Project of Graduate Course (A. Yger)

Trace, residue currents and multidimensional residues, duality and division

The aim of the course is to present from their origin (starting with the classical results by Macaulay, C. Jacobi, M. Nœther, D. Hilbert, G. Hermann, ..., which have been revisited all along the last century), up to the most recents developments (which are inspired by geometric considerations), the tools from analytic geometry which play a major operational part in effective transcendental geometry (multidimensional residue theory, from the cohomogical Poincaré point of view as well as from the "currential" De Rham-Leray point of view, duality theory, Hilbert-Samuel polynomials with attached notions of multiplicities). The course will focuse on remaining open questions as well as on those where the algebraic "translation" of analytic methods involved is not yet well understood. Such course needs as perequisite a solid basic background in algebraic geometry (master 1 level) as well as some knowledge in complex analysis and differential geometry ; reference [HY], completed with a basic master course in algebraic geometry, could for example provide such a preliminary background.

Program :

- Integration over analytic sets ; Lelong-Poincaré formula
- Positive closed currents and proper intersection theory
- Vogel-Stückrad and Cygan-Tworzewski algorithms in improper intersection theory
- Residue currents, multidimensional residue calculus, multidimensional residues and amoebas
- Division and Ehrenpreis-Palamodov noetherian operators : from classic approaches up to M. Andersson's recent results
- Duality, effective division formulaes in algebraic geometry
- The notions of integral closure, tight closure ; Briançon-Skoda or Lipman-Teissier type theorems (with consequences in effective algebraic geometry))
- The concept of infinity in projective or toric geometry.

Some references :

[A] M. Andersson, The membership problem for polynomial ideals in terms of residue currents, to appear in Ann. Inst. Fourier.
[AW] M. Andersson, E. Wulcan, Noetherian residue currents, manuscript (accessible on arkiv).
[B] J.E. Björk, Residues and $\mathcal{D}$-modules, in *The Legacy of Niels Henrik Abel*, Oslo, 2002
[D] J.P. Demailly, Courants positifs et théorie de l'intersection, Gazette Math. 53, 1992, 131-159.
[BGVY] C.A. Berenstein, R. Gay, A. Vidras, A. Yger, *Residue currents and Bézout identities*, Progress in Math. 114, Birkhäuser, 1993.
[GH] P. Griffiths, P. Harris, *Principles of Algebraic Geometry*, Wiley Interscience, 1978.
[GKZ] I.M. Gelfand, M.M. Krapanov, A.V. Zelevinsky, *Discriminants, resultants, and multidimensional determinants*, Birkhäuser, Boston, 1994.
[HH] M. Hochster, C. Huneke, Tight closure, invariant theory and the Briançon-Skoda theorem, J. Amer. Math. Soc. 3, 1990, 31-116.
[HY] A. Hénaut, A. Yger, *Eléments de géométrie niveau* $M1$, Ellipses, Paris, 2001.
[K] J. Kollár, Effective Nullstellensatz for arbitrary ideals, J. European Math. Soc. 1, 1999, 313-337.
[PT] Mikael Passare, A. Tsikh, Residus and Amoebas, book in preparation.
[T] A. Tsikh, *Multidimensional residues and their applications*, Transl. Amer. Math. Soc 103, 1992.
[TY] A. Tsikh, A. Yger, Residue currents, J. Math. Sci, 120 (6), 2004, 1916-1971.
[TW] P. Tworzewski, Intersection theory in complex analytic geometry, Ann. Polon. Math. 62, 1995, 177-191.