Blind Deblurring Using a Simplified Sharpness Index

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LABORATOIRE MAP5

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Outline

1. Measuring image quality
2. Phase coherence and sharpness indices
3. Application to blind deblurring

Sharpness

How can we define image quality or sharpness?

$\rightarrow$ effective resolution

$\rightarrow$ perceptually correlated

$\rightarrow$ sensitive to well-known artifacts

$\rightarrow$ independent of the context

Early works based on ...

- gray-level distribution (variance, kurtosis, entropy...)
- analysis of the Fourier spectrum
- analysis of the local features

No-reference quality measures

- Edges width analysis in [Marziliano et al., 2004].
- Comparison of the edges width to a perceptually-defined threshold called *Just Noticeable Blur* in [Ferzli, Karam, 2009].
- Metric Q of [Zhu, Milanfar, 2010] (singular values of local gradients).
- Sharpness by local phase coherence of a complex wavelet transform in [Wang, Simoncelli, 2003], [Hassen, Wang, Salama, 2010].
- Spectral and spatial sharpness measure S3 in [Vu, Chandler, 2009].
- Sharpness by DFT phase coherence in [Blanchet, Moisan, Rougé, 2008], [Blanchet, Moisan, 2012].

For a much more detailed discussion about image quality assessment, see the recent [Chandler, 2013].

Outline

Definitions

Let $u : \Omega \to \mathbb{R}$ be an image (Ω rectangle of $\mathbb{Z}^2$).
We denote by $\dot{u} : \mathbb{Z}^2 \to \mathbb{R}$ the periodic extension of u.
We introduce its periodic total variation

$$\begin{aligned}\mathrm{TV}(u) &= \sum_{\mathbf{x}\in\Omega} |\dot{u}(x_1+1, x_2) - \dot{u}(x_1, x_2)| + |\dot{u}(x_1, x_2+1) - \dot{u}(x_1, x_2)| \\ &= \|\partial_x \dot{u}\|_1 + \|\partial_y \dot{u}\|_1 \ .\end{aligned}$$

Discrete Fourier Transform

Let $\hat{u}$ be the Discrete Fourier Transform (DFT) of u.
Modulus of u : $|\hat{u}(\xi)|$.
Phase of u : $\mathrm{Angle}(\hat{u}(\xi)) \in (-\pi, \pi]$.

House

Lena

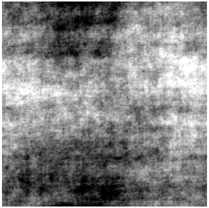
Phase of House with modulus of Lena

Random phase noise

Let $(\psi(\xi))_{\xi\in\hat{\Omega}}$ a random phase function, i.e. the coefficients are independent uniform in $(-\pi,\pi]$ except that $\psi(-\xi) = -\psi(\xi)$.
Let us introduce the random phase noise

$$U_\psi = \mathrm{DFT}^{-1}\left(|\hat{u}| \cdot e^{i\psi}\right) .$$

u



A realization of U_ψ

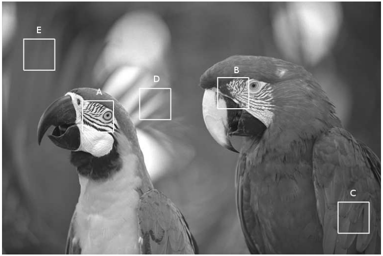
Global Phase Coherence

Definition (Blanchet, Moisan, Rougé 2008)

The Global Phase Coherence of u is defined by

$$\mathrm{GPC}(u) = -\log_{10} \mathbb{P}(\mathrm{TV}(U_\psi) \leq \mathrm{TV}(u)) \, .$$

Numerical approximation is available through Monte-Carlo simulations.



Values of GPC for these patches :
A : 206
B : 388
C : 240
D : 2.56
E : 0.33

From GPC to SI

Instead of U_ψ, let us consider the Gaussian field

$$u * W(\mathbf{x}) = \sum_{\mathbf{y} \in \Omega} \dot{u}(\mathbf{x} - \mathbf{y}) W(\mathbf{y})$$

where $W \sim \mathcal{N}\left(0, \frac{1}{|\Omega|}\right)$.

From GPC to SI

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where $W \sim \mathcal{N}\left(0, \frac{1}{|\Omega|}\right)$.
A (reasonable) Gaussian approximation of $\mathrm{TV}(u * W)$ yields

$$\mathbb{P}\Big(\mathrm{TV}(u * W) \leq \mathrm{TV}(u)\Big) \simeq \Phi\left(\frac{\mu - \mathrm{TV}(u)}{\sigma}\right)$$

where $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} e^{-t^2/2} dt$ and...

The theorem of [Blanchet, Moisan, 2012]

Theorem (Blanchet-Moisan 2012)

$$\mu := \mathbb{E}[\mathrm{TV}(u * W)] = (\alpha_x + \alpha_y)\sqrt{\frac{2}{\pi}}\sqrt{|\Omega|}\ ,$$

$$\sigma^2 := \mathbb{V}\mathrm{ar}(\mathrm{TV}(u * W))$$

$$= \frac{2}{\pi}\sum_{\mathbf{z}\in\Omega} \alpha_x^2 \cdot \omega\left(\frac{\Gamma_{xx}(\mathbf{z})}{\alpha_x^2}\right) + 2\alpha_x\alpha_y \cdot \omega\left(\frac{\Gamma_{xy}(\mathbf{z})}{\alpha_x\alpha_y}\right) + \alpha_y^2 \cdot \omega\left(\frac{\Gamma_{yy}(\mathbf{z})}{\alpha_y^2}\right)$$

$$\alpha_x = \|\partial_x \dot{u}\|_2 \quad \alpha_y = \|\partial_y \dot{u}\|_2$$

$$\forall t \in [-1, 1], \quad \omega(t) = t\ \mathrm{Arcsin}\ t + \sqrt{1 - t^2} - 1$$

$$\forall \mathbf{x} \in \mathbb{Z}^2, \quad \Gamma(\mathbf{z}) = \begin{pmatrix} \Gamma_{xx} & \Gamma_{xy} \\ \Gamma_{yx} & \Gamma_{yy} \end{pmatrix}(\mathbf{z}) = \sum_{\mathbf{y}\in\Omega} \nabla\dot{u}(\mathbf{y}) \cdot \nabla\dot{u}(\mathbf{z}+\mathbf{y})^T\ .$$

Variance approximation

We have seen that

$$\sigma^2 = \frac{2}{\pi} \sum_{\mathbf{z} \in \Omega} \alpha_x^2 \cdot \omega\left(\frac{\Gamma_{xx}(\mathbf{z})}{\alpha_x^2}\right) + 2\alpha_x\alpha_y \cdot \omega\left(\frac{\Gamma_{xy}(\mathbf{z})}{\alpha_x\alpha_y}\right) + \alpha_y^2 \cdot \omega\left(\frac{\Gamma_{yy}(\mathbf{z})}{\alpha_y^2}\right) .$$

where

$$\forall t \in [-1, 1], \quad \omega(t) = t \,\mathrm{Arcsin}\, t + \sqrt{1 - t^2} - 1 \qquad .$$

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where

$$\forall t \in [-1, 1], \quad \omega(t) = t \,\mathrm{Arcsin}\, t + \sqrt{1 - t^2} - 1 \approx \frac{t^2}{2} .$$

Therefore, σ^2 can be approximated by

$$\sigma_a^2 = \frac{1}{\pi} \sum_{\mathbf{z} \in \Omega} \alpha_x^2 \cdot \left(\frac{\Gamma_{xx}(\mathbf{z})}{\alpha_x^2}\right)^2 + 2\alpha_x\alpha_y \cdot \left(\frac{\Gamma_{xy}(\mathbf{z})}{\alpha_x\alpha_y}\right)^2 + \alpha_y^2 \cdot \left(\frac{\Gamma_{yy}(\mathbf{z})}{\alpha_y^2}\right)^2 .$$

Definition of the Simplified sharpness index S

After simplifications, we have

$$\sigma_a^2 = \frac{1}{\pi}\left(\frac{\|\Gamma_{xx}\|_2^2}{\alpha_x^2} + 2\cdot\frac{\|\Gamma_{xy}\|_2^2}{\alpha_x\alpha_y} + \frac{\|\Gamma_{yy}\|_2^2}{\alpha_y^2}\right) .$$

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Definition

Replacing σ by σ_a, we obtain the simplified sharpness Index

$$S(u) = -\log_{10} \Phi\left(\frac{\mu - \mathrm{TV}(u)}{\sigma_a}\right) .$$

to be compared to

$$\mathrm{SI}(u) = -\log_{10} \Phi\left(\frac{\mu - \mathrm{TV}(u)}{\sigma}\right) .$$

Properties and remarks

- $0 \leq \frac{\sigma^2 - \sigma_a^2}{\sigma_a^2} \leq \pi - 3 \approx 0.142$.
- These indices are affine-invariant, e.g. $S(a \cdot u + b) = S(u)$.
- If U is a random phase field, then

$$\forall t > 0, \quad \mathbb{P}(\mathrm{GPC}(U) \geq t) \leq 10^{-t} .$$

- We used TV to measure the quantity of oscillations of u. Subsequently, these indices will favor images with localized discontinuities.
- Singular points of TV will be singular for SI and S.
- These indices are not convex nor concave.

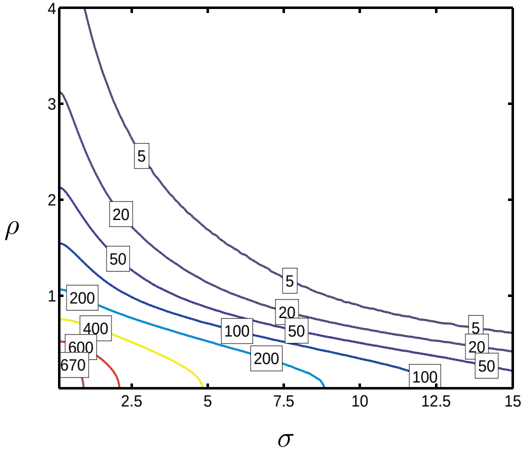
Practical remarks

- SI requires 4 FFTs whereas S requires 1 FFT.
- Dealing with a border-to-border discontinuity : one may take the periodic component of u (see [Moisan, 2011]).
- Dealing with the quantization bias : dequantization by subpixel translation (see [Desolneux, Ladjal, Moisan, Morel, 2002]).

$$T_{(1/2,1/2)}u = \mathrm{DFT}^{-1}\left(\hat{u}(\xi)e^{-i\langle(\frac{1}{2},\frac{1}{2}),\xi\rangle}\right) .$$

Sensitivity to blur and noise

$$(\rho, \sigma) \mapsto S(\kappa_\rho * u + \sigma N)$$



$$\widehat{\kappa_\rho}(\xi) = e^{-\pi\rho^2|\xi|^2}$$
ρ : level of blur

$N \sim \mathcal{N}(0, I)$
σ : noise std

Sensitivity to ringing (I)

Let us apply H^1-regularization to an image u :

$$\forall \boldsymbol{\xi} \in \Omega, \quad \widehat{u_{\lambda,\rho}}(\boldsymbol{\xi}) = \hat{u}(\boldsymbol{\xi}) \cdot \frac{\hat{\kappa_\rho}^*(\boldsymbol{\xi})}{|\hat{\kappa_\rho}|^2(\boldsymbol{\xi}) + \lambda\|\boldsymbol{\xi}\|^2} ,$$

where $\hat{\kappa_\rho}(\boldsymbol{\xi}) = \exp(-\pi\rho^2\|\boldsymbol{\xi}\|^2)$ is the Gaussian blur filter.

If ρ is too large, ringing artifacts appear on the image $u_{\lambda,\rho}$.

Sensitivity to ringing (II)

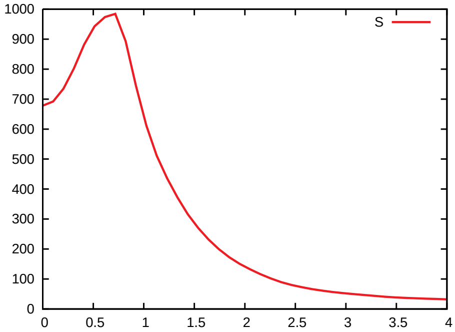
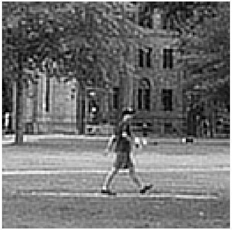


Figure: Evolution of $S(u_{0.01,\rho})$ when varying parameter ρ. Original image u is “Yale”.

Sensitivity to ringing (III)

$S(u^c_{0.01,0}) = 20.2$

$S(u^c_{0.01,0.7}) = 25.6$



$S(u^c_{0.01,1}) = 16.3$

Figure: Results of H^1-regularization when varying the regularization parameter ρ . Crops.

Outline

1. Measuring image quality
2. Phase coherence and sharpness indices
3. Application to blind deblurring
   - General scheme
   - Restriction to kernels with radial unimodal DFT

Blind deblurring

- We want to remove blur from a single image u.
- Linear space-invariant blur $\Rightarrow$ convolution by a kernel κ.
- When κ is unknown: blind deconvolution (see e.g. the recent [Levin et al., 2011]).

Stochastic optimization of a sharpness criterion

Goal : find a deconvolution kernel that maximizes

$$F_u(k) = S(k * u) \ .$$

The authors of [Blanchet, Moisan, 2012] suggested the following stochastic optimization scheme.

Algorithm 1

- Begin with $k = \delta_0$
- Repeat N times
  - ▷ Define k' from a random perturbation of k
  - ▷ If $S(k' * u) > S(k * u)$ then $k \leftarrow k'$
- Return k and $k * u$

Restriction to kernels with compact support

This scheme can be adapted to different sets of kernels.

Original | 5×5 | 21×21 separable

$\rightarrow$ failure cases when F_u is too singular.

Restriction to kernels with radial unimodal DFT (I)

$\rightarrow$ To avoid these degenerated cases, we suggest to consider kernels with radial DFT :

$$\forall \xi \in \Omega, \quad \hat{k}_r(\xi) = L_r\left(\|\xi\|\right) \; ,$$

where L_r is the piecewise affine interpolate of

$$r(0) = 1, r(1), r(2), \ldots, r(n_r - 2), r(n_r - 1) = 0 \; .$$

Restriction to kernels with radial unimodal DFT (I)

→ To avoid these degenerated cases, we suggest to consider kernels with radial DFT :

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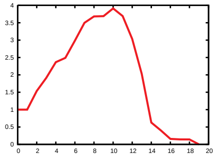
$$r(0) = 1, r(1), r(2), \ldots, r(n_r - 2), r(n_r - 1) = 0 \ .$$

→ Besides, the sequence r is supposed to be unimodal i.e.



Restriction to kernels with radial unimodal DFT (II)

So we have to maximize

$$\mathcal{F}_u : r \mapsto S(k_r * u) - \lambda_1 \, \mathrm{dist}(r, U) - \lambda_2 \, \|r\|_{H^1} \ ,$$

where λ_1 and λ_2 are weighting parameters.

- → The unimodality constraint is incorporated through the $\mathrm{dist}(r, U)$ term. U is the set of unimodal sequences.
- → We also include a regularity term

$$\|r\|_{H^1}^2 = \sum_{i=0}^{d-2} (r(i+1) - r(i))^2 \ .$$

Restriction to kernels with radial unimodal DFT (III)

Algorithm 2

- Initialize r with a piecewise-affine profile
- Repeat N times
  - ▷ Pick a random index i
  - ▷ Draw a random value $\epsilon \in [-a/2, a/2]$
  - ▷ Change $r(i)$ in $r(i) + \epsilon$
  - ▷ If $\mathcal{F}_u(r)$ decreases then refuse the change
- Return r, k_r and $k_r * u$

We checked convergence to a non-random profile.

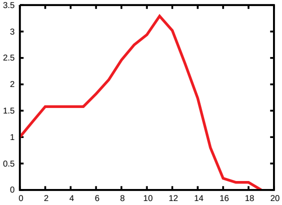
Results of blind deblurring (*Parrots*)

Input, $S = 727$

Output, $S = 963$

Input cropped

Output cropped



Final profile

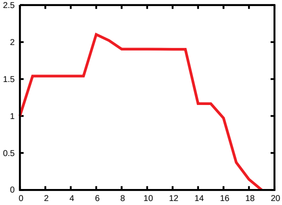
Results of blind deblurring (*Capitol*)

Input, $S = 359$

Output, $S = 428$

Input cropped

Output cropped



Final profile

Comparisons

- We will apply the algorithm on $u = \kappa * u_0 + n$ where κ is the Gaussian blur kernel (of std 1) and $n \sim \mathcal{N}(0, I)$.
- It allows us to compute PSNR values with respect to u_0.
- It enables computation of the oracle

$$k_0 = \operatorname*{Arg\,min}_{\hat{k} \text{ radial}} \mathbb{E}\left(\| u_0 - k * (\kappa * u_0 + N) \|^2 \right) .$$

- We can compare it to the blind deconvolution algorithm of [Levin et al., 2011].

Results of blind deblurring (*blurred Parrots*)

Original u_0
S = 727

Blurred and noisy input u
PSNR = 30.5, S = 140

Levin et al.
PSNR = 32.7, S = 591

Oracle output
PSNR = 36.0, S = 370

Blind deblurring ($\lambda_2 = 0$)
PSNR = 24.8, S = 440

Blind deblurring ($\lambda_2 = 10$)
PSNR = 34.2, S = 394

CONCLUSION

- The phase coherence indices GPC, SI and S can be used to measure sharpness.
- They favor sharp edges surrounding uniform zones.
- The proposed blind deblurring algorithm is a non-linear technique to select the best linear filtering of the image.
- Perspectives :
  - $\rightarrow$ extension to motion blur,
  - $\rightarrow$ include S in a purely non-linear deconvolution process.

Merci !

THANK YOU FOR YOU ATTENTION !

Acknowledgments
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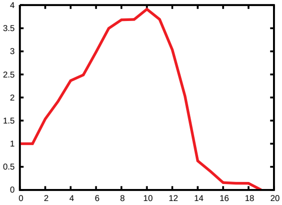
Results of blind deblurring (*Lena*)

Input, $S = 956$

Output, $S = 2010$

Input cropped

Output cropped



Final profile

Results of blind deblurring (*blurred Capitol*)


Original
S = 359


Blurred and noisy input
PSNR = 25.2, S = 78.9


Levin et al.
PSNR = 24.2, S = 218


Oracle output
PSNR = 31.0, S = 144


Deblurred ($\lambda_2 = 0$)
PSNR = 28.8, S = 185


Deblurred ($\lambda_2 = 10$)
PSNR = 27.7, S = 160