**PROGRAMME JEUNES CHERCHEUSES ET JEUNES CHERCHEURS**

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**Projet XXX**

**DOCUMENT SCIENTIFIQUE**

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| **Acronyme** | **ARIVAF** | | |
| **Titre du projet en français** | Arithmétique des variétés en familles | | |
| **Titre du projet en anglais** | Arithmetic of varieties in families | | |
| **Comité d'Evaluation référence (CE)**[1] | 5 (Mathématiques et Interactions) | | |
| **Aide totale demandée** | 138 400 € | **Durée du projet** | 48 mois |

## SOMMAIRE



[1] Indiquer la référence du CE choisi pour l'évaluation du projet (cf. tableaux page 3 et 4 du texte de l'appel à projets)

## 1. Contexte et positionnement du projet / Context and positionning of the proposal

The main topics of the project under submission is how properties of arithmetic nature vary in families of varieties.

There are several natural definitions of what a family is.

From a geometric point of view, a family is a morphism of schemes $Y \to X$ regarded as the collection of its fibers $Y_{x} \to spec(k(x))$. However, to get a reasonable notion of family, one usually considers only flat morphisms $Y \to X$. Flatness ensures that some numerical invariants (such as the dimension, the Hilbert polynomial, the dimension of the cohomology vector spaces etc.) be locally constant or, at least, vary semi-continuously ; in this sense, it provides a natural formulation of what a « continuous family » should be in the frame of arithmetic geometry.

From a categorical point of view, given a scheme S, a family is a fibered category that is a functor $F:X \to Sch/S$ from a category X to the category of S-schemes which satisfies additional axioms [FGAex, Chap. 3], [LMB00, Chap. 2]. Given a S-scheme $X \to S$, any geometric family $Y \to X$ defines a categorical family (whose fiber category over a S-scheme $U \to S$ is the discrete category with objects the $Y \times_{X,x} U \to U$, where x describes the set of all S-morphisms $U \to X$) but, conversely, it almost never happens that a categorical family originates from a geometric family. So, to try and study categorical families with geometric methods, one has to modify the problem, for instance:

- construct fiber categories $F_L:X_L \to Sch/S$ which are geometric family and cover $F:X \to Sch/S$; these methods are called rigidification methods;
- enlarge the definition of geometric family. This is done by endowing Sch/S with a Grothendieck topology (say the étale topology) and requiring that the functor $F:X \to Sch/S$ be a stack for this topology or, even better, a stack that admits a smooth (resp. an étale) covering by a scheme; such stacks are called algebraic (resp. Deligne-Mumford) stacks [FGAex, Chap. 4], [LMB00, Chap. 3, 4].

The two approaches are closely related. For instance, consider the Deligne-Mumford stack $A_g$, which associates to a S-scheme $U \to S$ the set of all g-dimensional and principally polarized abelian schemes over U. Instead of $A_g$, one can consider the functor $A_{g,n}$, which associates to a S-scheme $U \to S$ the set of all g-dimensional, principally polarized abelian schemes $A \to U$ together with a fixed isomorphism between $A[n]$ and $(Z/n)^g$. Then $A_{g,n}$ is a scheme for n large enough and the morphism $A_{g,n} \to A_g$ gives an explicit étale covering of $A_g$ by a scheme.

Such construction of explicit smooth (resp. étale) presentations of algebraic (resp. Deligne-Mumford) stacks by rigidification methods are not always possible but the notion of algebraic stack is rich enough to develop a powerful arithmetico-geometrical frame (which includes the former theories of schemes and algebraic spaces).

So, the fundamental question which is at stake in this project is: *to what extend geometry (being an element of a geometric family) determines arithmetic?*

The researchers involved in the project under submission are working on special aspects of this fundamental question using approaches of different nature in the general frame of arithmetic geometry, such as deformation theory, general theory of stacks, rational points on moduli spaces (modular curves, Shimura varieties, Hurwitz spaces *etc.*), étale fundamental groups of schemes and stacks, l-adic representations, diophantine geometry *etc.,* all of which are very active areas of investigation worldwide. Modern arithmetic geometry is often considered to be born in France, under the impulse of A. Grothendieck in the 60's, who elaborated, together with the French school, its fundations in the EGA and SGA. The theory of stacks, Galois categories, l-adic cohomology etc. were formalized during this period and have been developped since then, being fundamental theoretic tools to understand the objects of arithmetic geometry. Around the same period, diophantine geometry, which focusses on 'varieties', that is reduced schemes of finite type and separated over fields and stems from problem of more effective nature, started to develop powerful technical tools such as heights to investigate questions related to rational points on varieties. The theoretical tools of Grothendieck's formalism and the more technical ones of diophantine geometry are now intimately mixed and the conjugation of both has maintained the impressive development of modern arithmetic geometry for the past 50 years.

The purpose of this project is to understand better some of the main problems at stake in the study of variation of arithmetic properties in families as well as the different approaches and technics that were developped to try and tackle them. Hopefully, this sharing of knowledges should bring new insights to the specific problems studied by each of the researchers and help develop collaborations between them.

Our basic strategy is to organize series of short workshops (averagely three days workshops, twice or three times a year) around common themes of interest. Half of the talks will be given by researchers involved in the project, as in a working group, whereas the other half will be given by specialists of the topics. In the fourth year of the project, we plan to organize a one week international research conference with advanced research talks of both researchers involved in the project and exterior specialists. Part of the budget may also be used as a complement to other sources (laboratories, other projects, foreign grants *etc.*) to support the participation to international conferences. We give more details about these organizational aspects in sections 3-2, 3-3 and 3-4. In order to motivate the themes selected for the workshops, we explain below the main scientific features of the research programs of the researchers involved in the project under submission. To make the exposition clearer, we distinguish three main orientations:

1 - Fundamental groups and algebraic stacks;
2 - l-adic representations of the étale fundamental group;
3- Diophantine geometry.

Due to its transversal dimension and its learning purposes, we hope that our research project will interact constructively with other more specialized projects in arithmetic or diophantine geometry and help maintain or develop collaborations initiated in the frame of terminating projects.

# 2. Description scientifique et technique / Scientific and technical description

## 2.1 État de l'art / Background, state of art

1-

This part of the project is concerned with the study of arithmetic and geometric properties of varieties from the point of view of coverings. For this, one central object of focus is the fundamental group.

a) Review of various fundamental groups. The introduction of a notion of fundamental group in algebraic geometry was one of the early breakthroughs of Grothendieck, and over the years it has inspired the introduction of several analogues. Today, a broad view on this topic includes :
– the original topological fundamental group : it remains a relevant object over the complex numbers or in the p-adic (Berkovich) setting,
– the étale fundamental group of schemes (Grothendieck) or of algebraic stacks (Noohi, Zoonekynd),
– the fundamental group scheme (Nori, Gasbarri),
– the motivic fundamental group (Deligne),
– the tempered fundamental group of p-adic varieties (André).

Each of these variants has particular features. To mention just a few properties intrinsic to the fundamental groups themselves, let us say that the topological fundamental group is the most « computable » of all, the étale fundamental group incorporates more arithmetic information, the fundamental group scheme also takes into account inseparability of covers, and the tempered fundamental group reflects the p-adic topology of the base field while correcting the lack of topological p-adic coverings.

Other features vary : the category of (suitably-valued) representations of some of these groups may be described as equivalent to categories of local systems, or categories of vector bundles with connection. This leads to tannakian definitions of variants of the fundamental group. But for p and l distinct primes, l-adic representations of the étale fundamental group of a scheme of characteristic p cannot be described by bundles with connection. Also, some of these groups come with an interesting universal cover, and some do not.

The fundamental group scheme was initially introduced by Nori to compensate for the lack of étale p-covers of a proper scheme over an algebraically closed field of characteristic p (accordingly, the group scheme of p-torsion points of an abelian variety in characteristic p is better behaved than the group of p-torsion points). It is still an object of active research (Biswas-dos Santos, Langer). Recently, it was discovered by Esnault, in relationship with her study of the section conjecture in anabelian geometry, that this concept is also interesting for a curve over a non algebraically closed field of characteristic zero (especially, a number field).

b) Link between fundamental group and ramification theory. Quite naturally, the study of the loci where finite morphisms fail to be étale raises as much interest as étale coverings themselves. In most situations, one can not avoid ramification to come up ; in degenerating families, some special fibres may even be generically ramified.

When the ramification is tame, the ramification can be avoided by the introduction of new geometric structures. For example, log geometry allows to view a cover of a scheme X ramified along a divisor D as an unramified log-cover of (X,D). Another such device is the use of some stack structures ; for instance, twisted curves, introduced by Abramovich and Vistoli, have been successful in various places of algebraic geometry and may be used to absorb ramification. Sometimes both tools are used simultaneously: Olsson has defined useful stacks in terms of extension of log structures. In these cases, the étale fundamental group or some of its variants are relevant.

When the ramification is wild, ramification can not be avoided and becomes the main point of interest. Local ramification theory has recently been given a new impetus thanks to work of Abbes and Saito defining a ramification filtration on the Galois group of a discretely valued field with possibly imperfect residue field. This theory has implications both for the arithmetic and the geometric ramification. For example, in the geometric setting, this theory is needed when considering degenerating families, because a generic point of a special fibre has a local ring whose fraction field is such a field.

The case of generically ramified (i.e. inseparable) morphisms f:X→Y between schemes or varieties is also an important topic of current research. Such morphisms occur naturally as degenerations of separable morphisms f*:X*→Y* of the nicest kind (isogenies of abelian varieties, covers between smooth curves). When the original separable f* is Galois of group G*, there is a canonical group scheme degeneration G acting on f, and no ramification theory is known to describe such objects. These group scheme degenerations have been introduced and studied recently by various authors (Abramovich, Romagny) under the name of *effective models*. In these cases, the fundamental group scheme should naturally come into play.

2/3-

This part of the project is essentially devoted to the following problem. Let X be a normal and geometrically connected k-scheme of finite type. Let \Pi(X) denote the étale fundamental group of X and recall that any closed point x in X induces a section (well defined up to inner automorphisms) G_{k(x)}→\Pi(X) of the canonical restriction morphism \Pi(X)→G_{k} (here, k(x) denotes the residue field of x and G_{k(x)} its absolute Galois group). So, any l-adic representation r:\Pi(X)→GL_d(Z_l) gives rise to a family of l-adic representations r_x:G_{k(x)}→GL_d(Z_l), x in X. Write G, G^0 and G_x for the images of \Pi(X), \Pi(X_{\bar{k}}) and G_{k(x)} respectively. Then, what can be said about the variation of G_x, x in X? The l-adic representations we are interested in are of the following form:

1. If Y→X is a smooth proper and geometrically connected morphism of integral schemes then \Pi(X) acts on the ith etale cohomology group of the l-adic sheaf Z_l over the geometric generic fiber of Y→X. A special case is when Y→X is an abelian scheme and i=1, then one recovers the usual representation of \Pi(X) on the l-adic Tate module of the generic fiber of Y→X;
2. If Y→X is a smooth proper and geometrically connected morphism of integral schemes admitting a section then \Pi(X) acts on the outer automorphism group of the pro-l completion of the fundamental group \Pi_l of the geometric generic fiber of f:Y→X. In general, this representation is not l-adic but if we replace \Pi_l by its nilpotent quotient \Pi_{l,n} by the nth term of the lower central series, then the resulting representation is l-adic;
3. General l-adic representations (as the ones constructed in [CT09c] and [CT09d]).

Representations of type (1) have the GSRP property (i.e. the abelianization of the Lie algebra of the image of the geometric fundamental group of X is trivial). Whether representations of type (2) satisfy such a property is still unclear. The main conjectures concerning those representations are the following:

**<u>Conj (l,d,D)</u>**: Assume that X has dimension d. Then for any GSRP l-adic representation r of the etale fundamental group of X and for any integer D the set X_{r,l,D} of all closed points x in X with residue field of degree less than D over k and such that G_x is not open in G is not Zariski dense and there exists an integer N_{r,l,D} such that for any closed points x in X with residue field of degree less than D over k and not in X_{r,l,D} one has [G:G_x]<N_{r,l,D}.

Applied to a representation of type (1), raising from an abelian scheme A→X, Conj (l,d,D) implies the uniform boundedness of the l-primary torsion in the special fibers of A→X. In particular, Conj (l,d,D) for all D and such representations implies the celebrated l-primary torsion conjecture:

**<u>Conj (l-primary) torsion (g,D)</u>**: For any integers D and g there exists an integer N_{(l),g,D} such that for any g-dimensional abelian variety A defined over any number field k of degree less than D the (l-Sylow of the) torsion subgroup of A(k) has order smaller than N_{(l),g,D}.

The full proof of Conj (1,D) was achieved in 1996 after works of B. Mazur [M77], D. Abramovic, L. Merel [Me96] and others. An effective form of this result is given in [P99]. Similarly, conj (l,d,D) is the natural generalization of Serre's uniform image theorem [S68]. Actually, Serre showed that for any elliptic curve E over \Q without CM, G=GL_{2}(Z_l) for l larger than a bound say C_E depending *a priori* on E. He asked whether this bound C_E could be taken independently of E. This question is still partly open but one of the two remaining cases was recently proved in [BiP09a], [BiP09b]. Both problems (uniform boundedness of the (l-primary) torsion and uniform openness of the Galois image) are widely open for g>1.

In [CT09c] and [CT09d], Conj (l,d,D) is proved for d=1, D=1 and d=1, D arbitrary respectively. In particular, the l-primary torsion conjecture (g,D) holds for any family of g-dimensional abelian varieties parametrized by curves. This is a striking result in favour of the l-primary torsion conjecture and the uniform openness conjecture for higher dimensional abelian varieties. This result was extended to stacks in [C09a]. The technics developped in [CT09d] actually yield an unconditionnal variant of Conj (l,1,D). More precisely, for any l-adic representation, the set X_{r,l,D,>2} of all closed points x in X with residue field of degree less than D over k and such that G_x has codimension at least 3 in G is finite.

Question 1: Is the union of all X_{r,l,D,>2}, D>0 finite?

Concerning the set of points where G_x has codimension at least 1 (even in the GSRP case) or 2 in G, one has counter-examples to this question [CT09c], [CT09d]. But the generalization of these counter-examples to the codimension at least 3 case fail due to deep arithmetic results such as **Manin-Mumford conjecture** for semi-abelian varieties or some proved cases of **Andre-Oort conjecture**. So either constructing such a counter-example or formulating the general arithmetic conjecture that would answer positively question 1 is also one of our purposes. A positive answer to this question might lead to a Hodge theoretic formulation of Conj (l,d,D).

Question 2: For representations of type 1, are the special loci X_{r,l,D}, X_{r,l,D,>2} independent of l?

This question is closely related to the theory of (pure) **motives**. The **standard conjectures** (or, more precisely, the **Mumford-Tate conjecture** for motives) would yield a positive answer to it.

## 2.2 Objectifs et caractère ambitieux/novateur du projet / Rationale highlighting the originality and novelty of the proposal

1-

We sketch four types of questions that we would like to explore concerning fundamental groups.

1- 1 Stacks and wild ramification:

Until now, stacks have been used to absorb tame ramification along a normal crossing divisor. The general recipe is the following: if one starts from a Galois cover Y→X of group G, tamely ramified along D, it gives rise to an étale cover Y→[Y/G], where [Y/G] is the quotient stack, that has the usual schematic quotient Y/G=X as a moduli space. Abhyankar's Lemma then says that one can reconstruct [Y/G] only from X,D, and the ramification indices, it is a stack of roots (Vistoli). These stacks are nicely described in terms of the toric stack [A^1/G_m], quotient of the affine line by the multiplicative group, that has a convenient moduli interpretation as the stack classifying line bundles endowed with sections. When one considers wild ramification, the situation is much more involved, even in the case of a wildly ramified cover of curves. One reasonable aim is to describe the quotient stack [Y/G] in this situation. In simple cases, it seems that one can replace the toric stack [A^1/G_m] by the stack [P^1/G_a] quotient of the projective line by the additive group. This stack has also a nice moduli interpretation, although more complicated (Andreatta-Gasbarri). Since the fundamental group of affine curves in positive characteristic is very large, it is unlikely that one can construct an analog of the stack of roots that would encapsulate all types of wild ramification, but it is very plausible one can master part of it.

1-2 Fundamental group scheme of stacks:

Nori's definition of the fundamental group scheme is valid for a proper scheme X , reduced over a perfect field, equipped with a rational point. Its finite quotients classify torsors on X under finite group schemes endowed with a rational point lifting the given one. Since such torsors appear naturally when X is an Artin stack, it is a very natural question to ask if one also can define a fundamental group scheme in this setting. This was worked out in the very special case of the stack of roots in [Bo6]. Moreover, the largest abelian quotient of such a fundamental group scheme corresponds to the torsion of the Picard scheme of X, that was the object of study of [Br09a], [Br09b]. Among the applications of such a formalism, one would get a fundamental group scheme for the generalized stack of roots defined by Olsson in terms of log structures. This would extend the definition of the "tame fundamental group scheme" defined in [Bo6] for a scheme endowed with a simple normal crossings divisor, to a general normal crossings divisor.

1-3 Galois definition of the fundamental group gerb:

Esnault and Hai recently refined Nori's definition, avoiding the use of a rational point by using non neutral tannakian categories (Deligne). This formalism works for a smooth scheme X of finite type over a field of characteristic zero k and produces a gerb over the étale site of k. The main example is when X is a curve and k a number field, since this gerb is related to Grothendieck's section conjecture in anabelian geometry: this is the gerb of sections of the exact sequence linking the geometric fundamental group, the arithmetic fundamental group, and the absolute Galois group of k. It would be very useful, though, to find a more general construction of this fundamental group gerb, to make it work for any scheme X over any base S. One idea to do this is to replace the tannakian formalism by non abelian cohomology (Giraud) that also produces gerbs in a very general setting. More precisely, the fundamental group gerb would correspond to the extension of Galois toposes defined by X/S (Leroy, these Galois toposes are simply defined as the categories of local systems of sets for the étale topology). This definition, much more in the spirit of SGA1 , would also be more satisfying than the original one; one difficulty, however is that it would not prove that the fundamental group gerb is algebraic. The applications would be numerous; for instance, it is not even clear if one has by now a satisfying formalism for the fundamental group scheme when S is the spectrum of a discrete valuation ring.

1-4 Ramification and inseparability:

By degeneration to characteristic p of an étale or tamely ramified cover $f:Y\to X$, one cannot avoid inseparability to appear (examples are classical in the case of covers of smooth curves with monodromy group a p-group). If f is Galois with group G, then, as proven in [Ro09a], locally on X this inseparability is encoded in an effective model G'. This is a finite flat group scheme G' with a surjective morphism $G\to G'$. It is very desirable to study the action of G' and its relation with the usual invariants of Y and X ; however, the geometry and arithmetic of such finite flat group scheme actions has not received much attention from algebraic geometers (notable exceptions are a few papers by Serge Skryabin). For example, G' should be related to the fundamental group scheme $\pi$ of the degenerate fibre of X : examples in [Ro09a] show that it will not be a quotient of $\pi$ in general, but one can expect that there is a largest quotient of G' that is a quotient of $\pi$. Another feature of the action of G' that we would like to describe is its "ramification", which could ideally be measured by the stabilizers S of generic points in the degenerate fibre of Y and by a divisor of Y away from which $f:Y\to X$ is as close as can be to a G'-torsor (or better a G'/S-torsor).

## 2-

In this section, we only consider representation of type (1).

The main idea underlying Cadoret Tamagawa's approach is to consider l-adic Galois representations in families, which can be done thanks to the specialization isomorphisms of l-adic cohomology. More precisely, if $Y\to X$ is a smooth proper and geometrically connected morphism of integral schemes and if gen denotes the generic point of X then, for any closed point x in X, there are canonical isomorphisms (called specialization isomorphisms):

$$H^i(X_{\bar{gen}},\Z_l)\to H^i(X_{\bar{x}},\Z_l),\ i>=0,$$

which are compatible with the Galois actions of $\Pi(X)$ and $G_{k(x)}$ (viewed as a subgroup of $\Pi(X)$ *via* the section $s_x$) . Hence, considering the family of l-adic representations $r_{i,x}:G_{k(x)}\to GL(H^i(X_{\bar{gen}},\Z_l))$, x in X associated with $r_i:\Pi(X)\to GL(H^i(X_{\bar{gen}},\Z_l))$ is exactly the same as considering the family $G_{k(x)}\to GL(H^i(X_{\bar{x}},\Z_l))$. However, this new point of view allows to use specific étale fundamental group technics and, in particular, to translate information of number theoretic nature - the size of the Galois images - in terms of information of arithmetico-geometric nature – the existence of rational points on certain finite étale covers $X_{n}$ of X (depending on the problem considered) (see [CT08], [CT09b], [CT09c] and [CT09d] for more details). As already mentioned, the method culminated in the proof of Conj (l,d,D) [CT09d]. A key ingredient in this proof in Falting's theorem (Mordell conjecture).

2-1 Higher dimensional base schemes X:

To extend those kind of results to higher dimensional base schemes X, a key seems to study the growth of the Kodaira dimension along the projective systems of varieties $X_{n+1}\to X_{n}$ (with the notation of the preceding paragraph). We conjecture that for n large enough all the possible $X_{n}$ are of general type. Assuming this, the weak **Lang conjecture** (and an easy induction argument) would show that Conj (l,d,1) holds and hence, in particular, the l-primary torsion conjecture for (g,1). This would thus relate two of the main classical conjectures of arithmetic geometry: Lang conjecture and the l-primary torsion conjecture. Some partial results have been obtained for surfaces in [C09b]. The proofs here rely heavily on the classification of surfaces ; a main problem is to find arguments working in all dimensions. Another option would be to try and find a proof of [CT09c] which does not resort to Mordell conjecture bu t exploit more deeply the projective system structure $X_{n+1}\to X_{n}$. Such an alternative proof might hopefully be generalized unconditionnally to higher dimensional base schemes X.

2-2 Mod l representations:

As already mentioned, conjecture (l,d,D) is related to the l-primary torsion conjecture for abelian varieties. To tackle the full torsion conjecture, the natural objects to study are the families of mod l representations, l:prime, of $\Pi(X)$ acting on the l-torsion of the generic fiber of $A\to X$. To each non trivial l torsion point v of the generic fiber of $A\to X$, one can associate a finite etale cover $X_v\to X$ (defined over a finite exnsion of k) with the property that v specializes to a k-rational torsion point of $A_x$ for some k-rational point x on X if and only if x lifts to a k-rational points on $X_v$. Thus, writing $X_l$ for the disjoint union of the $X_v$, the main conjecture is:

**Conj (d,D)**: Assume that X has dimension d. Then for any family of mod l representations, l:prime, raising from an abelian scheme $A\to X$ whose generic fiber contains no isotrivial subvariety and for any integer D, $X_{l}(K)$ is empty for any field extension K of k of degree less than D.

This conjecture is currently out of reach even for the case (d,D)=(1,1). Let us however mention a more accessible one, which would follow from the torsion conjecture over function fields:

**Conj (genus)**: Assume that X is a curve and, for each prime l, let g(l) denote the minimal genus of the X_v. Then g(l) goes to infinity with l.

From Mordell conjecture (Faltings' theorem), this would at least show that X_l(k) is finite for l large enough. But this is only an evidence for the (d,D)=(1,1) case of Conj (d,D). In [CT09d], Conj (genus) is proved when X has genus larger than 1 or genus 0 and A→ X has semi-stable reduction over all but possibly one points. We aim at completing this proof (i.e. remove the semi-stable assumption on A→X when X has genus 0).

Proving the emptiness of the X_l(k) for l large enough motivates should rely on a deep understanding of the arithmetic of the "modular" curves X_v, in particular, interpretating arithmetico-geometrically their moduli properties (is there a purely functorial analog of the theory of Hecke operators?) or constructing nice arithmetic models of them with a moduli interpretation.

2-3 Other themes of interest are:

- The application of Cadoret-Tamagawa's results to controlling the variation of arithmetic data encoded in l-adic cohomology such as motivized motivic Galois groups or Néron-Severi rank.
- The extension of Cadoret-Tamagawa's method to stacks. In particular, understand better the action of the étale fundamental group of stacks on l-adic-cohomology of stacks.
- The extension of their results to finitely generated base fields of positive characteristic.

3-

In this section, we are more particularly interested in bounding uniformely the torsion subgroup in families of abelian varieties over number fields that is Conj torsion (g,1). The theory of heights is particularly adapted to thi purpose. For instance, the proof of Mordell-Weil theorem, which implies the finiteness of the torsion subgroup of an Abelian variety over a number field relies basically on this theory. The current proof of the existence of an explicit and uniform bound for the order of the torsion subgroup of elliptic curves over number field resorts to other technics (formal immersion, winding quotient of the modular jacobian etc.). However, for some subfamilies of elliptic curves (in which the bad reduction is controlled via Szpiro quotient), heights provide a quicker proof for the existence of a uniform and explicit bound and the bound provided is much better than the general one [HS01], [P04].

For higher dimensional abelian varieties, height-theoretic technics can provide explicit and uniform bounds for the torsion subgroup in families of abelian varieties over number fields provided the bad reduction of the abelian varieties in the family is controlled. Explicit examples of this kind of bounds can be found in [Mas92] or [Da93]. See also [Pa08] for an exhaustive study in the case of abelian surfaces.

Uniform bounds for the torsion subgroup are usually obtained as corollaries of statements about the minoration of the Neron-Tate height of the Abelian variety. The uniformity for the torsion subgroup in a family of Abelian varieties follows from the uniformity for the minoration of the Neron-Tate heigt in the family. This is a main motivation to try and control the theory of heights on moduli stacks for abelian varieties. In this frame, the key conjecture is the so called "Lang-Silverman conjecture", which predicts that the Neron-tate height of an abelian variety is bounded from below by its Faltings' height. In proving this conjecture, comparing Faltings' height with the Theta height plays a crucial part. The Theta height is the canonical height on moduli space - here the moduli space of pincipally polarized Abelian varieties (possibly with level structures to rigidify). Works of J.-B. Bost and S. David (letter to Masser and Wustholtz, 1995), investigated in more details by F. Pazuki (2009), provide explicit comparison between those heights.

Understanding better the theory of heights on moduli spaces of Abelian varieties should help develop the required tools to tackle the Lang-Silverman conjecture, which, in turn, would provide uniform boundedness statements for the torsion subgroup of Abelian varieties over number fields.

# 3. Programme scientifique et technique, organisation du projet / Scientific and technical programme, project management

## 3.1 Programme scientifique et structuration du projet / Scientific programme, specific aims of the proposal

### 1- Introduction:

Our aim is to gather around a common theme – the variation of arithmetic properties in families of curves and abelian schemes – young researchers belonging to different but complementary streams of arithmetic geometry.

We intend to exploit the diversity and complementarity of the participants' culture to extend each participant's knowledge. We hope that this sharing will help each participant adquire new skills and apply them to his own field of research in an appropriate way.

As a result, our project will emphasize short (1-to-3 days) workshops devoted to present in details several tools and technics. The themes selected below have been chosen after discussing with all the participants and according to their research interests (see section 2). The scientific skills and interests of each participants (see section 5.3 and annex 7.2 for more details) have determined the attribution of the organizational tasks. The aims of these workshops are, on the one hand, to introduce new tools and technics to participants of the project that are not familiar with them but might need them for their own research purposes and, on the other hand, help participants who are familiar with them to tackle further questions concerning those tools and their applications. Concerning this latter aspect, the invitation of external experts will play an important part.

Aside from these short workshops, we plan to organize:
- An introductory meeting (2 days);
- A closing conference (5 days);

Note that all the participants involved in the project are expected to attend all the organizational events (opening meeting, research workshops and closing conference).

Eventually, in the same spirit, we plan to fund the participants of the research project to attend research conferences or schools related to the project (in complement to other sources).

## 2- Tasks (Research workshops):

Our definition of a task is the organization of a series of workshop in the frame of each of the three main orientations:

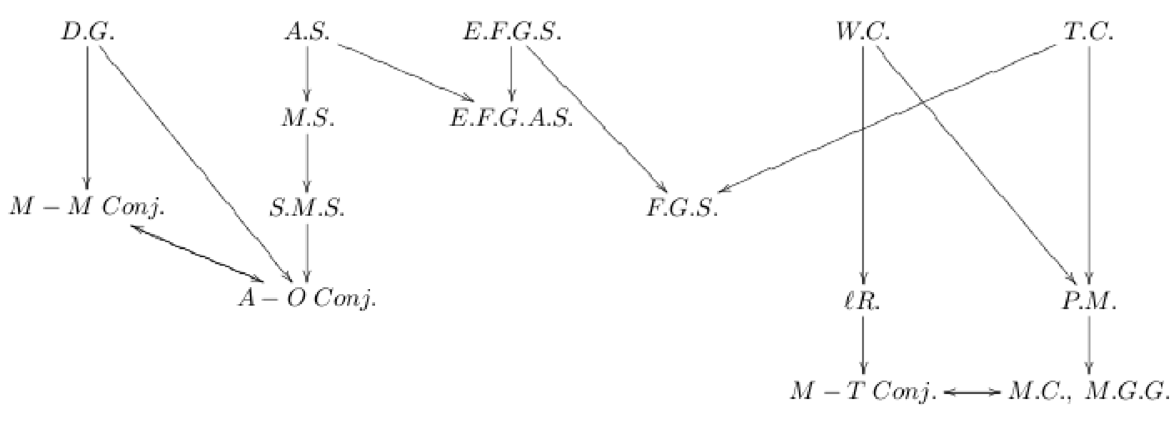
1- Fundamental groups and algebraic stacks;
2- l-adic representations of étale fundamental groups;
3- Diophantine geometry.

As already mentioned, half of the talks will be given by researchers involved in the project, as in a working group, whereas the other half will be given by specialists of the topics. This choice depends on whether the lectures are aimed at learning basics (in which case, we will privilegiate the “working group spirit” and most of the lectures will be given by participants to the research project) or more advanced topics (in which case, we will privilegiate the “workshop spirit” and most of the lectures will be given by external experts).

The organizers were designated according to their scientific skills. Their role will be to prepare the scientific program of the workshops they are in charge with, that is select the speakers and attribute them a precise content for their talk.

The coordination of these meetings obeys the scientifical coherence summarized in the following leitfaden.

**LEITFADEN**




A.S.: Algebraic stacks;
E.F.G.S.: Etale fundamental group of schemes;

E.F.G.A.S.: Etale fundamental group of algebraic stacks;
D.G.: Basics of diophantine geometry (height theories);
M.S.: Moduli spaces and moduli schemes;
S.M.S.: specific moduli schemes (Siegel, Hilbert, Shimura \textit{etc.});
T.C.: Tannakian categories;
P.M.: Pure motives (construction, standard connjetures, Hodge and Tate conjecture);
M.C., M.G.G.: Motivized cycles and motivic Galois groups;
M-T Conj.: Mumford-Tate conjecture;
F.G.S.: Fundamental group scheme;
W.C.: Weil cohomologies;
l- R.: $\ell$-adic representations;
M-M Conj.: Manin-Mumford conjecture;
A-O Conj.: Andr\'e-Oort conjecture.

Practical organizational aspects will be rather light since, though these workshops will be open to public, the audience should remain within 15-20 participants. Depending on where these workshops are organized (Bordeaux or Paris), the local administrative staff and participants to the project will be expected to reserve a conference room, accomodations etc.

We give more details about the three tasks, in particular, the length, organizers, topics and suggested external experts for the workshops organized in the frame of each task in section 3.3.

3- Opening meeting and closing conference:

Opening meeting:

Length: 2 days.
Organizers: Cadoret, Romagny.

Aim: The aim of this meeting is to gather all the participants of the project in order to give them the opportunity to present their own research interests and the connections of these with the themes of the project; in particular why they expect that some new insights may arise from applying the tools we plan to study to their own problems.

Closing conference:

Length: 5 days.
Organizers: Cadoret, Romagny.

Aim: The aim of this closing conference is to present some of the main achievements obtained during the duration of the project (2010-2013). It should gather both the participants of the project, some of the previously invited external experts and researchers (both French and foreign) having obtained striking results in the area of the project.

## 3.2 Coordination du projet / Project management

The project will be structured as follows:

1- Opening meeting;
2- 1-to-3 days research workshops;
3- Closing conference.

For the aims and scientific purposes of 1-, 2- and 3- see section 3.1 above. We only summarize here the organizational aspects of these events:

1- Opening meeting:

Organizers: Cadoret, Romagny.

Place: I.M.B. - Univ. Bordeaux 1.
Date: September 2010.

Organizational description: This meeting will start with a short introduction presenting the project in its globality. Each participants will then be given a one hour slot to describe his motivation and expectations (5 talks/day) . We also expect interesting informal discussions.

Audience: This event is the only one which we do not intend to open to public since we would like to preivilegiate informal discussions between the members of the project.

2- 1-to-3 days research workshops:

Organizers: The scientific skills of each participants (see section 5.3 and annex 7.2 for more details) have determined the attribution of the organizational tasks.

Places and dates: The dates given in section 3.4 are only approximative since they may depend on the disponibility of the participants and expected external experts *etc.* We tried and avoid holydays periods (July, August, December) as well as period with heavy teaching loads (february, march, october, november). The coordination of these meetings, however, is fixed and obeys the scientifical coherence summarized in the leitfaden. Also, we occasionally gathered together, for instance, a 1 day and 2 day workshops in a 3 days workshops in order to avoid too many meetings per year. The places are not fixed yet but, averagely, half of the workshop will be hold in Bordeaux and the other half in Paris.

Audience: These workshops will be opened to the public (see section 4).

3- Closing conference:

Organizers: Cadoret, Romagny.

Place: Paris (I.H.P.?).
Date: Late September 2013.

Organizational description: This meeting should be a standard research conference with one hour talks, 4 to 5 talks/day. We expect tha t half of the talks will be given by the participants to the project in order to present new results they obtained during the four years of the project. The other half of the talks will be given by researchers having obtained striking results on the variation of arithmetic properties in families.

Audience: We expect an audience of 50-70 participants. In particular, we would like to encourage young participants to attend this conference and will provide partial funding to support them (see section 6.2)

## 3.3 Description des travaux par tâche / Detailed description of the work organised by tasks

We give here more details about the workshops organized in the frame of each of the three main tasks. For global articulation between those workshops, see the leitfaden and section 3.4.

### 3.3.1. Tâche 1 / Task 1 Fundamental groups and algebraic stacks (Borne, Brochard, Cadoret, Maugeais, Romagny, Zapponi)

Meeting 1:

Length: 3 days.
Organizers: Borne, Cadoret, Romagny, Zapponi.
Topics:
- Topological and étale fundamental groups, general theory (SGA1);
- Weil cohomologies 1: definition and comparison of classical Weil cohomologies (Betti, de Rham and l-adic);
- Comparison of various actions of fundamental groups (locally constant system, étale and l-adic cohomology);
- Computations and examples (rationally connected varieties, curves and Abhyankar's conjecture).

Suggested external experts: None.

Meeting 2:

Length: 3 days.
Organizers: Borne, Brochard, Romagny.
Topics:
- Descent theory;

- Algebraic spaces and stacks, coarse moduli spaces;
- Examples.

Suggested external experts: None.

Meeting 3:

Length: 3 days.
Organizers: Maugeais, Brochard, Romagny.
Topics:
- Other fundamental groups : fundamental group scheme, motivic / tempered fundamental group;
- Fundamental group of stacks;
- Recent results on the fundamental group scheme.

Suggested external experts: H. Esnault, Y. André or E. Lepage.

Meeting 4:

Length: 2 days.
Organizers: Maugeais, Romagny.
Topics:
- Ramification theories;
- Inseparability.

Suggested external experts: A. Abbes.

### 3.3.2. Tâche 2 / Task 2 L-adic representations (Borne, Cadoret, Pépin, Ratazzi):

Meeting 1:

Length: 3 days.
Organizers: Cadoret, Pépin.
Topics: Information encoded in l-adic cohomology:
- Basics of the theory of Lie groups and their Lie algebras;
- Weil cohomologies 2: general formalism, connexion with cycles;
- Semi-stable reduction theorems;
- Motivations, in particular, uniform boundedness results: torsion of abelian varieties, Galois image on the l-adic cohomology, « Motivized » motivic Galois group, Néron-Severi ranks etc. (overview of results already achieved and expected ones. See also meeting 4).

Suggested external experts: None.

Meeting 2:

Length: 2 days.
Organizers: Cadoret, Pépin.
Topics: Weil conjectures;
Suggested external experts: L. Illusie.

Meeting 3:

Length: 3 days.
Organizers: Borne, Cadoret.
Topics:
- Tannakian categories;
- Pure motives;
- Standard conjectures, Hodge and Tate conjecture;
- Motivic Galois groups and Mumford-Tate Groups;
- André's theory of motivized cycles;
- Deligne fix part theorem and application to l-adic cohomology (variation of « motivized » motivic Galois groups, rank of motivized cycles etc.)

Suggested external experts: Y. André, B. Kahn.

Meeting 4:

Length: 1 day.
Organizers: Ratazzi.
Topics: Mumford-Tate conjecture for abelian varieties – known cases.
Suggested external experts: R. Noot, R. Pink.

### 3.3.3. Tâche 3 / Task 3 Diophantine geometry (Parent, Pazuki, Ratazzi):

Meeting 1:

Length: 2 days.
Organizers: Pazuki.
Topics: Introduction to height theory and comparison between the different notion of heights.
Suggested external experts: S. David, J.-B. Bost, C. Soulé or G. Rémond.

Meeting 2:

Length: 1 days.
Organizers: Ratazzi.
Topics: How Lang-Silverman conjecture implies uniformity results in arithmetic geometry.
Suggested external experts: S. David, M. Hindry.

Meeting 3:

Length: 2 days.
Organizers: Parent.
Topics:
– Modular and Shimura Curves;
– André-Oort and Manin-Mumford conjecture.
Suggested external experts: A. Yafaev.

Meeting 4:

Length: 3 days.
Organizers: Pazuki.
Topics:
– André-Oort conjecture in connexion with dynamics on abelian varieties and Shimura curves;
– Extension of the theory of height to the case of varieties endowed with a polarizable morphism.
Suggested external experts: D. Ghioca, T. Tucker, E. Ullmo.

## 3.4 Calendrier des tâches, livrables et jalons / Planning of tasks, deliverables and milestones

| year | | | | |
|---|---|---|---|---|
| 1 | Opening Meeting - Late-September | | | |
| 2 | Task 2, Meeting 1 – early January | Task 1, Meeting 1 – mid-May | Task 3, Meeting 1+ Task 3, Meeting 2 – Late September. | |
| 3 | Task 1, Meeting 2 – early January | Task 2, Meeting 2+Task 3, Meeting 3 – mid-May | Task 2, Meeting 3 – Late September. | |
| 4 | Task 1, Meeting 3 – early January | Task 3, Meeting 4 - mid-May | Task 1, Meeting 4+Task 2, Meeting 4 - late September | Closing conference - Mid-December |

# 4. Stratégie de valorisation des résultats et mode de protection et d'exploitation des résultats / Data management, data sharing, intellectual property and results exploitation

Concerning data management and data sharing, we will distinguish between the personal scientific production of each researcher of the team and common production raising from organizational events such as workshops or conferences. In order to centralize the information, we propose that a webpage devoted to the project be created. On this webpage, the following data should be posted:

1. Synthetical presentation of the project, its goals, informations about the recent or forecoming events related to the project etc. (basically, the content of the scientific document B).
2. List of the participants with links to their personal webpages;
3. Concerning personal production:

(1) A list of title and abstracts of articles published by the researchers involved in the project and with, when available, links to electronic versions.
(2) A list of preprints related to the project. Posting preprints seems important to us since they reflect the most recent advancements. However, to avoid problems related to the validity of such free online posting, we will adopt the following policy:
(i) Preprints should be sent to the webmaster (no free access to an online posting serveur) and could only be posted by him;
(ii) Concerning preprints by researchers who are not directly involved in the project, they should be submitted through one of the researcher involved in the project. This researcher will be in charge of checking the mathematical correctness of the text before sending it to the webmaster (the mention of the referee and date of submission will be posted together with the preprint).
(iii) If a posted preprint appeared to be incorrect even after the check of (ii), it will be immediately deleted from the list of posted preprints. Actualized versions will be accepted through the process of (ii).

4. Concerning organizational events:

(1) Schedule of the events planned (workshops and conferences organized in the frame of the project but also national and international events related to the project);
(2) For each event organized in the frame of the project:
(i) Before the event: schedule, title and abstract of talk, participants, lecture notes, practical information;
(ii) After the event: short report synthetizing the achievements discussed in the workshop;
(iii) Also, for the conference we plan to organize during the fourth year of the project, we aim at publishing proceedings describing the main achievements of the projects.

The three days workshops and the final conference will be all open to the public. A mailing list will be created to let the researchers that might be interested in these events.

Also, part of the budget will be used as a complement to other sources (laboratories, other projects or institutes, foreign grants *etc.*) to support the participation to international conferences or special semesters. We will support in priority missions which involve communications (research talks in conferences, advanced courses *etc.*)

# 5. Organisation du projet / Proposal organisation

## 5.1. Description, adéquation et complémentarité des participants / Relevance and complementarity of the partners within the consortium

Our aim is to gather around a common theme – the variation of arithmetic properties in families of curves and abelian schemes – young researchers belonging to different but complementarity streams of arithmetic geometry such as:

- Deformation theory (Maugeais, Romagny, Zapponi);
- General theory of stacks (Brochard, Maugeais, Romagny);
- Rational points on moduli spaces (modular curves, Shimura varieties, Hurwitz spaces *etc.*) (Cadoret, Parent, Pazuki);
- Etale fundamental groups of schemes, tannakian groups (Borne, Cadoret);
- l-adic Galois representations (Cadoret, Pépin, Ratazzi);
- Diophantine geometry (Pazuki, Ratazzi).

The scientific skills of each participants (see section 5.3 and annex 7.2 for more details) have determined the attribution of the tasks (see section 3.3). We refer to sections 5.2 and 7.2 for more details about the specific skills of each participants.

In constructing our team, diversity of skills and cultures was an important factor, but we also took care of avoiding isolated researchers and, even, for each orientation, we tried and gather several young experts that already know each other quite well. Also, we believe it's important to have researchers belonging to several cultures in order to make the connections between them. For instance,

- For connection between 1 and 2: N. Borne knows about l-adic cohomology and representations whereas A. Cadoret is familiar with the theory of étale fundamental group (she is teaching M2 lectures on this topics this year), Tannakian categories and algebraic stacks.
- For connection between 1 and 3: P. Parent has been working for years on specific moduli schemes such as modular and Shimura curves.
- For connection between 2 and 3: N. Ratazzi's knowledge of Galois representations over number fields might be useful to sudy those over function fields. Also, he and F. Pazuki are expert in Abelian varieties and their knowledge should be useful to understand better what is at stake in the study of l-adic representations of the fundamental group over the generic l-adic Tate module of an abelian scheme.

We also tried and mix both young researchers (S. Brochard, F. Pazuki, C. Pépin) and more experienced ones with national and international established collaborations and experience in organizing international conferences (see section 7.2 for more details).

## 5.2. Qualification du porteur du projet / Qualification of the principal investigator

A. Cadoret, maître de conférences at I.M.B. - Univ. Bordeaux 1, was designated to be the principal investigator of the project both for technical and scientifical reasons:

Technical: 4/10 members of the team are administratively attached to I.M.B. (whereas 2/10 are attached to I.M.J. - Univ. Paris 6/7, 1/10 to Univ. Montpellier 2, 1/11 to Univ. Le Mans, 1/10 to Univ. Paris 11) so it is important that the main investigator be attached to I.M.B. as well. Also, A. Cadoret has adquired some administrative experience in organizing the Number Theory Seminar in Bordeaux since September 2007 and coordinating a previous joint French-Japanese research project, the P.H.C. Sakura « *Torsion of abelian schemes and rational points on moduli spaces »,* with A. Tamagawa from R.I.M.S. - Kyoto Univ. Also, together with A. Tamagawa, she has organized an international workshop on the same theme at I.M.B. on January 25th-29$^{th}$, 2010. See: http://www.math.u-bordeaux1.fr/~cadoret/SakuraBis.html . She is also responsible, together with A. Tamagawa, for the submission of a joint research project CNRS/JSPS « Représentations of étale fundamental groups and applications ».

Scientific: A. Cadoret's scientifical background is at the intersection of several main orientations of the research project. After studying the arithmetic of moduli stacks for G-covers and G-curves during her Ph.D. (see [C05a], [C05b], [C08a], [C08b], [CD09], [CT09a]), she has turned, in tight collaboration with A. Tamagawa, to the study of l-adic representations of the etale fundamental groups with applications to the arithmetic of abelian schemes and, more generally, to l-adic cohomology (see [CT08], [C09a], [C09b], [CT09b], [CT09c], [CT09d], [CT09e]). Now, she keeps on investigating this track and extending the technics she developped with A. Tamagawa to the motivic frame. In particular, using the main result of [CT09d], she could improve previous results of André (variation of motivized motivic Galois groups), Maulik-Poonen-Voisin (variation of the rank of Néron-Severi groups and, more generally, motivized motivic groups). She is also trying to extend their method to stacks (see [C09b]).

At the international level, A. Cadoret was recently invited to give talks in several conferences (Workshop « Anabelian Geometry », August 2009 - Isaac Newton Institute, Cambridge (U.K.), workshop « Higher dimensional algebraic geometry », December 2009 - R.I.M.S., Kyoto (Japan), Workshop Sakura « Torsion of abelian schemes and rational points on moduli spaces », January 2010 - Univ. Bordeaux 1, workshop « PIA2010 », February 2010 – Heidelberg (Germany), Conference « Arithmetic and differential Galois groups », April 2010 – C.I.R.M., Luminy etc). She also visits regularly R.I.M.S. (Kyoto), she visited T.I.F.R. - Mumbai (india) (November 2008) in the frame of the special semester on « p-adic geometry » and Isaac Newton Institute – Cambridge (U.K.) (October 2009) in the frame of the special semester on « Non abelian fundamental groups in arithmetic geometry ». In spetember 2010, she will visit Morningside Center of Mathematics – Beijin (China). These invitations gave her the opportunity to develop a network of scientifical relashionships that should be helpful in such a project as the one under submission.

### 5.3 Qualification, rôle et implication des participants / Contribution and qualification of each project participant

| | Nom | Prénom | Emploi actuel | Unité de rattachement et Lieu | Discipline | Personne. mois | Rôle/Responsabilité dans le projet 4 lignes max |
|---|---|---|---|---|---|---|---|
| Coordinateur | CADORET | Anna | Maître de Conférences | Univ. Bordeaux 1 | Mathématiques | 36 | Etale fundamental groups (of schemes), l-adic representations, abelian schemes, algebraic stacks (for curves and abelian varieties). |
| Autres membres | BORNE | Niels | Maître de Conférences | Univ. Lille 1 | Mathématiques | 38 | Etale fundamental groups (of schemes and stacks), fundamental groups schemes. |
| | BROCHARD | Sylvain | Maître de Conférences | Univ. Montpellier 2 | Mathématiques | 38 | Algebraic stacks, Picard scheme (and Picard stack), flatness on Artin rings. |
| | MAUGEAIS | Sylvain | Maître de Conférences | Univ. Le Mans | Mathématiques | 38 | Covers of curves, deformation, moduli spaces, etale cohomolohy. |
| | PARENT | Pierre | Maître de Conférences | Univ. Bordeaux 1 | Mathématiques | 14 | Arithmetic of modular forms: modular curves and Shimura curves. |
| | PAZUKI | Fabien | Maître de Conférences | Univ. Bordeaux 1 | Mathématiques | 24 | Heights, rational points, torsion points, moduli spaces for abelian varieties. |
| | PEPIN | Cédric | Doctorant | Univ. Bordeaux 1 | Mathématiques | 48 | Jacobian of algebraic curves, Néron models. |
| | RATAZZI | Nicolas | Maître de Conférences | Univ. Paris 11 | Mathématiques | 18 | Abelian varieties, l-adic representations, Galois representations, bound of the torsion, Neron-Tate height, Mumford-Tate group. |
| | ROMAGNY | Matthieu | Maître de Conférences | Univ. Paris 6 | Mathématiques | 38 | Moduli of algebraic curves, algebraic stacks, reduction, group schemes. |
| | ZAPPONI | Leonardo | Maître de Conférences | Univ. Paris 7 | Mathématiques | 38 | Covers of curves, Dessins d'enfants, moduli spaces of curves, lifting and reduction, fields of definition. |

## 6. Justification scientifique des moyens demandés / Scientific justification of requested budget

The only requested support will be for :

– Sabbatical.

– Missions/Invitations.

### 6.1 Sabbaticals:

As all the researchers (except Cédric Pépin, who is a Ph. D. Student) involved in the project under submission are Maîtres de Conférences hence have to devote a large part of their time to teaching, we intend to ask for one sabbatical (half service= 96h equivTD) a year. Hence 4x10= **40Keuros**. They will be attributed according to the respective teaching loads (and proportionally to the involvement of each researcher in the project).

### 6.2 Missions/Invitations:

We divide missions/invitations according to whether or not they are directly related to the events organized in the frame of the project (opening meeting, 3 days research workshops, closing conference).

The estimated costs of  missions/invitations were estimated as follows:
- Per diem*: 100 euros/day;
- Travel**: 200 euros (inside France), 500 euros (from abroad).

*Living expenses (accomodation + meals).
** Includes connexion to/from airport or railway stations (taxi, shuttles, metros *etc.*).

For these computations, one could consider whether the workshop is expected to be held in Bordeaux (funding for 6 participants) or Paris (funding for 7 participants). However, since, as already mentioned, the places (and dates) of the planned research workshops might be changed due to organizational or scientific factors (such as adequation with other scientific events), the computations below are based on a number of 7 funded participants.

For the closing conference, we expect an average audience of 50-70 persons. We would like to encourage young researchers (PhD and Post-Doc) to attend it. As a result, we plan to fund 10 of them up to 600 euros.

<u>1- Missions/Invitations related to the workshops (except closing conference):</u>

These computations rely on section 3.3:

Number of meetings: 10;
Number of days: 9x3+2=29;
Number of  French external experts:14;
Number of foreign external experts:5;
Number of local travels: 14+10x7= 84;
Number of travels from abroad: 5;
Total Cost: 100(29x7+3x14+5)+200x84+500x5=**45,3KEuros**.

<u>2- Closing conference:</u>

Number of days: 6;
Number of funded participants (participants to the ANR project+external experts): 17;
Number of local travels: 11;
Number of travels from abroad: 5;
Total Cost: 100x6x17+200x11+500x5=15,1KEuros.
Additional expenses: 2KEuros;
Additional funding for young participants:10x600=6KEuros

The total estimated cost for the closing conference is thus:. 15,1+2+6=**23,1 KEuros**.

<u>3- Missions not directly related to the  workshops:</u>

Aside from the Missions/Invitations related to the project, an average amount of 3000 euros/4years will be attributed to each participant to attend conferences or schools related to the project (as a complement to other fundings such as laboratories, other research projects *etc.*) or, possibly, to buy books or computer goods. Hence **30 Keuros**.

As a result, the total requested amount is: 40+45,3+23,1+30=**138,4 KEuros.**

# 7. Annexes

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## 7.2. Biographies / CV, Resume

### 7.2.1. Borne Niels:

39 years. H.D.R. Univ. Lille 1 (2008).
Post-Doc :

- 2000-2001 Regensburg (U.Jannsen) ;
- 2001-2003 Bologna (A.Vistoli).

Current position: Maître de conférences, Univ. Lille1.
Current academic/scientific responsibilities: Member of the hiring committee of Univ. Lille 1.
Workshops, conferences etc. :

- Co-organizer of GTEM Workshop « Arithmetic of Curves and Covers », November 21-23, 2007 – Lille (France) (http://math.univ-lille1.fr/~gtem/);
- Co-organizer of GTEM Workshop on log-geometry, June 2010 – Bordeaux (France).

Publications: [Bo1], [Bo2], [Bo3], [Bo4], [Bo5], [Bo6], [BE] (number of publications: 7).
Collaborators: M. Emsalem, A. Vistoli.
Details: In his Phd Thesis, N. Borne described the equivariant K_0 of a projective curves in terms of cycles in

coefficients in representations. During his postdoc, he extended this formalism to make it more efficient in positive characteristic. In between, he used modular representation theory to solve embedding problems for the étale fundamental group of a projective curve over an algebraically closed field of positive characteristic, with kernel a p-group. He also studied similar problems in characteristic zero, and employed étale cohomology to give an algebraic construction of covers of an affine curve with solvable Galois group (j.w. M.Emsalem). Recently, he turned his attention to parabolic vector bundles, and used stacks of roots to define a tame fundamental group scheme for a projective variety endowed with a simple normal crossings divisor. Currently, N. Borne's main topic of interest is the étale fondamental group of schemes and of orbifolds, its (finite) representations, and the associated vector bundles. More specifically he is currently working on two projects. The first one (joint with A.Vistoli) aims to study the category of essentially finite parabolic vector bundles on a log scheme, and show it is in fact tannakian, to get a fundamental group scheme à la Nori in this context. The purpose of the second project (joint with M.Emsalem) is to get a better understanding of Esnault-Hai's work on packets in the section conjecture. In both cases algebraic stacks are an essential tool.

### 7.2.2. BROCHARD Sylvain:

30 years. Ph.D. Univ. Rennes 1 (2007).
Post-Doc : 2008-2009 Leiden Univ.
Current position: Maître de conférences, Univ. Montpellier 2.
Current academic/scientific responsibilities: Organizer of the "Algebraic Geometry, Algebraic Topology and Algebra" seminar in Montpellier 2 University.
Workshops, conferences etc. : Organizer of a workshop for young researchers about the algebraic fundamental group in spring 2007 in Rennes (France).
Publications: [Br09a], [Br09b], [BrM09] (number of publications: 2).
Collaborators: A. Mézard.
Details: In his PhD thesis, Sylvain Brochard studied the Picard functor and the Picard stack of an algebraic stack. He generalized to the framework of stacks a lot of results that were known for the Picard functor of a scheme (representability, smoothness, separation, finiteness properties...). He also computed explicitly the Picard scheme of a few common stacks (root stacks, smooth Abramovich-Vistoli curves, weighted projective spaces). Apart from this work, in collaboration with Ariane Mézard, he took an interest in a surprising conjecture formulated by Bart de Smit concerning flatness on Artin rings.

### 7.2.3. CADORET Anna:

32 years. Ph. D. Univ Lille 1 (2004).
Post-Doc : None.
Current position: Maître de conférences, Univ. Bordeaux1.
Current academic/scientific responsibilities:

- co-organiser of the Number theory seminar, I.M.B. – Univ. Bordeaux 1.
- Member of the committee for Agrégation externe de mathématiques.
- Main coordinator of P.H.C. Sakura « Torsion of abelian schemes and rational points on moduli spaces » (2008-2010).
- Member of the C.C.S.U. (ex C.S.) of Univ. Bordeaux1 (2010-).

Workshops, conferences etc. : Co-organizer of the Sakura workshop «torsion points of abelian schemes and rational points on moduli spaces », January 25-29, 2010 - Bordeaux (France) (http://www.math.u-bordeaux1.fr/~cadoret/SakuraBis.html).
Publications: [C05a], [C05b], [C08a], [C08b], [CT08], [CD09], [C09a], [C09b], [CT09a], [CT09b], [CT09c], [CT09d] (number of publications: 7).
Collaborators: P. Dèbes, A. Tamagawa.
Details: See section 5.2.

### 7.2.4. MAUGEAIS Sylvain:

33 years. Ph.D. Univ. Bordeaux 1 (2003).
Post-Doc: Univ. of Muenster (2003-2006).
Current position: Maître de conférences, Univ.du Maine.
Current academic/scientific responsibilities: None.
Workshops, conferences etc. : None.
Publications: [BMa05], [Ma03], [Ma06], [Ma09a], [Ma09b] (number of publications: 3).
Collaborators: J. Bertin.
Details: In his Ph.D. thesis, S. Maugeais studied the equivariant deformation of stables curves, focusing on the case where the action is trivial on a nonempty open subscheme. The results he obtained then gave some informations on the moduli space of G-equivariant curves for a p-cyclic groupe G, a particular attention being given to a compactification of the moduli space of hyperelliptic curves. He is now studying separatly the equivariant deformation of smooth curves (with a faithfull action) and the inseparable morphisms. His interests also include uniformisation in the p-adic setting and fundamental groups.

### 7.2.5. PARENT Pierre:

37 ans. H.D.R. Univ. Bordeaux 1 (2009).
Current position: Maître de conférences, I.M.B. Univ. bordeaux 1.
Post-Doc: None.
Current academic/scientific responsabilities: Member of the hiring committees of Univ. Bordeaux 1, Univ. Montpellier 2, Univ. Clermont-Ferrand.
Workshops, conferences etc. : Co-organiser of the Workshop « p-adic representations », April 24-26, 2006, Bordeaux (France) (http://www.math.u-bordeaux.fr/~parent/repPadiques2006.html).
Publications: [P99], [P00], [P03], [P03b], [P05], [PY07], [BiP08], [BiP09a], [BiP09b] (number of publications: 7).
Collaborators: Y. Bilu, A. Yafaev.
Details: P. Parent's research mainly concerns the arithmetic of modular and Shimura curves. He tries to combine techniques from arithmetic geometry (of relative curves and abelian schemes over discrete valuation rings) with tools borrowed from number theory (like special values of $L$-functions) and, more recently, methods of diophantine approximations, in order to study rational points.

### 7.2.6. PAZUKI Fabien:

29 years. Ph.D. Univ. Bordeaux 1 and Paris 7 (2008).
Post-Doc: None.
Current position: Maître de conférences, Univ. Bordeaux 1.
Current academic/scientific responsibilities: None.
Workshops, conferences etc. : None.
Publications: [Pa09a], [Pa09b],[Pa09c], [Pa09d], [Pa09e], [PaCo09] (number of publications: 2).
Collaborators: H. Cohen.
Details: F. Pazuki's research focuses on the arithmetic of abelian varieties. His PhD thesis deals with the Lang-Silverman conjecture, predicting a strong uniform inequality between the height of non-torsion rational points on an abelian variety and the Faltings' height of the variety itself. He is still active in the field, trying to generalize the results he obtained in dimension 2. He also published some work on a new explicit 3-descent technique on elliptic curves. More recently, he worked with D. Ghioca and T. Tucker and found general counter-examples for the dynamical Manin-Mumford conjecture.

### 7.2.7. PEPIN Cédric:

25 years. Ph.D. Student (Qing Liu).
Details: In SGA7 IX, Grothendieck formulates a conjecture concerning the behavior of the duality of abelian varieties at the level of their Néron models. It has been proved in different cases, such as in the semi-stable reduction case (Grothendieck), or the unequal characteristic case (Bégueri). The case of equal characteristic positive is still open. C. Pépin studies this problem for Jacobians, where the Néron model has been discribed by Raynaud in terms of the Picard functor of a regular model of the curve defining the Jacobian.

### 7.2.7. RATAZZI Nicolas:

32 years. Ph.D Univ. Paris 6 (2004).
Current position: Maître de conférences, Univ. Orsay – Paris 11.
Post-Doc: None.
Current academic/scientific responsibilities:

- Member of selection committee of I.M.B. – Univ. Bordeaux 1 and Insitut Camille Jordan – Univ. Lyon 1 (2009).
- Member of the C.C.S.U. (ex C.S.) of univ. Orsay – Paris 11 (2006-).
- Member of ANR Jeunes « Diophante » (2006-2010).

Workshops, conferences etc. :

- Co-organizer of the international conference «Développements récents en approximation diophantienne » at C.I.R.M. – Luminy (France) in October 2007 (http://math.univ-lyon1.fr/~adamczew/cirm07.html).
- In the frame of the ANR project « Diophante », organizer of a three days meeting on Zilber-Pink conjecture at Univ. Orsay – Paris 11 in April 2008 (http://people.math.jussieu.fr/~divizio/ANR/paris.html).

Publications: [R04a], [R04b], [R04c], [R07a], [R07b], [NR08], [R08], [ER09], [HR09a], [HR09b] (number of publications:8).
Collaborators: M. Hindry, M. Nakamaye, E. Ullmo.

### 7.2.8. ROMAGNY Matthieu:

33 years. Ph.D. I.J.F. Univ. Grenoble 1 (2002).
Current position: Maître de conférences, Univ. Paris 6.
Post-Doc :
- 2003 Univ. Stockholm (T. Ekedahl) ;

- 2003-2004 M.P.I.M. - Bonn, Univ. Bonn (F. Pop).
Academic/scientific responsabilities: Member of the hiring committees of Univ. Paris 6, Univ. du Maine, Univ. Lille 1.
Workshops, conferences etc. :
- Co-organizer of the conference « Développements récents sur les courbes algébriques » in Versailles, April 3rd-4th 2007 (http://people.math.jussieu.fr/~romagny/alg_curves_2007/)
- Co-organizer of the conference « Geometry and Arithmetic of Moduli Spaces of Coverings » in Istanbul, June 9th-20th, 2008 (http://math.gsu.edu.tr/GAMSC/)
Publications: [Ro04], [Ro05], [RoWe06], [BRo08], [ChRo08], [Ro09a], [Ro09b] (number of publications: 4).
Collaborators: J. Bertin, P.-E. Chaput, D. Tossici, A. Mézard, D. Abramovich.
Details: In his Ph.D. thesis, M. Romagny devised various tools based on algebraic stack techniques in order to study moduli spaces. He applied this in particular to moduli spaces of covers of curves (Hurwitz spaces). In collaboration with José Bertin, he wrote a comprehensive memoir on Hurwitz spaces at "good" characteristics, bringing together old and new results and making use of the power of algebraic stacks. More recently, he turned his attention to "bad" characteristics and studied the degeneration of the Galois group of an n-sheeted cover at a prime divisor of n. This is bearing fruit in a collaboration with Dan Abramovich, leading to a proper moduli space for p-cyclic covers. His interests also include the general theory of group schemes, their invariants and coinvariants.

### 7.2.9. ZAPPONI Leonardo:

40 years. Ph.D Université de Franche-Comté (1997).
Current position: Maître de conférencs, Université Pierre et Marie Curie (Paris 6).
Post-Doc :

- 20001-2004 E.P.F.L. (E. Bayer) ;
- 2000-2001 M.P.I.M – Bonn;
- 1999-2000 Several short-term invitations (Univ. Bonn, Univ. G. Mercator – Duisburg, M.S.R.I.).

Publications: [BouWZ], [LZ], [Z1], [Z2], [Z3], [Z4], [Z5], [Z6] (number of publications: 8)
Collaborators: I. Bouw, S. Wewers.

## 7.3. Implication des personnes dans d'autres contrats / Involvement of project particpants to other grants, contracts, etc...

| Part. | Nom de la personne participant au projet | Personne . mois | Intitulé de l'appel à projets<br>Source de financement<br>Montant attribué | Titre du projet | Nom du coordinateur | Date début &<br>Date fin |
|---|---|---|---|---|---|---|
| N°1 | Borne, Niels | | | | | |
| N°2 | Brochard, Sylvain | | Joint research program CNRS/JSPS<br>CNRS<br>15 000 euros/year (under request) | Representations of étale fundamental groups and applications | Cadoret, Anna | 01/01/2010<br>-<br>31/12/2011 |
| N°3 | Cadoret, Anna | | Joint research program CNRS/JSPS<br>CNRS<br>15 000 euros/year (under request) | Representations of étale fundamental groups and applications | Cadoret, Anna | 01/01/2010<br>-<br>31/12/2011 |
| N°4 | Maugeais, Sylvain | | | | | |
| N°5 | Parent, Pierre | 34 | ANR Blanc<br>ANR | Iwasawa | Anglès, Bruno | 01/01/2010<br>-<br>31/12/2013 |

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | under request | | | |
| | | | Joint research program CNRS/JSPS<br>CNRS<br>15 000 euros/year (under request) | Representations of étale fundamental groups and applications | Cadoret, Anna | 01/01/2010 - 31/12/2011 |
| N°6 | Pazuki, Fabien | 24 | ANR Blanc<br>ANR<br>under request | Hamot (Hauteurs, modularité, transcendance) | Bilu, Yuri | 01/01/2010 - 31/12/2013 |
| | | | Joint research program CNRS/JSPS<br>CNRS<br>15 000 euros/year (under request) | Representations of étale fundamental groups and applications | Cadoret, Anna | 01/01/2010 - 31/12/2011 |
| N°7 | Ratazzi, Nicolas | 24 | ANR Blanc<br>ANR<br>under request | Hamot (Hauteurs, modularité, transcendance) | Bilu, Yuri | 01/01/2010 - 31/12/2013 |
| N°8 | Romagny, Matthieu | | Joint research program CNRS/JSPS<br>CNRS<br>15 000 euros/year (under request) | Representations of étale fundamental groups and applications | Cadoret, Anna | 01/01/2010 - 31/12/2011 |
| N°9 | Zapponi, Leonardo | | | | | |