Ex 3 a) $\int \cos(3x-5)\,dx$ $\qquad u = 3x-5 \rightsquigarrow \frac{du}{dx} = 3$

$\rightsquigarrow du = 3\,dx$

$$= \frac{1}{3}\int \cos(u)\,du$$

$$= +\frac{1}{3}\sin(u) = +\frac{1}{3}\sin(3x-5).$$

La fonction est défini sur tout $\mathbb{R}$.

b) $$\int \frac{x^2-3x+4}{x}\,dx = \int x\,dx - 3\int dx + \int \frac{4}{x}\,dx$$

$$= \frac{1}{2}x^2 - 3x + 4\ln(x)$$

La fonction est défini sur $\mathbb{R}\setminus\{0\}$
$=]-\infty, 0[\cup]0, \infty[$.

c) $$\int \frac{1}{x-2}\,dx = \int \frac{1}{u}\,du = \ln|u| = \ln|x-2|.$$

$u = x-2$, $\frac{du}{dx} = 1$ $\quad du = dx$

La fonction ~~est~~ défini pour $x \neq 2$, donc sur $]-\infty, 2[\cup]2, \infty[$.

Ex 4: a) $$\int_0^1 e^{-x}\,dx = \left[-e^{-x}\right]_0^1 = -e^{-1} + 1 = 1 - \frac{1}{e}$$

b) $$\int_0^1 \underbrace{x\,e^{2x}}_{u\cdot v'}\,dx \overset{\text{p. parties !}}{=} \Big[\underbrace{x\,\tfrac{1}{2}e^{2x}}_{u\cdot v}\Big]_0^1 - \int_0^1 \underbrace{\tfrac{1}{2}e^{2x}}_{v}\,dx$$

$$= \frac{e^2}{2} - \left[\frac{1}{4}e^{2x}\right]_0^1 = \frac{e^2}{4} + \frac{1}{4}.$$

c) $$\int_0^1 2x\,e^{x^2}\,dx = \left[\begin{matrix} u = x^2, & \frac{du}{dx} = 2x \\ 2x\,dx = du & \end{matrix}\right] = \int_0^1 e^u\,du = e - 1.$$

d) $\int_0^1 e^x \sqrt{e^x+3}\, dx$

$u = e^x + 3$

$\frac{du}{dx} = e^x$

$du = e^x\, dx$

$= \int_4^{3+e} \sqrt{u}\, du$

$= \left[\frac{2}{3} u^{3/2} \right]_4^{3+e}$

$= \frac{2}{3} \left((3+e)^{3/2} - 8 \right).$

Ex 5: a) $\int \frac{\log(x)}{x} dx = \int u\, du = \frac{1}{2} u^2 = \frac{1}{2} (\log x)^2$

$\log(x) = u$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{dx}{x}$

Remarque: $\log = \ln$
(faute de notation de ma part)

b) $A = \int \underbrace{\frac{1}{x}}_{u'} \cdot \underbrace{\log x}_{v}\, dx = \left[\log(x) \cdot \log(x) \right] - \int \underbrace{\log(x)}_{u} \underbrace{\frac{1}{x}}_{v'} dx$

On a $A = (\log(x))^2 - A$

donc $2A = (\log(x))^2$

donc $A = \frac{1}{2} (\log x)^2.$

Ex 6: Si $\frac{1}{x^2-4x+5} = \frac{a}{x+1} + \frac{b}{x-5}$, alors

$1 = (x-5) \cdot a + (x+1) \cdot b$ pour tout x.

On pose $x = 5 \rightsquigarrow 1 = 6b$

$x = -1 \rightsquigarrow 1 = -6a$

$\rightsquigarrow \frac{1}{x^2-4x+5} = \frac{1}{6} \left(\frac{1}{x-5} - \frac{1}{x+1} \right).$

donc $\displaystyle\int_0^2 \frac{1}{x^2-4x+5}\,dx = \frac{1}{6}\ln(x-5) - \frac{1}{6}\ln(x+1).$

comme Ex 2.c

Ex 7:

a) $\displaystyle\int_2^3 \frac{x}{x^2-3}\,dx = \frac{1}{2}\int_1^6 \frac{1}{u}\,du$

$x^2-3=u$

$\frac{du}{dx} = 2x$

$= \left[\frac{1}{2}\ln(u)\right]_1^6$

$= \ln(\sqrt{6}).$

b) $\displaystyle\int_1^2 \frac{x}{\sqrt{5-x^2}}\,dx$

$5-x^2=u$

$\frac{du}{dx} = -2x$

$du = -2x\,dx$

$\displaystyle = -\frac{1}{2}\int_4^1 \frac{1}{\sqrt{u}}\,du$

$\displaystyle = \int_1^4 \frac{1}{2}u^{-\frac{1}{2}}\,du = \left[\sqrt{u}\right]_1^4 = 1.$

c) $\displaystyle\int_0^1 \frac{\cos(x)}{1-\sin^2(x)}\,dx$

$\sin(x) = u$

$\frac{du}{dx} = +\cos(x)$

$du = +\cos(x)\,dx$

$\displaystyle = +\int_0^{\sin(1)} \frac{1}{1-u^2}\,du$

Comme Ex.6

$\displaystyle = \frac{1}{2}\cdot\int_0^{\sin(1)} \left(\frac{1}{1-u} + \frac{1}{1+u}\right)du$

$= \left[\frac{1}{2} - \ln(1-u) + \frac{1}{2}\ln(1+u)\right]_0^{\sin(1)}$

Attention !!!

$= \left[\frac{1}{2}\ln\left(\frac{1+u}{1-u}\right)\right]_0^{\sin(1)} = \ln\left(\sqrt{\frac{1+\sin(1)}{1-\sin(1)}}\right).$

d) $\displaystyle\int_0^3 \sqrt{9-x^2}\, dx$

$x = 3\sin(u)$! $\quad$ $x=0 \leadsto u=0$
$x=3 \leadsto u=\pi/2$!!!

<u>Attention :</u> les rôles de x et u sont à l'envers par rapport aux autres cas !

<u>Donc</u> $\dfrac{dx}{du} = 3\cos(u) \leadsto dx = 3\cos(u)\, du$

$$\int_0^{\pi/2} \underbrace{3\cos(u)}_{\sqrt{9-9\sin^2(u)}\ !!!} \cdot 3\cos(u)\, du$$

Comme dans le Cours du 12.9.

$$= 9\int_0^{\pi/2} \cos^2(u)\, du = 9 \cdot \frac{\pi}{4}$$

<u>Ex 8</u> : a) $\displaystyle\int x^2\sqrt{x^3+1}\, dx$ $\qquad$ $x^3+1 = u$

$\dfrac{du}{dx} = 3x^2$

$$= \frac{1}{3}\int \sqrt{u}\, du = \frac{2}{9}\cdot u^{3/2}$$

$$= \frac{2}{9}\left(1+x^3\right)^{3/2}.$$

b) $\displaystyle\int \frac{x+1}{x^2+2x+2}\, dx = \int \frac{x+1}{(x+1)^2+1}\, dx$

$x+1 = u \leadsto du = dx$ $\qquad$ $\displaystyle = \int \frac{u}{u^2+1}\, du$

$u^2+1 = v \leadsto \dfrac{dv}{du} = 2u \leadsto$ $\qquad$ $\displaystyle = \frac{1}{2}\int \frac{1}{v}\, dv$

$$= \frac{1}{2}\log|v| = \frac{1}{2}\log(1+u^2)$$

$$= \ln\sqrt{1+u^2} = \ln\left(\sqrt{1+(1+x)^2}\right)$$

(sonny: log = ln)

$$= \ln\left(\sqrt{x^2+2x+2}\right)$$

c) 1) $\int \underset{u}{\sin(x)}\ \underset{v'}{\cos(x)}\ dx$

$$= \sin^2(x) - \int \underset{u'}{\cos(x)}\ \underset{v}{\sin(x)}\ dx$$

Forme $A = \sin^2(x) - A$

donc

$$\int \sin(x)\cos(x)\,dx = \frac{1}{2}\sin^2(x).$$

Remarque: On a toujours $\int f(x)\, f'(x)\,dx$

$$= \frac{1}{2} f(x)^2 \ !$$

2) $2\sin(x)\cos(x) = \sin(2x)$!

Donc $\int \sin(x)\cos(x)\,dx = \int \frac{1}{2}\sin(2x)\,dx$

$$= -\frac{1}{4}\cos(2x).$$

Exercice: Demontrer que $\frac{1}{2}\sin^2(x) + \frac{1}{4}\cos(2x) = \text{const.}$!

Ex 9 $\quad I(T) = \int_0^T \lambda e^{-\lambda t}\,dt = \int_0^{\lambda T} e^{-u}\,du = 1 - e^{-\lambda T}.$

$$\lambda t = u \rightsquigarrow \frac{du}{dt} = \lambda$$

$$\rightsquigarrow du = \lambda\,dt$$

On a $e^{-x} \to 0$, $x \to +\infty$, donc

$$\lim_{T\to\infty} \int_0^T \lambda e^{-\lambda t}\, dt = 1 \;!$$

$$E(T) = \int_0^T \underset{u}{t} \cdot \underset{v'}{\lambda e^{-\lambda t}}\, dt \qquad \begin{pmatrix} v = -e^{-\lambda t} \\ u' = 1. \end{pmatrix}$$

$$= \left[-t\, e^{-\lambda t}\right]_0^T - \int_0^T 1\cdot\left(-e^{-\lambda t}\right) dt$$

$$= -T e^{-\lambda T} - \frac{1}{\lambda}\left[e^{-\lambda t}\right]_0^T$$

$$= \frac{1}{\lambda}\left(1 - e^{-\lambda T} - \lambda T e^{-\lambda T}\right)$$

lorsque $T \to +\infty$, $e^{-\lambda T} \to 0$, et $\lambda T e^{-\lambda T} \to 0$, donc

$$\lim_{T\to\infty} E(T) = \frac{1}{\lambda}.$$

<u>Ex 10</u>: a) $\displaystyle\int \frac{1}{\sin(x)}\, dx = \int \frac{\sin(x)}{\sin^2(x)}\, dx$

$\displaystyle = \int \frac{\sin(x)}{1-\cos^2(x)}$ et on procède comme pour

ex. 7c (vous voyez d'où 7c vient ?)

b) $\displaystyle\int \log\left(\tfrac{1}{x}\right) dx \overset{\text{truc méchant}}{=} \int \underset{u'}{1} \cdot \underset{v}{\log\left(\tfrac{1}{x}\right)}\, dx$

$u = x$

$v' = \dfrac{1}{\frac{1}{x}} \cdot \left(-\dfrac{1}{x^2}\right) = -\dfrac{1}{x}\;!$

$\displaystyle = x \log\left(\tfrac{1}{x}\right) - \int x \cdot \left(\frac{-1}{x}\right) dx = x\log\left(\tfrac{1}{x}\right) + x$