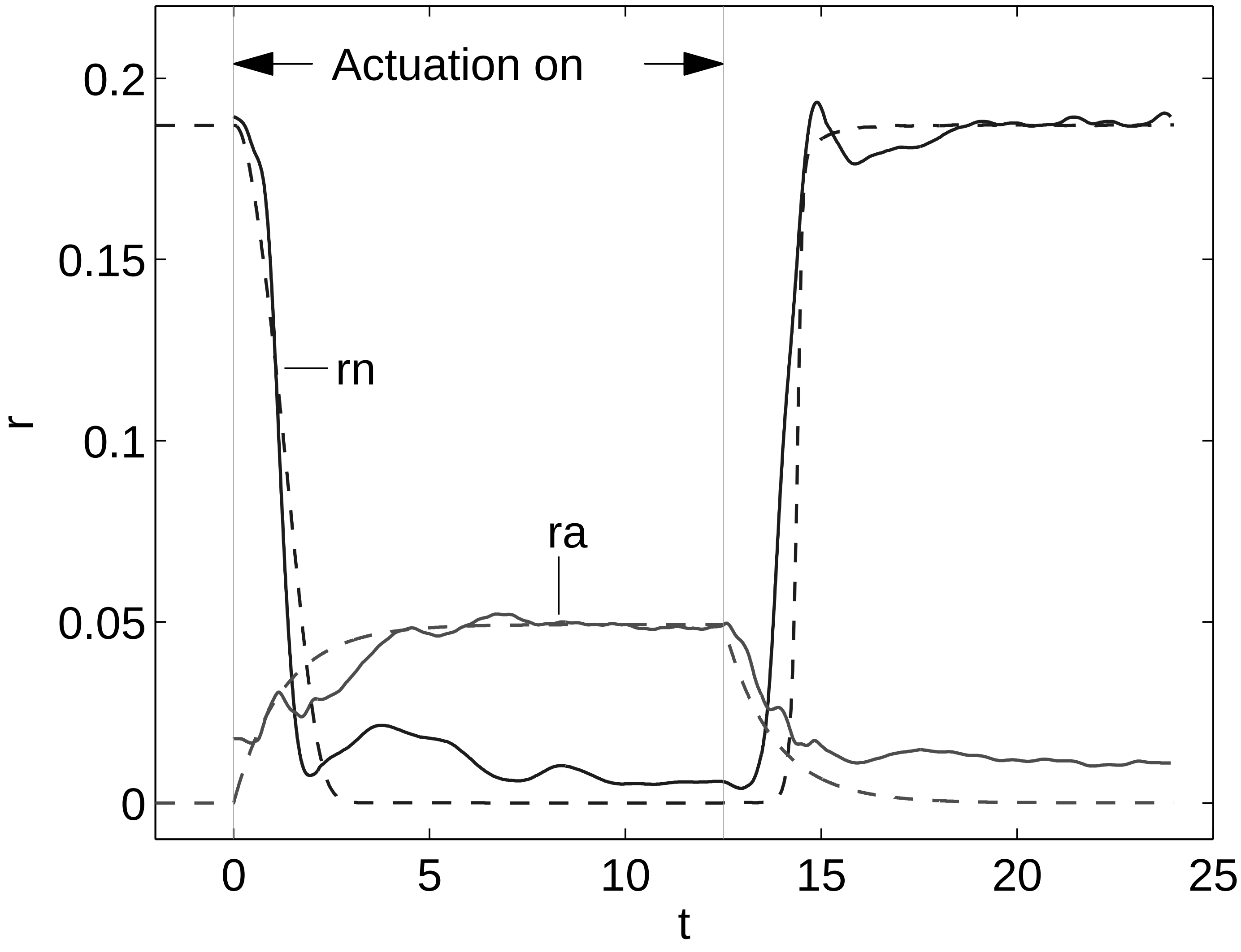


lock-in with actuation



Amplitudes of transient flow: URANS vs. generalized mean-field model

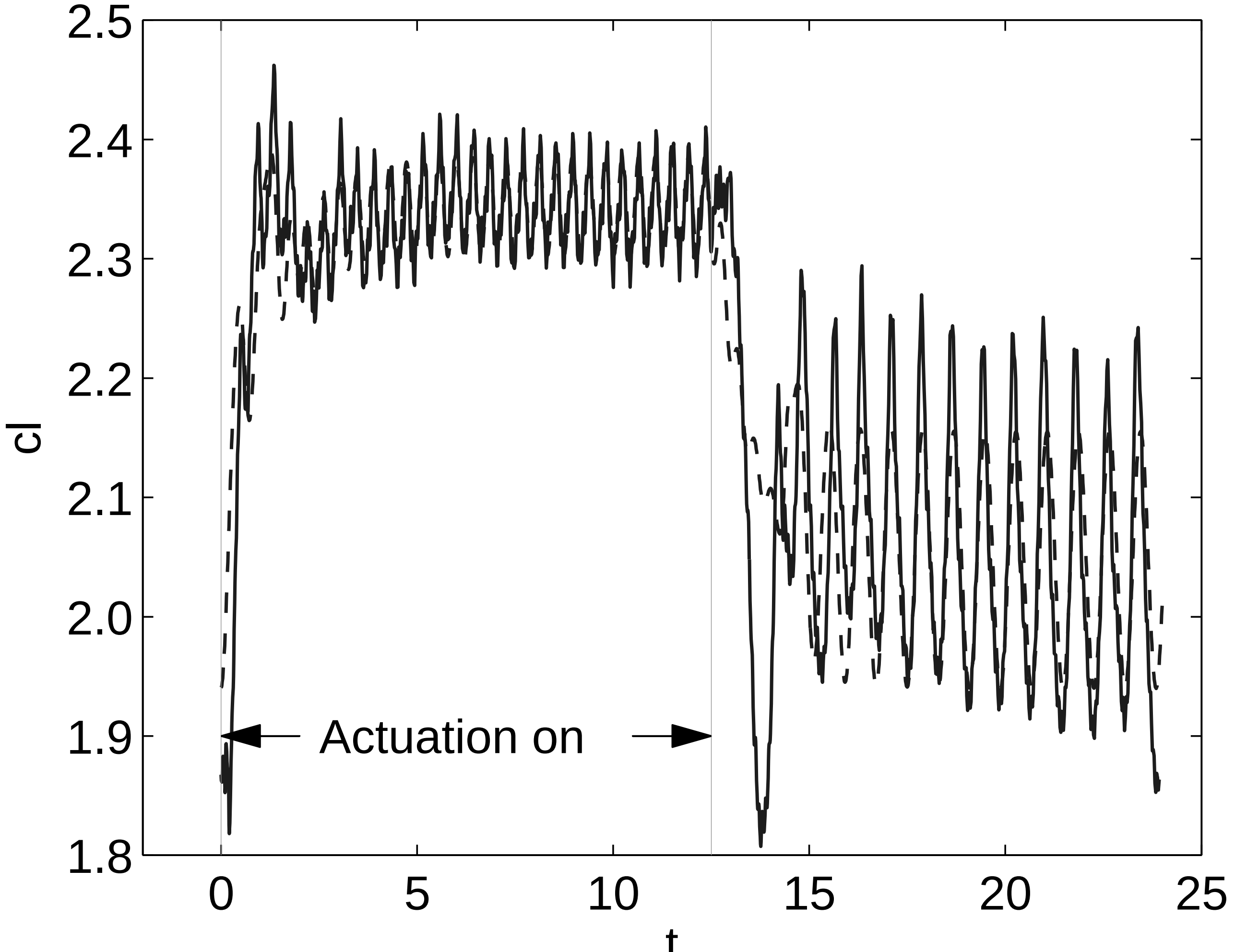
— Luchtenburg, Günther, Noack, King & Tadmor (2007) JFM preprint —



Lift coefficient of transient flow: URANS vs. generalized mean-field model

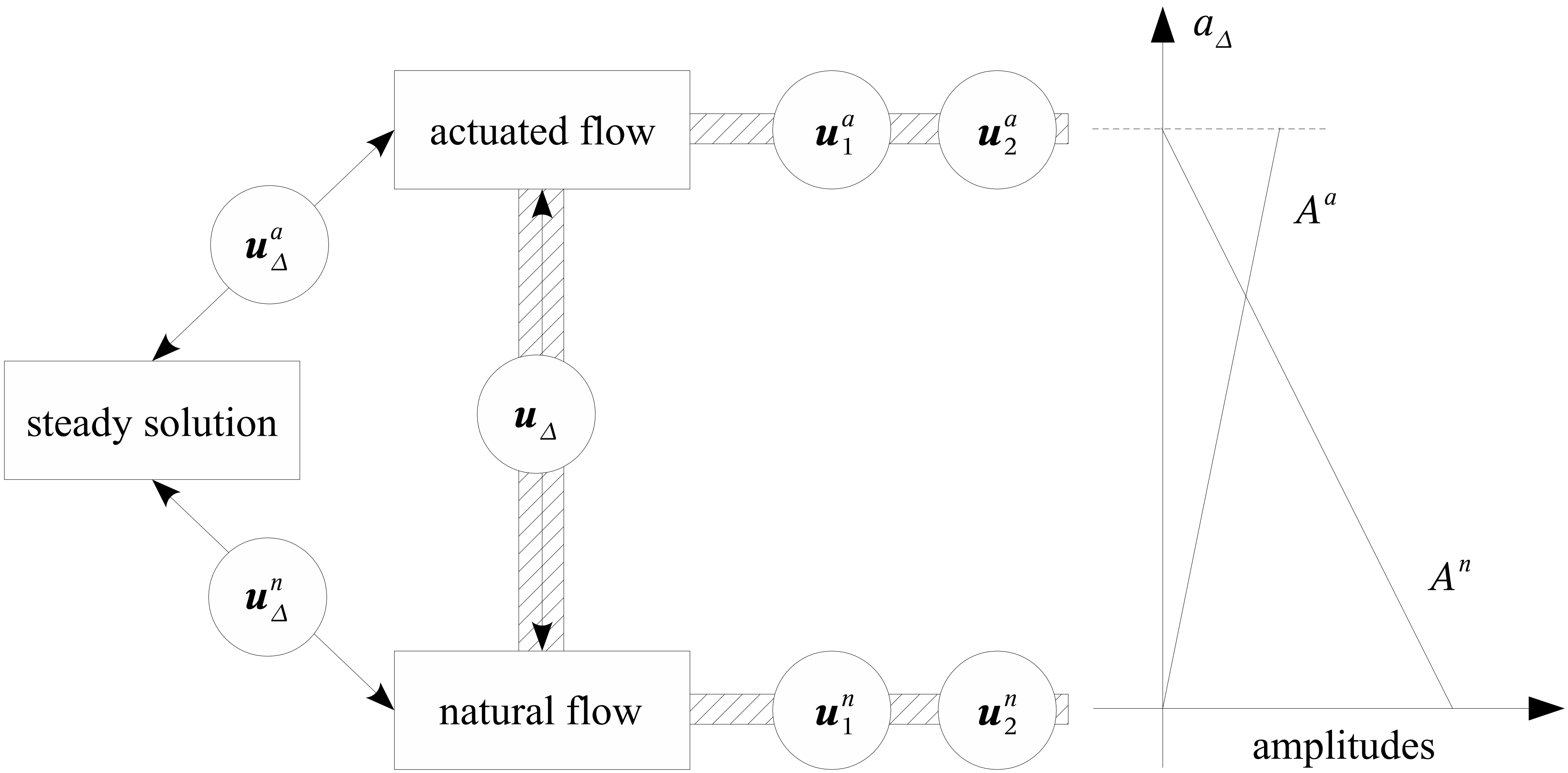
— *Luchtenburg, Günther, Noack, King & Tadmor (2007) JFM preprint* —

$$c_L(t) = c_{L0} + \sum_{i=1}^{4} k_i a_i(t) + k_5 (A^n)^2 + k_6 (A^a)^2 + k_7 (A^n)^4 + k_8 (A^a)^4$$



Energetic interpration as competing modes

— Luchtenburg, Günther, Noack, King & Tadmor (2007) JFM preprint —



Overview

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1. Introduction

— *low-order Galerkin modeling*

2. Control of laminar shear flow

— *low-order modeling of weakly nonlinear dynamics*

3. Control of turbulent shear flow

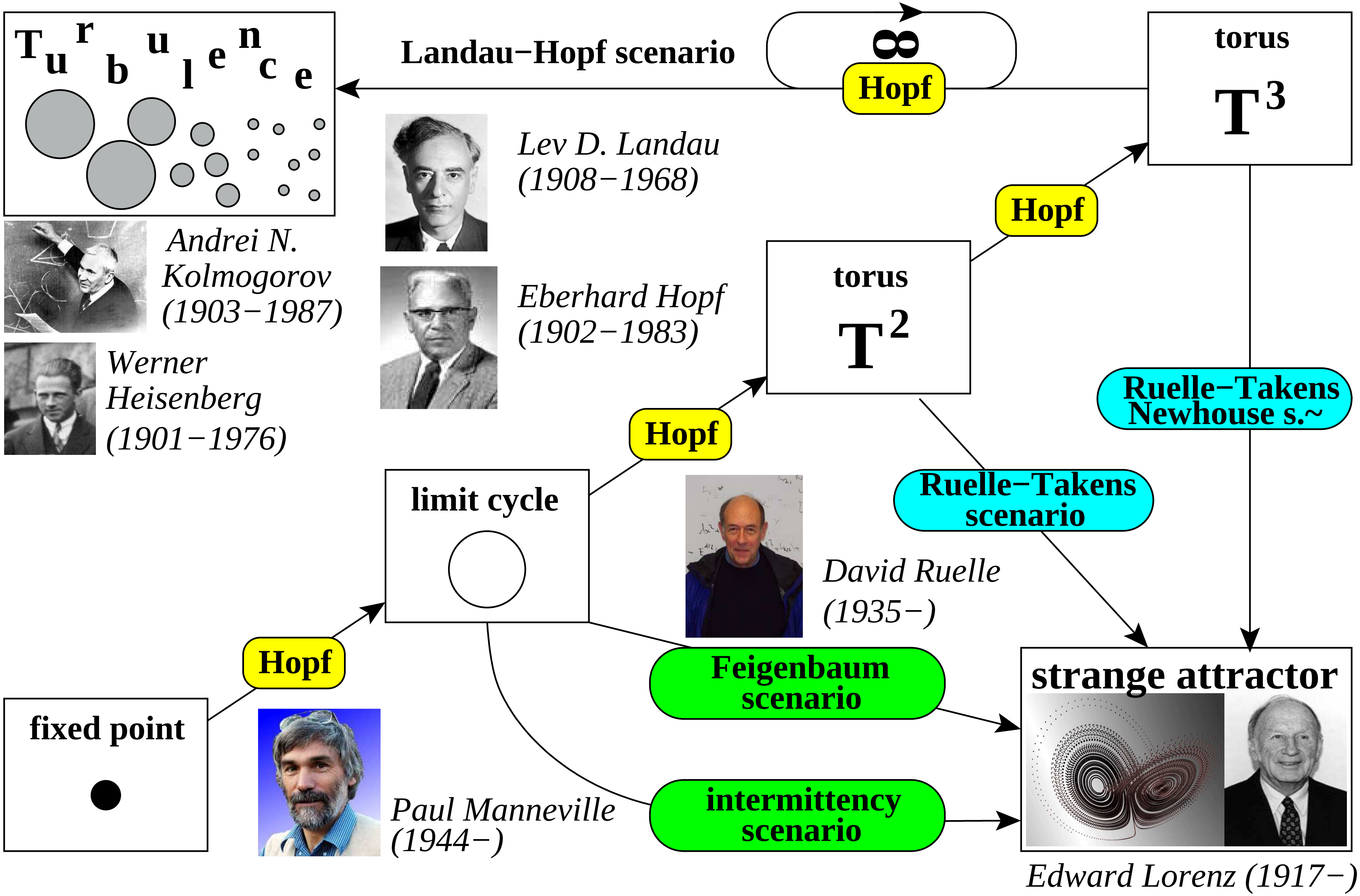
— *low-order modeling of strongly nonlinear dynamics*

4. Instabilities, turbulence and control

— *an emerging unifying theory*

5. Concluding remarks and outlook

Instabilities $\mapsto$ turbulence



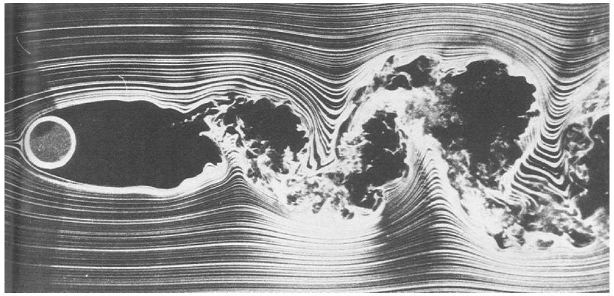
Statistical physics $\mapsto$ turbulence

Ludwig Boltzmann
(1840–1906)
Equivalent subsystems:
1877 Entropy

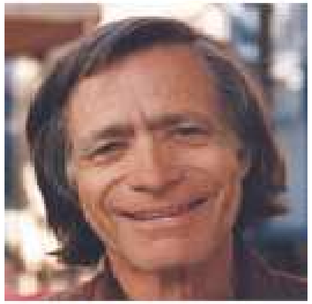
$S = k \, ln W$

Lars Onsager
(1903–1976)
Particle/vortex picture:
1949 point vortices in 2D flows
= thermodyn. degree of freedom

Ludwig Liepmann's
WARNING:



Robert H Kraichnan
Wave/Galerkin picture:
1955 Fourier modes = thermodyn. degrees of freedom
(absolute equilibrium ensemble)

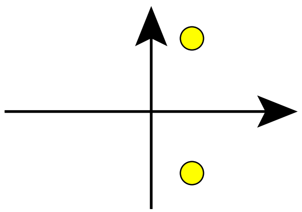


How to partition the flow in equivalent subsystems (atoms)
(= thermodynamic degrees of freedom)???

Control ↦ turbulence

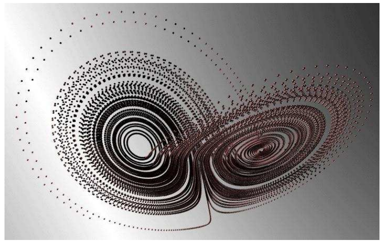
linear dynamics

da/dt = A a + B b



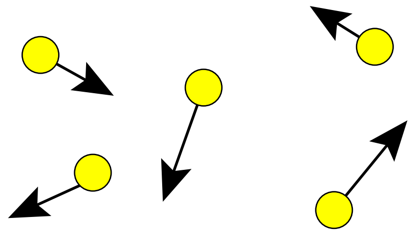
strange attractor

da/dt = f (a,b)



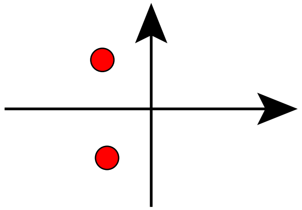
statistical physics

S = k ln W



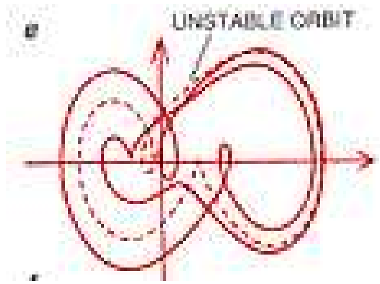
linear control

b =K a



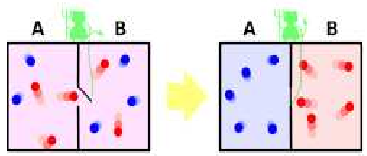
Anno Dazumal

chaos control



Ott, Grebogi, Yorke 1990 PRL

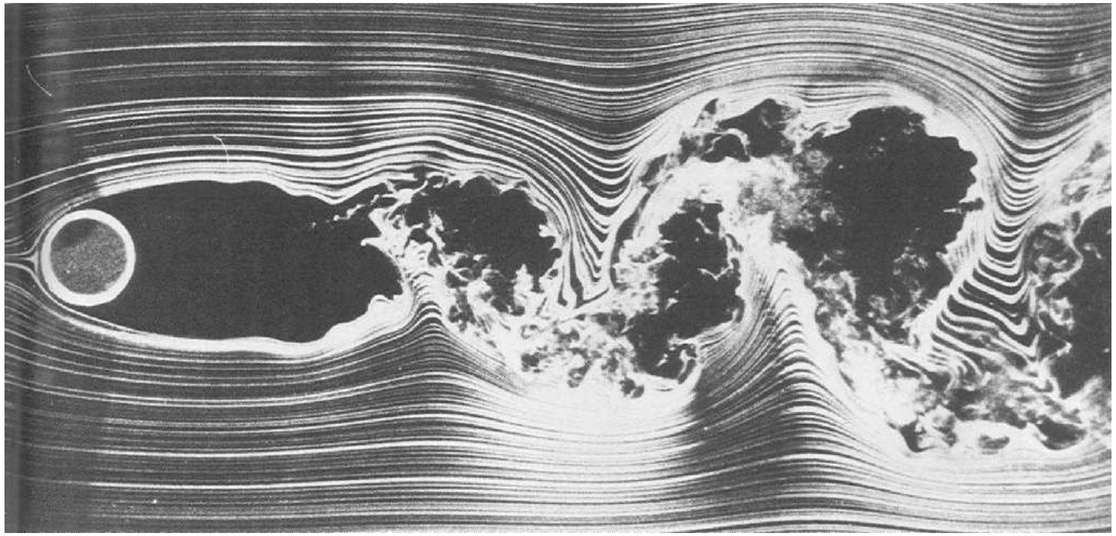
Maxwell's demon



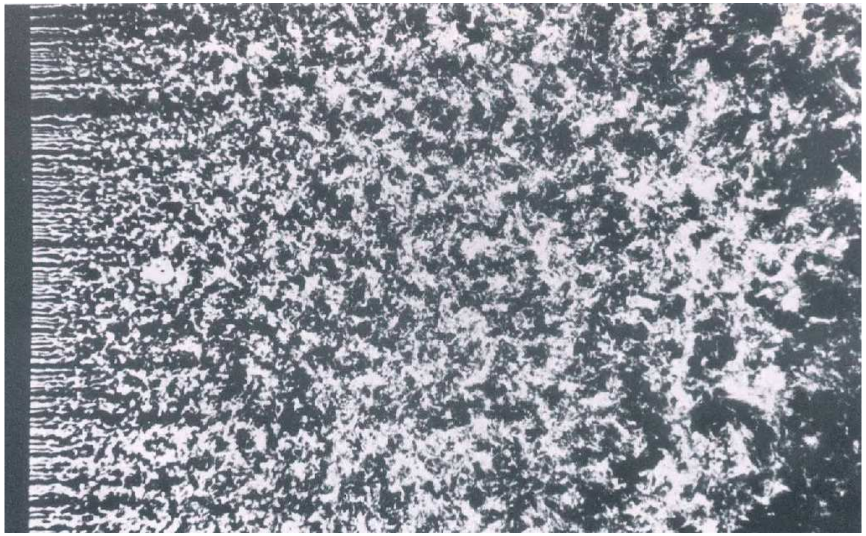
Maxwell 1867
Wiener 1948

Motivation for statistical physics approach

cylinder wake



grid turbulence



Van Dyke, Album of Fluid Motion

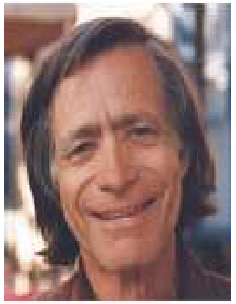
Goal: description in statistical physics / thermodynamics

⇒ powerful concepts of entropy, entropy principles, ...

First task: Define the thermodynamic degrees of freedom.

Ansatz: DoF = modes of a traditional Galerkin model

R.H. Kraichnan



L. Onsager

'Traditional' Galerkin method

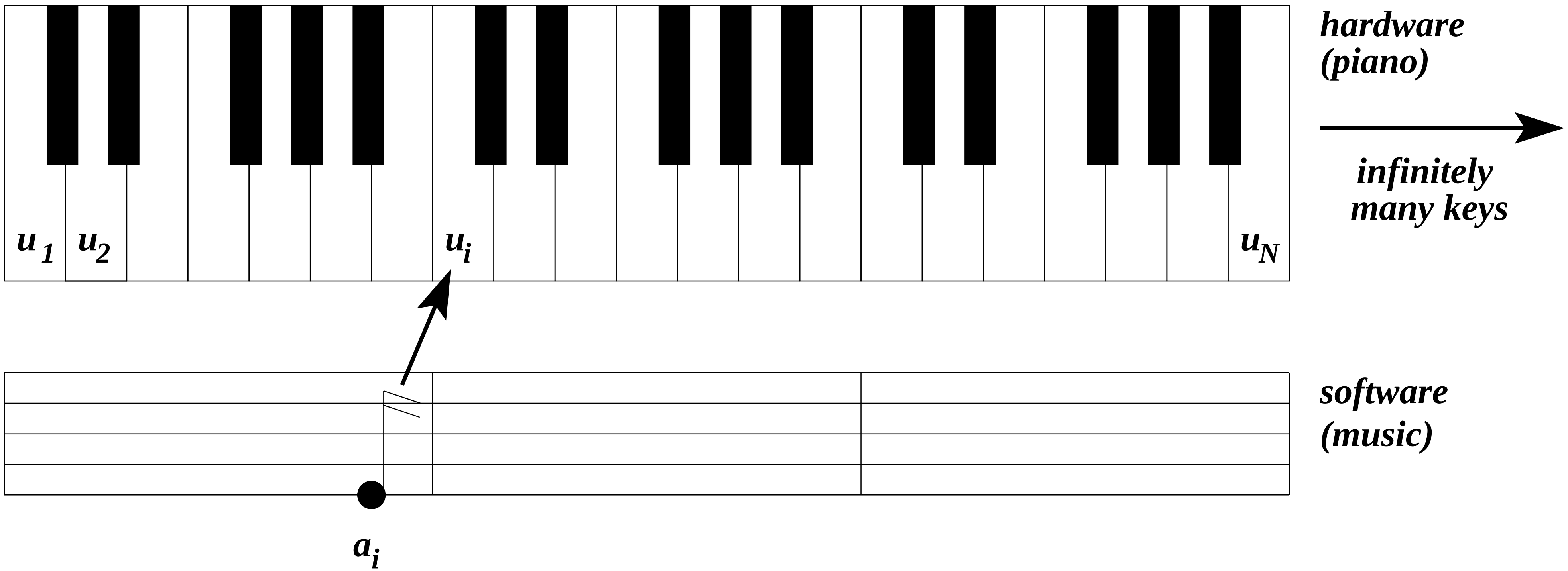
— *Fletcher 1984 Computational Galerkin Methods, Springer* —

Galerkin method

$$\mathbf{u}(\mathbf{x},\mathbf{t}) \quad \rightarrow \quad \partial_t \mathbf{u} = \frac{1}{Re}\triangle \mathbf{u} - \nabla(\mathbf{u}\mathbf{u}) - \nabla p$$

$$\downarrow \qquad\qquad \downarrow$$

$$\mathbf{u}^{[N]} = \sum_{i=0}^{N} a_i(t)\mathbf{u}_i(\mathbf{x}) \quad \rightarrow \quad \frac{da_i}{dt} = c_i + \sum_{j=1}^{N} l_{ij}\, a_j + \sum_{j,k=1}^{N} q_{ijk}\, a_j\, a_k$$



Finite-time thermodynamics formalism

— Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET —

dynamical system

constant
$\frac{da_i}{dt} = c_i + \ldots$

linear term
$+ \sum_j l_{ij} a_j + \ldots$

energy preserving

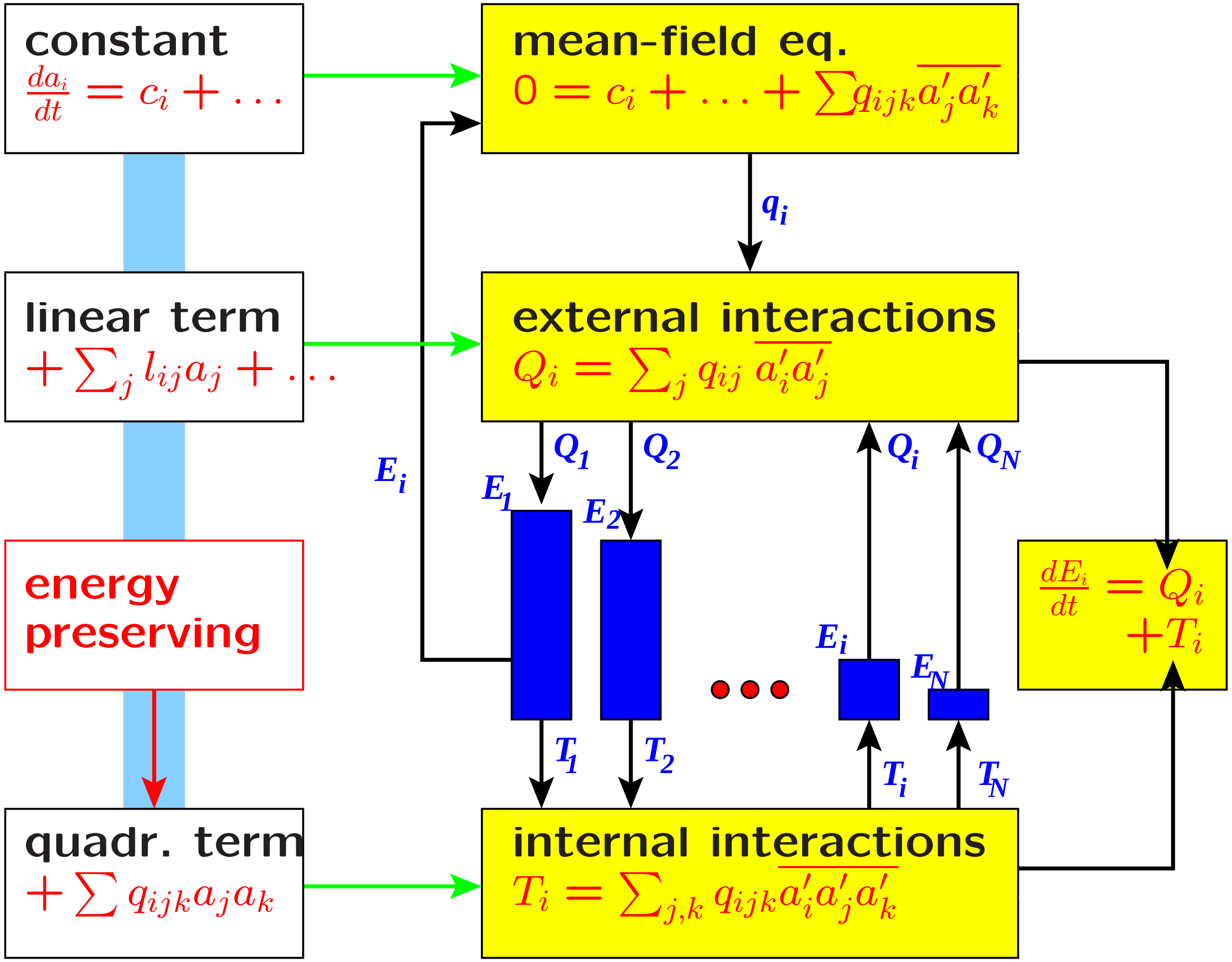
quadr. term
$+ \sum q_{ijk} a_j a_k$

Finite-time thermodynamics formalism

— ≡ Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET —

dynamical system | **averaged equations**

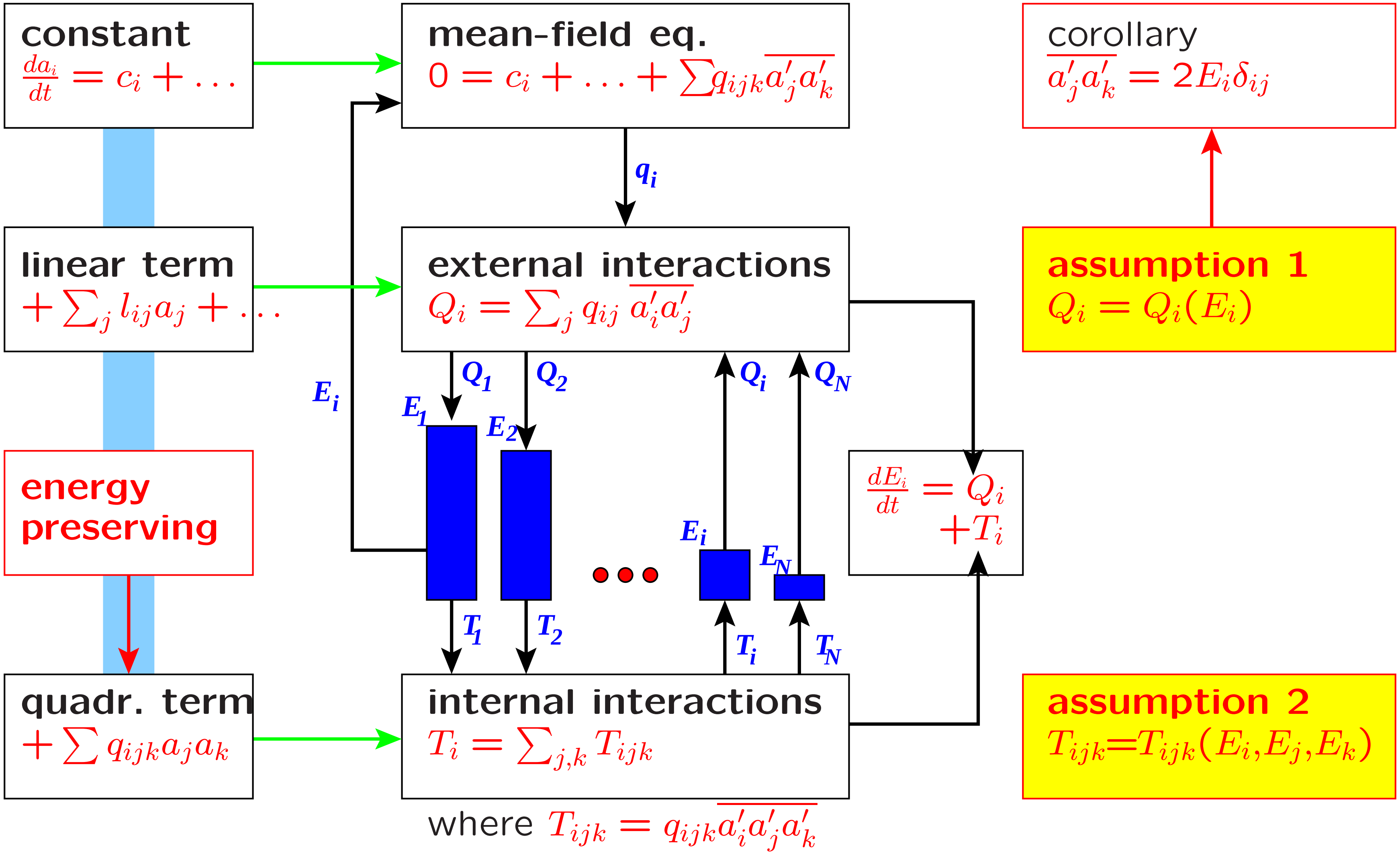
$a_i = \overline{a_i} + a'_i,\ E_i = \overline{(a'_i)^2}/2$



Finite-time thermodynamics formalism

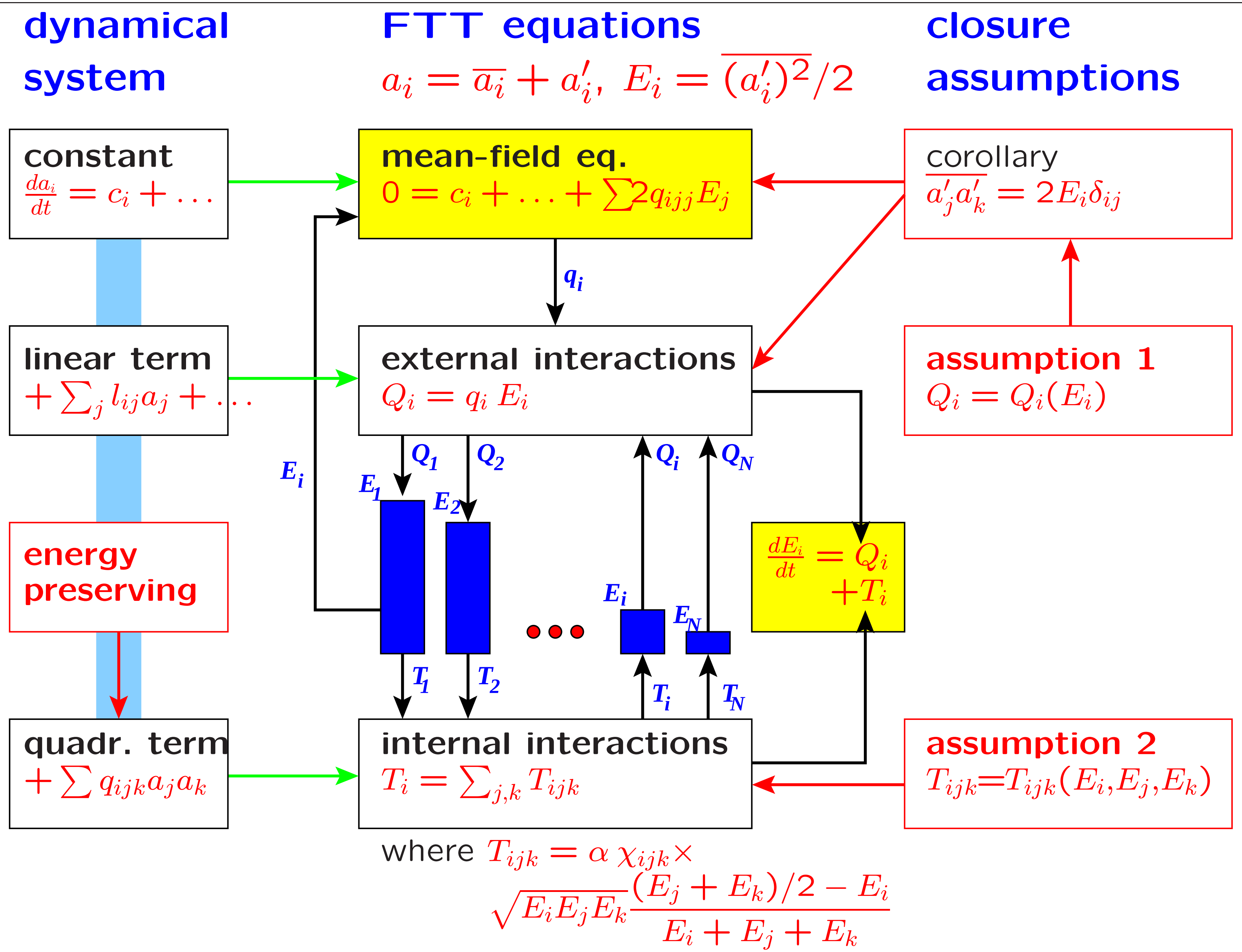
— ≡ Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET —

dynamical system | **averaged equations** $a_i = \overline{a_i} + a_i',\ E_i = \overline{(a_i')^2}/2$ | **closure assumptions**



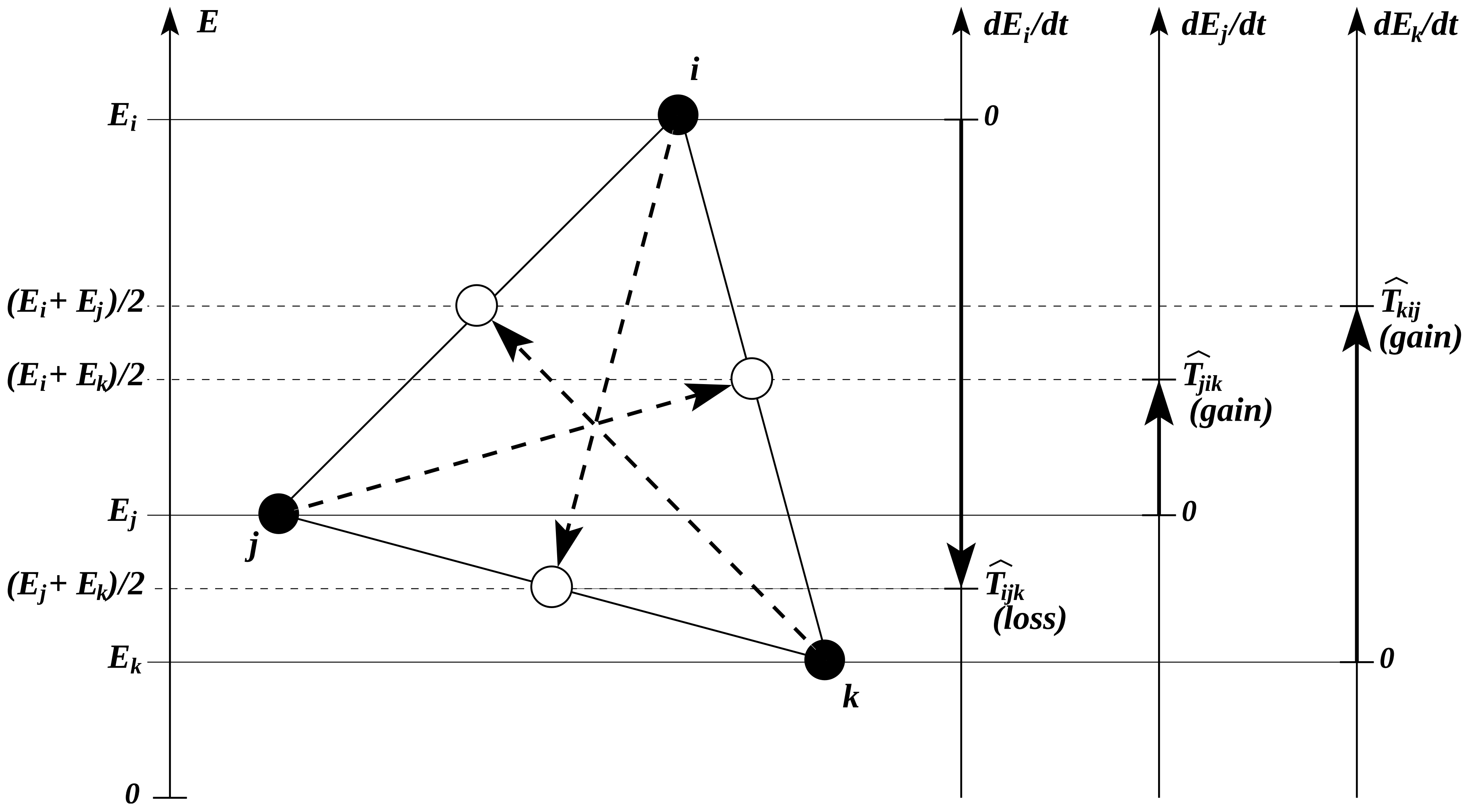
Finite-time thermodynamics formalism

— Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET —



Fick's law of triadic interactions

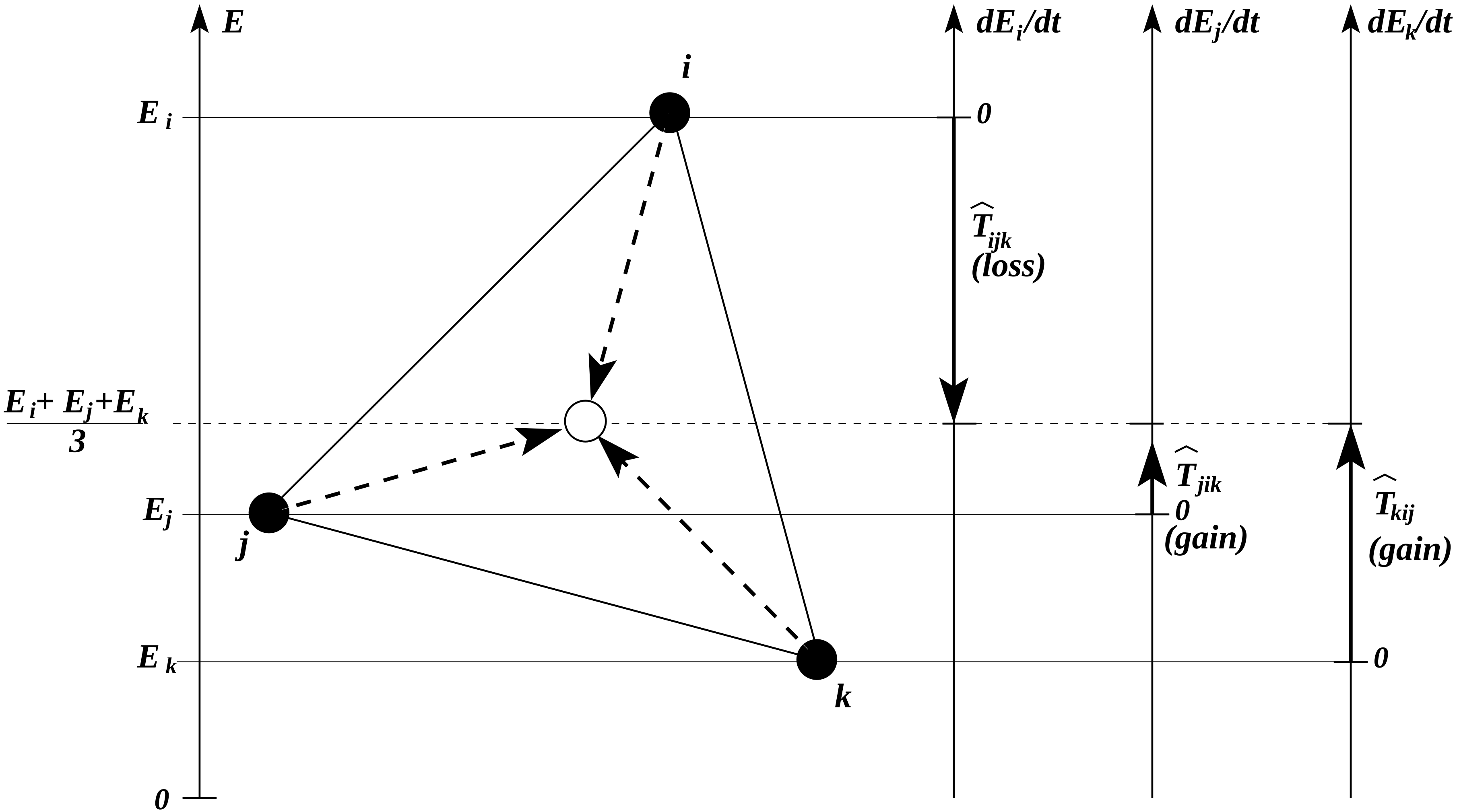
$$T_{ijk} = \alpha \chi_{ijk} \sqrt{E_i E_j E_k} \, \frac{\frac{1}{2}(E_j + E_k) - E_i}{E_i + E_j + E_k}$$



Fick's law of triadic interactions II

— ≡ Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET —

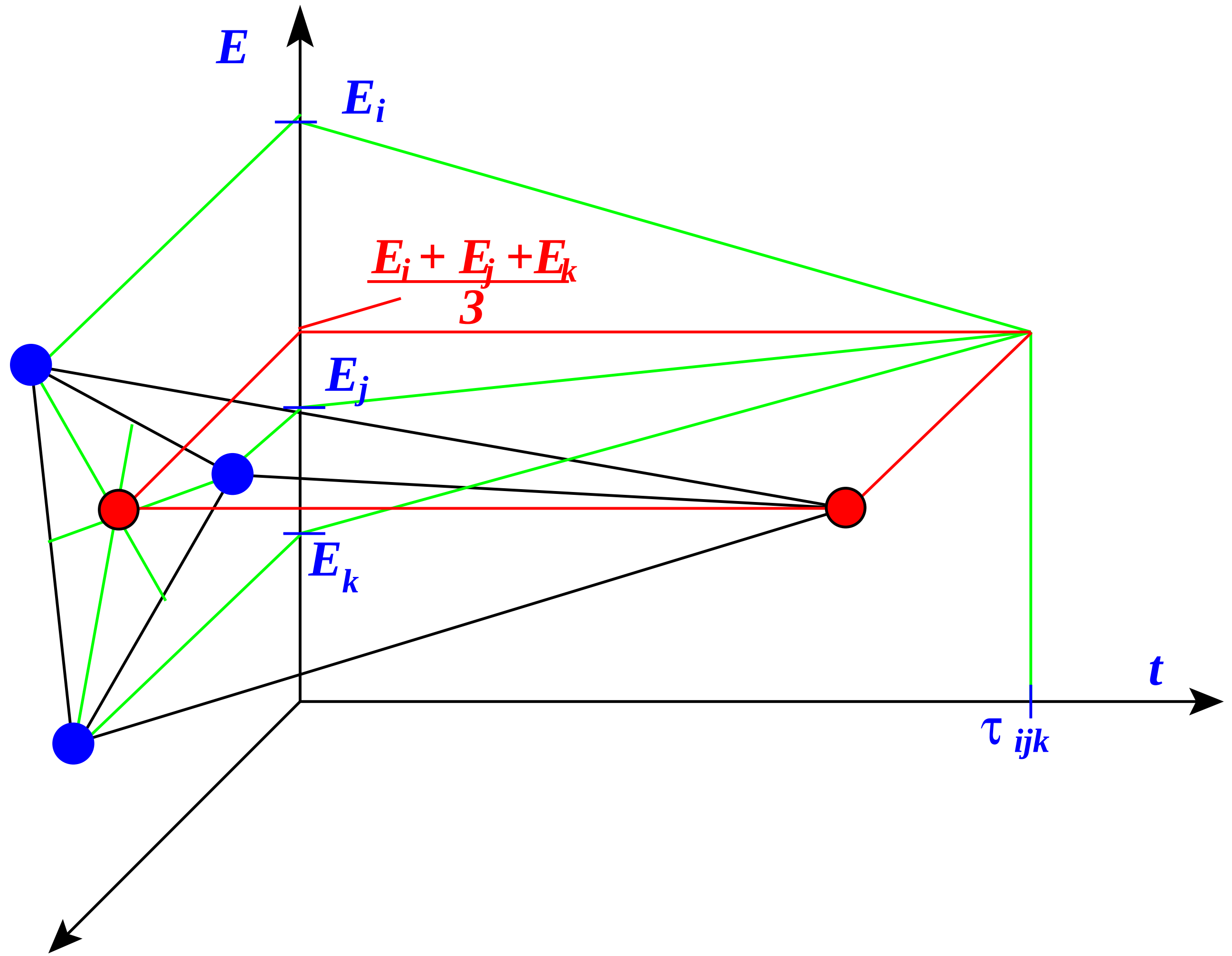
$$T_{ijk} = \sigma_{ijk} \left[1 - \frac{3E_i}{E_i + E_j + E_k} \right], \quad \text{where} \quad \sigma_{ijk} = \frac{3}{2} \alpha \, \chi_{ijk} \sqrt{E_i E_j E_k}$$



Fick's law of triadic interactions III

— ≡ Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET —

$$T_{ijk} = \sigma_{ijk} \left[1 - \frac{3E_i}{E_i + E_j + E_k} \right], \quad \text{where} \quad \sigma_{ijk} = \frac{1}{\tau_{ijk}} = \frac{3}{2} \alpha \chi_{ijk} \sqrt{E_i E_j E_k}$$



Fick's law for triadic interactions

— ≡ *Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET* —

Ansatz

$$T_{ijk} = T_{ijk}(E_i, E_j, E_k)$$

Properties from analysis of $T_{ijk} = q_{ijk}\,\overline{a_i a_j a_k}$

(1) Homogeneity . . . . $T_{ijk}(\lambda E_i, \lambda E_j, \lambda E_k) = \lambda^{3/2}\, T_{ijk}(E_i, E_j, E_k)$

(2) Zeros . . . . $T_{ijk}(E_i, E_j, 0) = T_{ijk}(E_i, 0, E_k) = T_{ijk}(0, E_j, E_k) = 0$

(3) Symmetry . . . . $T_{ijk} = T_{ikj}$

(4) Monotonicity . . . . $E_i < \min\{E_j, E_k\} \quad \Rightarrow T_{ijk}(E_i, E_j, E_k) < 0$

(5) Energy preservation . . . . $T_{ijk} + T_{ikj} + T_{jik} + T_{jki} + T_{kij} + T_{kji} = 0$

(6) Realizability (strictly: $|T_{ijk}| \le |q_{ijk}|\, |a_i|\text{max}\, |a_j|\text{max}\, |a_k|\text{max}$)

$$|T_{ijk}| \lesssim |q_{ijk}|\, \sqrt{E_i\, E_j\, E_k}$$

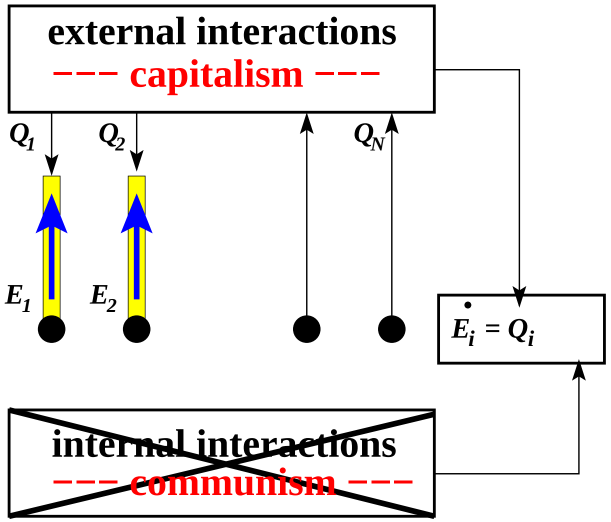
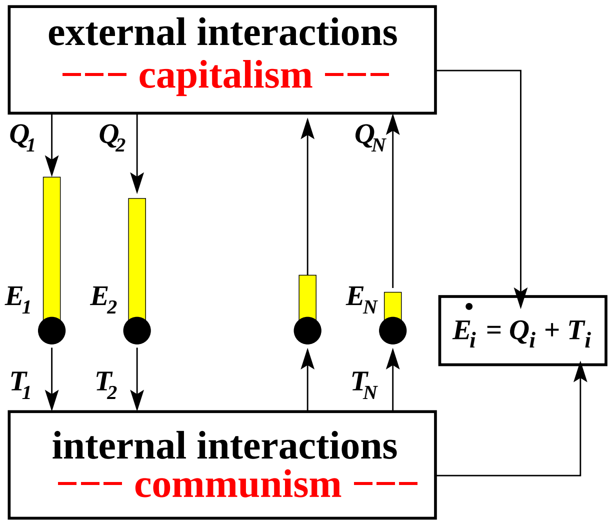
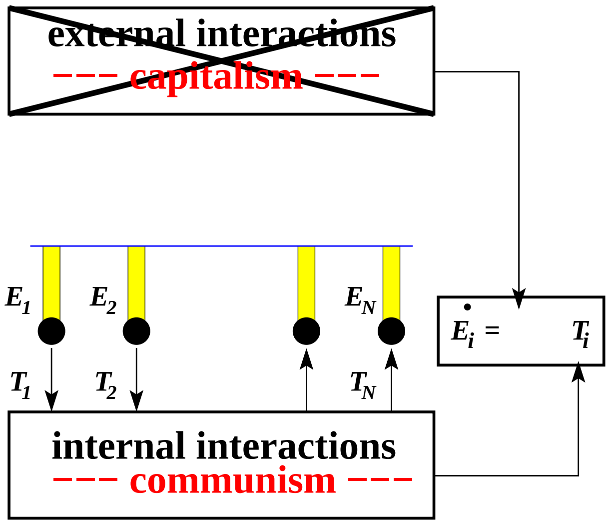
Solution

$$T_{ijk} = \alpha \chi_{ijk}\, \sqrt{E_i E_j E_k}\, \frac{\frac{1}{2}(E_j + E_k) - E_i}{E_i + E_j + E_k}$$

with the totally symmetric triadic structure function
$\chi_{ijk} := \frac{1}{6}\left(|q_{ijk}| + |q_{ikj}| + |q_{jik}| + |q_{jki}| + |q_{kij}| + |q_{kji}|\right)$ and α determined from energy flow consistency between donor and recipient modes.

FTT model — extremal limits

— Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET —

Linear dynamics	Partial LTE	LTE
		

theoretical fluid mechanics

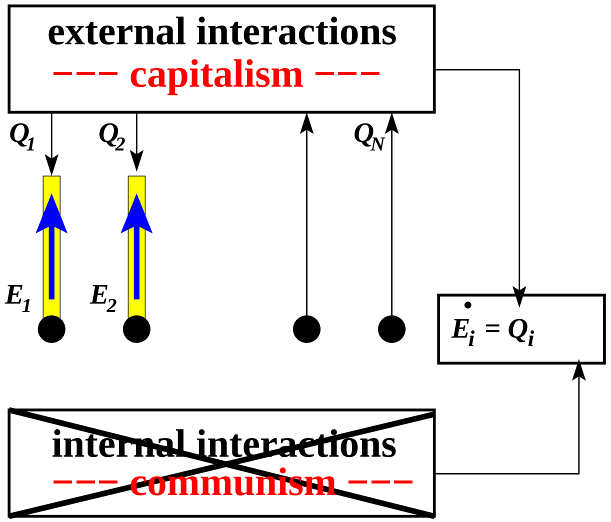
linear stability analysis	nonlinear dynamics	abs. equilibrium ensemble

FTT model — extremal limits

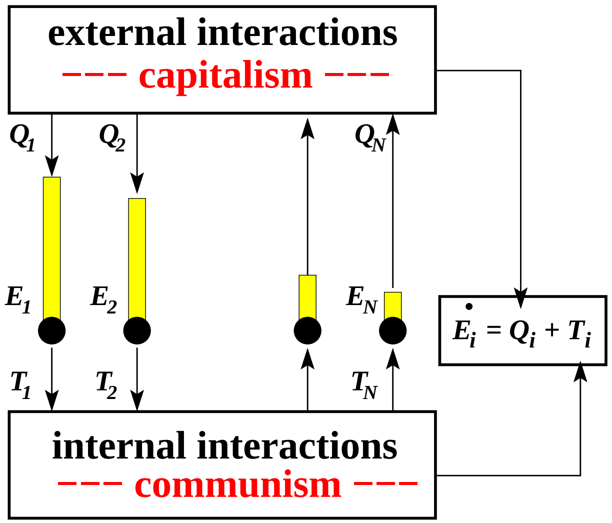
— ≡ *Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET* —

Linear dynamics	Partial LTE	LTE

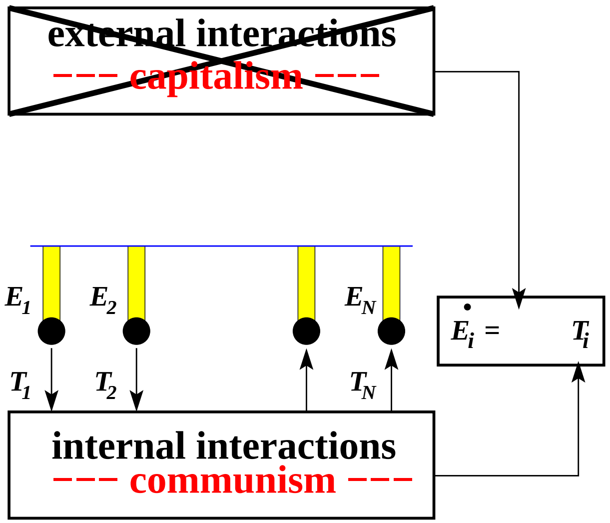
Linear dynamics



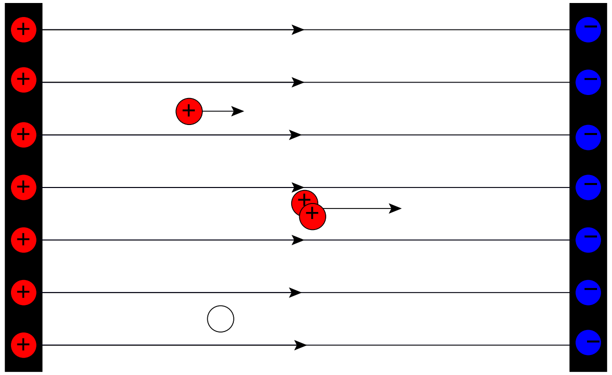
Partial LTE



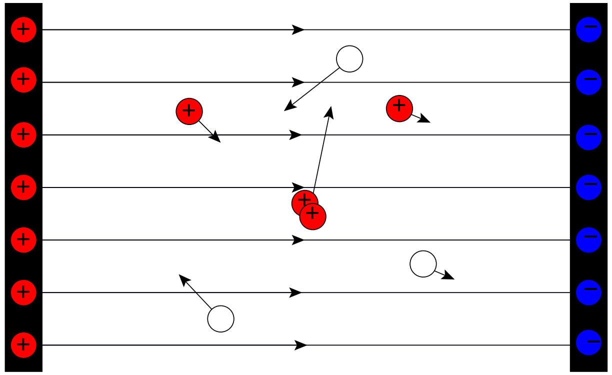
LTE

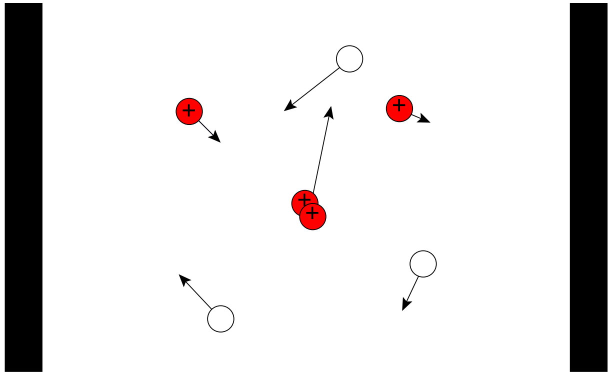


plasma physics analogy for charged particles in E-field



no collisions



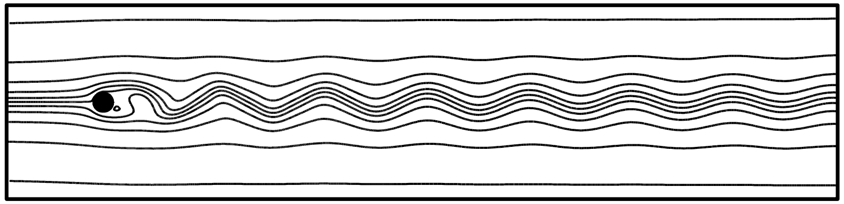


no E-field

Periodic cylinder wake ($Re = 100$)

— Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET —

2D flow around circular cylinder (DNS)

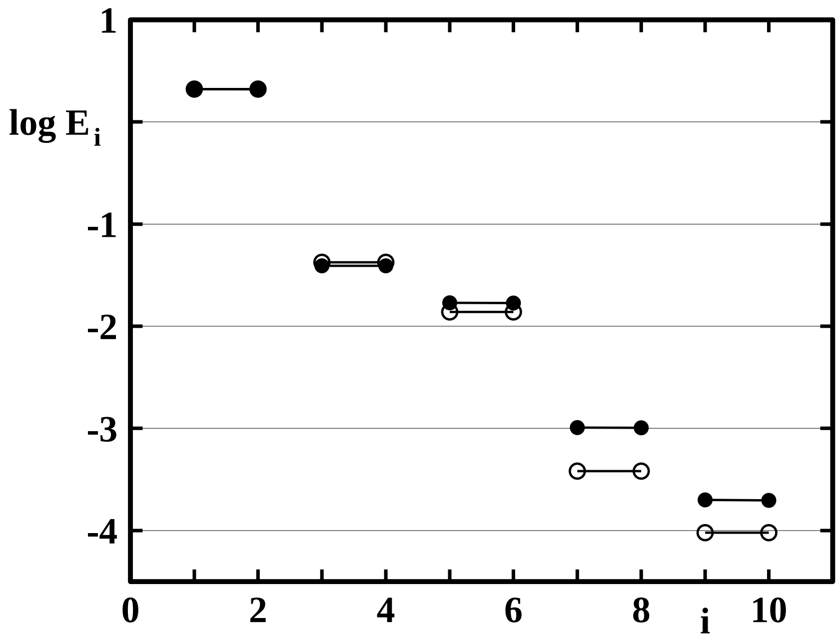


10-dim. Galerkin model

$$\mathbf{u} = \sum_{i=0}^{N} a_i \mathbf{u}_i \quad \text{(POD modes)}$$

$$\dot{a}_i = c_i + \sum_{j=1}^{N} l_{ij}\, a_j + \sum_{j,k=1}^{N} q_{ijk}\, a_j\, a_k$$

Energy distribution (computed and FTT predicted)



•: DNS; ○: FTT

Good agreement between DNS and FTT prediction!

Burgers' equation

— ≡ *Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET* —

Boundary value problem

$$\partial_t u + (U + u)\,\partial_x u = g(x,t) + \nu\,\partial^2_{xx} u$$

$U = 1$, $\nu = 1/100$, energy source $g(x,t) = \sigma\ (a_1\,\Theta_1 + a_2\,\Theta_2)$, $\sigma = 1/50$.

BC: $u(x + 2\pi, t) = u(x,t)$

Galerkin approximation (here: $N = 10$, 1st to 5th harmonics)

$$u(x,t) = a_0(t)\Theta_0(x) + a_1(t)\,\Theta_1(x) + \ldots a_N\,\Theta_N(x)$$

$\Theta_0 = \frac{1}{\sqrt{\pi}}$, $\Theta_1 = \frac{1}{\sqrt{2\pi}} \sin x$, $\Theta_2 = \frac{1}{\sqrt{2\pi}} \cos x$, $\Theta_3 = \frac{1}{\sqrt{2\pi}} \sin 2x$, $\ldots$

Galerkin system: $\dot{a}_0 = 0$

$$\dot{a}_i = \sum_{j=1}^{N} l_{ij}\,a_j + \sum_{j,k=1}^{N} q_{ijk}\,a_j\,a_k$$

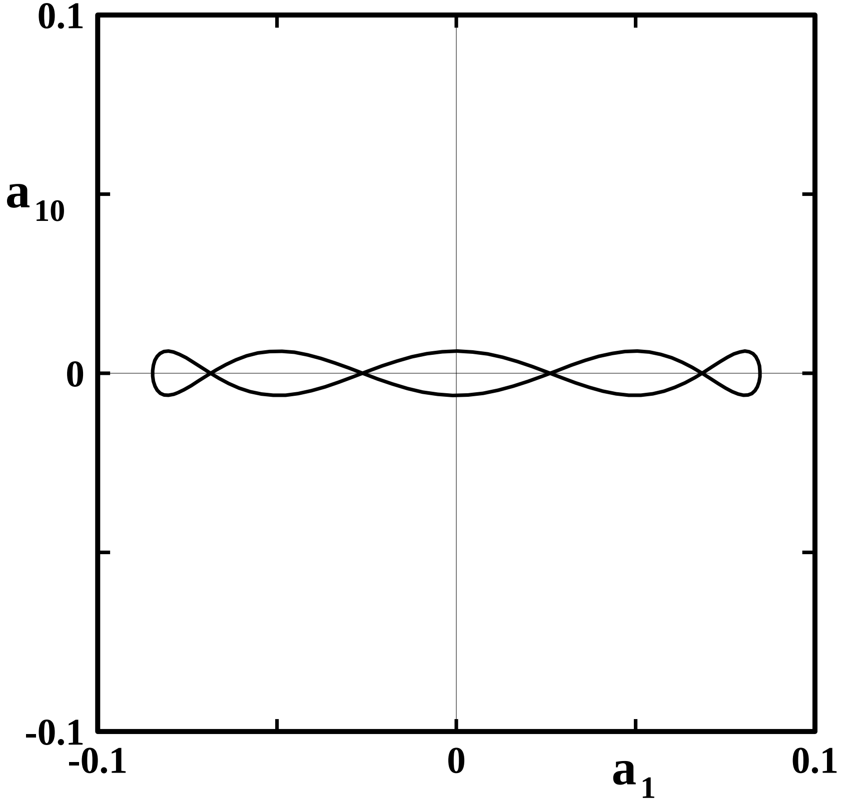
nonlinearly coupled oscillators ($i = 1, 2$: self-excited, $i \geq 3$: damped)

Burgers' equation II

— ≡ *Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET* —

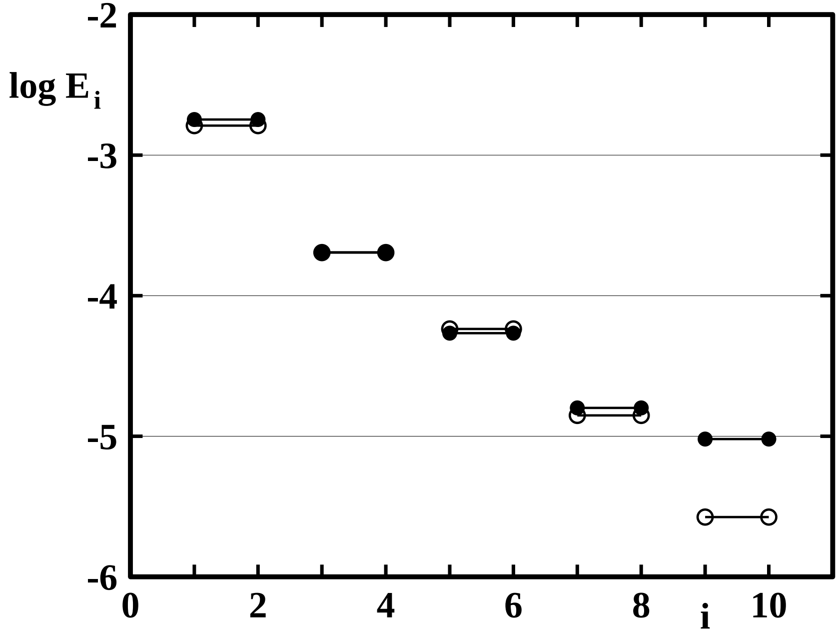
Travelling wave solution with energy source and diffusion term

phase portrait



$U = 1$, $\sigma = 1/50$, $\nu = 1/100$

energy distribution



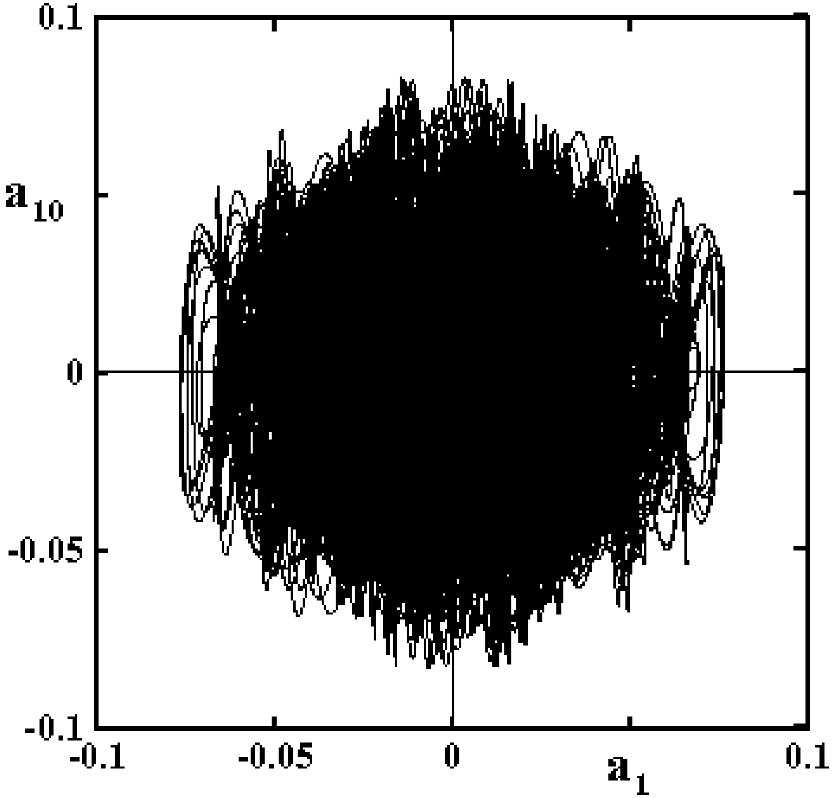
Good agreement between simulation • and FTT ○

Burgers' equation III

— ≡ *Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET* —

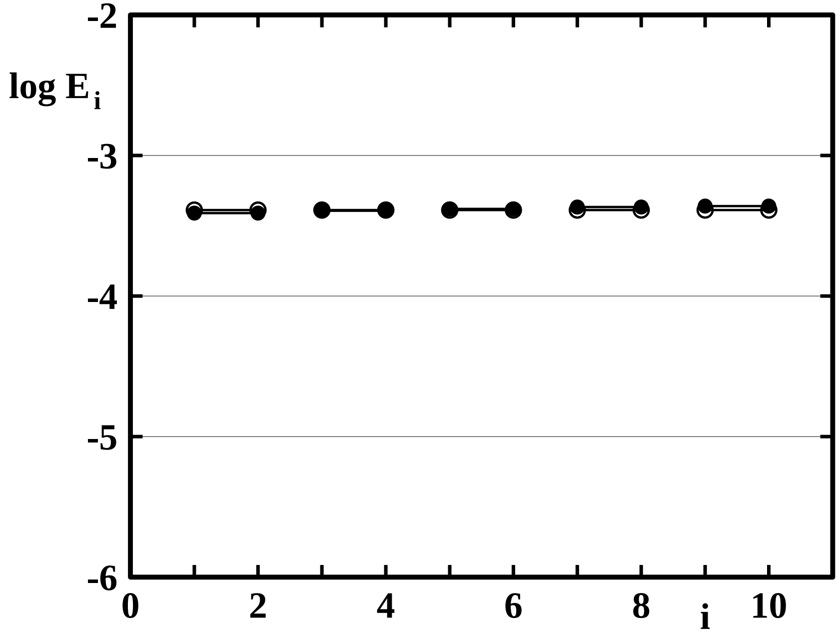
Truncated Burgers' solution **without** source and **without** diffusion term ≡ Majda & Timofeyev 2000

phase portrait



$U = 1,\ \sigma = 0,\ \nu = 0$

energy distribution



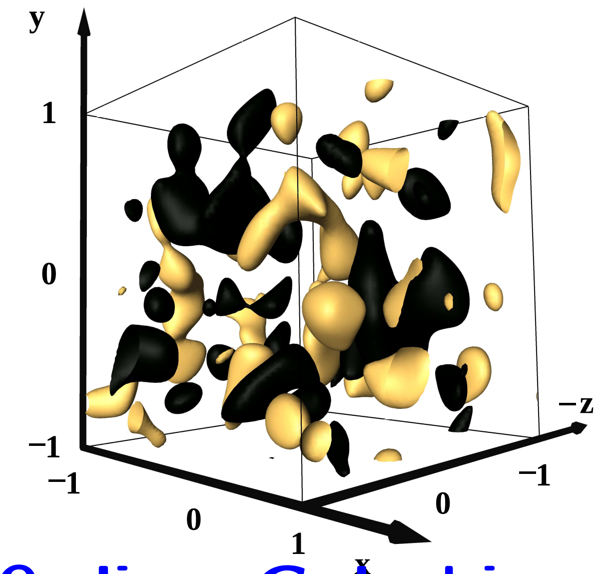
Equipartion of energy

in simulation • and FTT ○

Homogeneous shear turbulence ($Re = 1000$)

— ≡ *Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET* —

3D flow



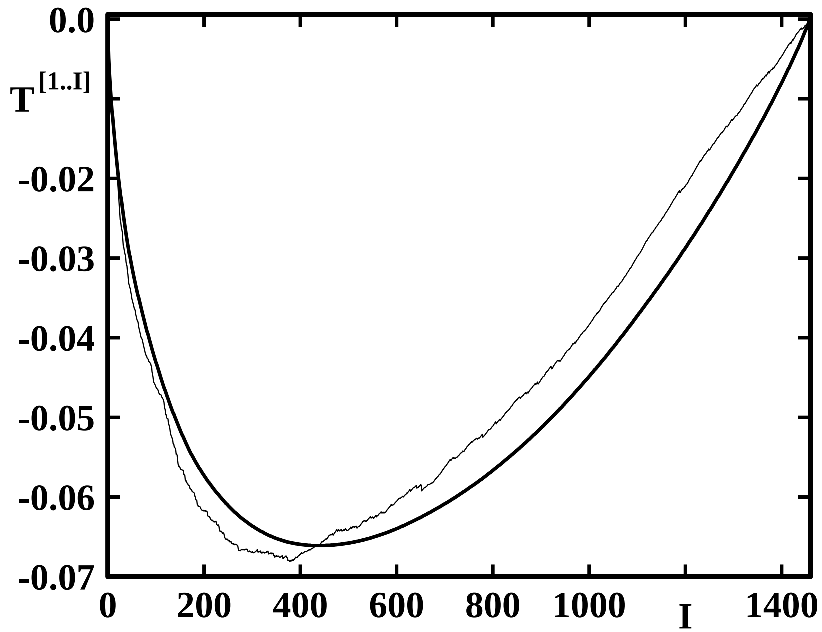
1459-dim. Galerkin model

$$\mathbf{u} = \sum_{i=0}^{N} a_i \mathbf{u}_i$$ (Stokes modes)

$$\dot{a}_i = c_i + \sum_{j=1}^{N} l_{ij}\, a_j + \sum_{j,k=1}^{N} q_{ijk}\, a_j\, a_k$$

Cumulative transfer term

(GM and FTT)



$$T^{[1..I]} := T_1 + \ldots + T_I$$

—: GM; — : FTT

Good agreement between GM and FTT prediction!

FTT applications

— ≡ *Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET* —

— *and many follow-up publications* ≡ ≡ ≡ —

- FTT $\Rightarrow$ mean-field model
- rigorous system reduction of evolution equation

$$\mathbf{u} = \mathbf{u}_{\text{dyn}} + \mathbf{u}_{\text{slaved}} + \mathbf{u}_{\text{stoch}}$$

- derivation of nonlinear subgrid turbulence model
- unified description of normal and inverse turbulence cascade
- fully nonlinear, infinite horizon control
- statistical mechanics & definition of entropy
- MaxEnt principle for attractor
- ...

FTT = eye opener + key enabler for many applications

Overview

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1. **Introduction**

   — *low-order Galerkin modeling*

2. **Control of laminar shear flow**

   — *low-order modeling of weakly nonlinear dynamics*

3. **Control of turbulent shear flow**

   — *low-order modeling of strongly nonlinear dynamics*

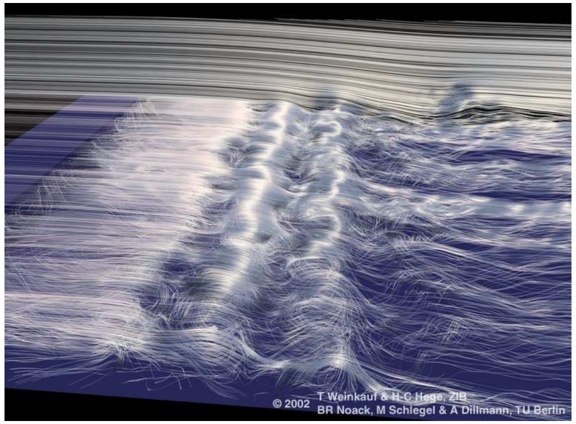
4. **Instabilities, turbulence and control**

   — *an emerging unifying theory*

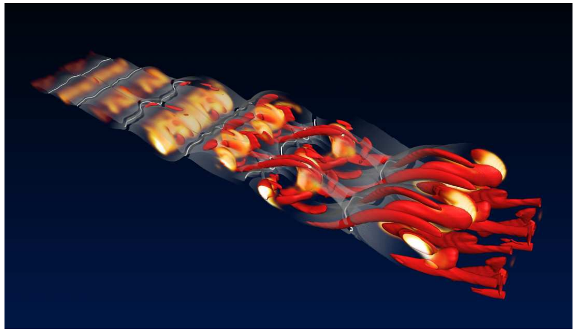
5. **Concluding remarks and outlook**

Configurations

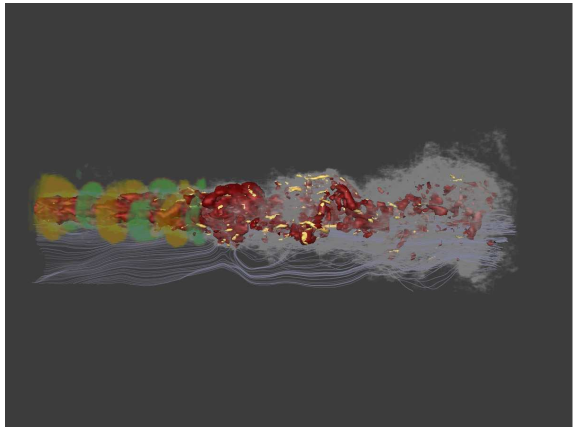
3D flow over a step



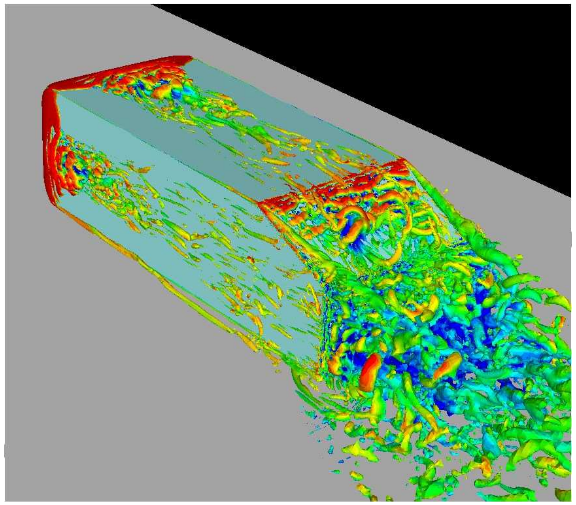
3D mixing layer



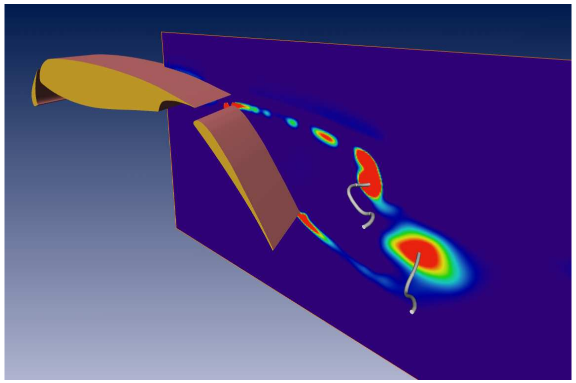
jet noise



Ahmed body



airfoil



wake

channel flow

combustor

cavity flow

. . .

Conclusions

≡ Noack, Cordier, King, Morzyński, Siegel & Tadmor (2009) Springer

- **Galerkin modelling** for **flow control** is a doable art!

$$\mathbf{u} = \sum_{i=0}^{N} a_i \, \mathbf{u}_i, \qquad \dot{a}_i = c_i + \sum_{j=1}^{N} l_{ij} \, a_j + \sum_{j,k=1}^{N} q_{ijk} \, a_j \, a_k + g_i \, b$$

More info: CISM Course in Udine, Sep. 15–19, 2008

- **Physics mechanisms** for turbulence control

  strongly nonlinear.

  - drag reduction of D-shaped body
  - lift increase of high-lift configuration
  - noise reduction of turbulent jet
  - ...

- **Model for natural and controlled attractor needed!**

  ⇒ **Upgrade Galerkin model with ergodic measure**

Conclusions

≡ Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET

- **Finite-time thermodynamics model** builds on GM

$$\mathbf{u} = \sum_{i=0}^{N} a_i \,\mathbf{u}_i, \qquad \dot{a}_i = c_i + \sum_{j=1}^{N} l_{ij}\, a_j + \sum_{j,k=1}^{N} q_{ijk}\, a_j\, a_k$$

$\Rightarrow$ **first and second moments** of unsteady flows

  - 1D Burgers' eq., • 2D wake, • 3D shear turbulence.

- **FTT $\mapsto$ Statistical physics (economics) link**
  - $\mathbf{u}_i$ . . . . . **person /thermodyn. degrees of freedom)**
  - E_i . . . . **wealth /order parameter**
  - $\sum q_{ijk} a_j a_k \Rightarrow T_i$ . . . . **pure communism /LTE**
  - $\sum l_{ij} a_j \Rightarrow Q_i$ . . . . . . **pure capitalism /lin. instability**
  - **Both terms** . . . . **social market /partial LTE**

- **FTT $\mapsto$ energy-based and nonlinear control design**

Outlook

- **Turbulence closure with a Finite-Time**

  Thermodynamics of the turbulence cascade

  Modes → energy distribution → mean flow & fluctuations

  (Alternative to eddy-viscosity ansatz and uRANS)

- **Non-equilibrium cybernetics**

  Non-linear infinite-horizon control of the attractor

  via a manipulation of the turbulence cascade

  Adds to control theory based on linearization and stabilization

- **Model-based feedback flow control**

  in experimental demonstrators

  Improvement benchmarked against black-box-model control

Publications

More information: call +49-30-314.24732 or read

- Noack, Afanasiev, Morzyński, Tadmor & Thiele (2003) **JFM** . . . . *generalized mean-field model of wake*
- Noack, Papas & Monkewitz (2005) **JFM** . . . . *pressure-term representation in shear-layer model*
- Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) **JNET** . . . . *Finite-Time Thermodynamics formalism*
- Noack, Cordier, King, Morzyński, Siegel & Tadmor (2009+) **Springer** . . *ROM for flow control*