# On a Galerkin–type of approach to wrinkled flames in gravity fields

Gaël BOURY, Guy JOULIN

Laboratoire de Combustion et de Détonique (UPR 9028 CNRS)

ENSMA - BP 40109 - Téléport 2 - 1 Avenue Clément Ader

86961 Futuroscope Cedex

Abstract prepared for the workshop
"Dynamique des fronts"

Bordeaux, April 22-23, 2003

# On a Galerkin–type of approach to wrinkled flames in gravity fields

Gaël BOURY, Guy JOULIN

Laboratoire de Combustion et de Détonique (UPR 9028 CNRS)

ENSMA - BP 40109 - Téléport 2 - 1 Avenue Clément Ader

86961 Futuroscope Cedex

In absence of any gravity, the spontaneous wrinkling of flat–on–average premixed gaseous flames is fairly well described (Boury, 2003) by an evolution equation (for the flame shape, $F(x,t)$ ) that has the same structure as the one first derived by Sivashinsky (1977) in the limit of small fractional density jumps across the front. Exact, $2\pi/k_{box}$–periodic, solutions to the equation are expressible in terms of elementary functions (Thual et al, 1985), provided the latter's singularities in the complex $x$–plane are located according to some (known) coupled O.D.E s. The limit of large periods $2\pi/k_{box}$ is even solvable analytically, via the resolution of a linear equation for the (then "quasi–continuous") distribution of poles of $F_x$ along the imaginary $x$–axis. When gravity is present, adapting Sivashinsky's analysis (Boury, 2003) leads to a new equation for $F(x,t)$ viz :

$$F_t + \frac{aS_L}{2}\left(F_x^2 - \langle F_x^2\rangle\right) = \Omega S_L \left(\frac{F_{xx}}{k_n} + I(F,x) - \kappa\left(F - \langle F\rangle\right)\right) \qquad (1)$$

where $a \geq 1$, $\Omega > 0$ and $S_L > 0$ are known physico–chemical constants, $\langle\cdot\rangle$ denotes an average over space $(x)$ and $k_n > 0$ is a cut–off wavenumber. The wavenumber $\kappa$ measures the intensity of gravity ($\kappa > 0$ for downward propagations). The linear operator $I(\cdot,x)$ has $|k|$ as symbol in Fourier $x$–space and is responsible for the Landau–Darrieus hydrodynamical mechanism of instability ; for $\kappa < 0$ (resp. $\kappa > 0$) gravity effects amplify (resp. counteract) it, and kill it if $\kappa > k_n/4$.

No exact, non–trivial solution to (1) is analytically available if $\kappa/k_n \neq 0$ (and $< 1/4$). Numerical resolutions of (1) are feasible, but encounter severe difficulties (relating to round–off background noise) whenever $|\kappa| \ll k_n/4$ and $k_{box} \ll k_n$ (long waves and weak gravity). By Murphy's law, this is unfortunately one of the most intriguing and interesting cases ! A projection method of the Galerkin–type is proposed to try and get approximate solutions to (1). Trial and test functions are chosen in the same class as that solving (1) when gravity is absent ($\kappa = 0$). In the interesting limit where $|\kappa|/k_n \to 0$, $|\kappa|/k_{box} = \mathcal{O}(1)$, a highly singular equation is obtained for the density $P(\cdot) \geq 0$ of imaginary singularities of $F_x$ in complex $k_{box}x$–plane. Solving it for $P(\cdot)$ would greatly help one understand and predict the statistical properties of gravity–affected wrinkled flames, with or without external forcing (Boury, 2003). Alas, nontrivial solutions for $P(\cdot)$ are analytically available only when gravity or Landau–Darrieus instability is absent ! Handling both simultaneously is still a numerical and mathematical challenge, especially for $0 < \kappa/k_{box} < \pi$. Is it a genuine difficulty (e.g. non–existence of solutions for $\kappa/k_n \neq 0$) or a mere technical one (then how to overcome it).

Helps from the mathematical community would be invaluable, for this $P(\cdot)$ approach might serve (if legitimate) as a basis to a new representation of wrinkled flames in even more general situations.