

# Power of Two as Sums of Three Pell Numbers

Joint work with J. J. Bravo, F. Luca

**Bernadette Faye**

Ph.d Student

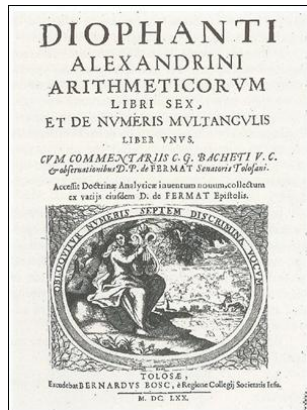
Journées Algophantiennes Bordelaises,

07-09 July 2017

# Motivation

Diophantine equations obtained by asking that members of some fixed binary recurrence sequence be

- ▶ squares,
- ▶ factorials,
- ▶ triangular,
- ▶ belonging to some other interesting sequence of positive integers.



# Motivation

**Problem:** Find all solutions in positive integers  $m, n, \ell, a$  of the equation

$$P_m + P_n + P_\ell = 2^a,$$

where

$$\begin{cases} P_0 = 0 \\ P_1 = 1 \\ P_{n+2} = 2P_{n+1} + P_n, \text{ for } n \geq 0 \end{cases}$$

# History

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$$\{n : s_a(n) < K \text{ and } s_b(n) < K\}$$

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- ▶ Bravo and Bravo(2015):  $F_n + F_m + F_\ell = 2^a$ .
- ▶ Bravo, Gómez and Luca(2016):  $F_n^{(k)} + F_m^{(k)} = 2^a$



## Most Recent results...

- ▶ Meher and Rout(Preprint):

$$U_{n_1} + \cdots + U_{n_t} = b_1 p_1^{z_1} + \cdots + b_s p_s^{z_s}$$

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- ▶ Chim and Ziegler(Preprint):

$$F_{n_1} + F_{n_2} = 2^{a_1} + 2^{a_2} + 2^{a_3}.$$

$$F_{m_1} + F_{m_2} + F_{m_3} = 2^{t_1} + 2^{t_2}.$$

## Theorem 1 (Bravo, F., Luca, 2017)

*The only solutions  $(n, m, \ell, a)$  of the Diophantine equation*

$$P_n + P_m + P_\ell = 2^a \quad (1)$$

*in integers  $n \geq m \geq \ell \geq 0$  are in*

$(2, 1, 1, 2), (3, 2, 1, 3), (5, 2, 1, 5), (6, 5, 5, 7),$

$(1, 1, 0, 1), (2, 2, 0, 2), (2, 0, 0, 1), (1, 0, 0, 0).$



# Strategy of the Proof

Assume  $n \geq 150$ ,  $n \geq m \geq \ell$

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- The iterated application of linear forms in logarithms...

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- ▶ Properties of the convergent of the continued fractions

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$$2^a < \alpha^{n-1} + \alpha^{m-1} + \alpha^{\ell-1} < 2^{2n-2}(1 + 2^{2(m-n)} + 2^{2(\ell-n)})$$

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$$\implies a \leq 2n.$$

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- ▶ Dividing both sides by  $\alpha^n/(2\sqrt{2})$ , we get

$$\left| 1 - 2^{a+1} \cdot \alpha^{-n} \cdot \sqrt{2} \right| < \frac{8}{\alpha^{n-m}}.$$

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- ▶ Bounding  $n - m$  in terms of  $n$

# Linear forms in Logarithms à la Baker

## Theorem 2 (Matveev 2000)

*Let  $\mathbb{K}$  be a number field of degree  $D$  over  $\mathbb{Q}$ ,  $\eta_1, \dots, \eta_t$  be positive real numbers of  $\mathbb{K}$ , and  $b_1, \dots, b_t$  rational integers. Put*

$$\Lambda = \eta_1^{b_1} \cdots \eta_t^{b_t} - 1 \quad \text{and} \quad B \geq \max\{|b_1|, \dots, |b_t|\}.$$

*Let  $A_i \geq \max\{Dh(\eta_i), |\log \eta_i|, 0.16\}$  be real numbers, for  $i = 1, \dots, t$ .*

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*Let  $A_i \geq \max\{Dh(\eta_i), |\log \eta_i|, 0.16\}$  be real numbers, for  $i = 1, \dots, t$ . Then, assuming that  $\Lambda \neq 0$ , we have*

$$|\Lambda| > \exp(-1.4 \times 30^{t+3} \times t^{4.5} \times D^2(1 + \log D)(1 + \log B)A_1 \cdots A_t).$$

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Then

$$|\Lambda| \geq \exp \left( -1.4 \times 30^6 \times 3^{4.5} \times 2^2 \times (1 + \log 2)(2 \log n) \times 1.4 \times 0.9 \times 0.7 \right)$$

$$\implies (n - m) \log \alpha < 1.8 \times 10^{12} \log n.$$

## Second Linear Forms in Logarithms

Rewriting the equation  $P_n + P_m + P_\ell = 2^a$  in a different way, we get to

$$\left| 1 - 2^{a+1} \cdot \alpha^{-n} \cdot \sqrt{2}(1 + \alpha^{m-n})^{-1} \right| < \frac{5}{\alpha^{n-\ell}}.$$

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Matveev's Theorem

$$\implies (n - \ell) \log \alpha < 5 \times 10^{24} \log^2 n.$$

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$$\implies n < 1.7 \times 10^{43}.$$

# Summary of the above finding

## Lemma 3

*If  $(n, m, \ell, a)$  is a solution in positive integers of equation  $P_n + P_m + P_\ell = 2^a$ , with  $n \geq m \geq \ell$ , then*

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- ▶  $n < 1.7 \times 10^{43}$ .
- ▶  $a < 2n + 1 < 4 \times 10^{43}$ .

# Reducing the bound on $n$

## Lemma 4 (Baker-Devenport reduction Algorithm)

*Let  $M$  be a positive integer, let  $p/q$  be a convergent of the continued fraction of the irrational  $\gamma$  such that  $q > 6M$ , and let  $A, B, \mu$  be some real numbers with  $A > 0$  and  $B > 1$ . Let  $\epsilon := ||\mu q|| - M||\gamma q||$ . If  $\epsilon > 0$ , then there is no solution to the inequality*

$$0 < |u\gamma - v + \mu| < AB^{-w},$$

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$$u \leq M \quad \text{and} \quad w \geq \frac{\log(Aq/\epsilon)}{\log B}.$$

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$$0 < (2a+3) \left( \frac{\log 2}{\log \alpha} \right) - 2n < \frac{20}{\alpha^{n-m}}.$$

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has the shape

$$|x\gamma - y| < 20/\alpha^{n-m}.$$

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We use properties of the convergents of the continued fraction to  $\gamma := [a_0, a_1, a_2, \dots] = [0, 1, 3, 1, 2, \dots]$ .

Since  $2a + 3 < 9 \times 10^{43}$ , with a quick computation with  
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Furthermore  $a_M := \max\{a_i : i = 1 \dots, 88\} = 100$ . Then, from the properties of the continued fractions,

$$\frac{1}{(a_M + 2)(2a + 3)} < (2a + 3)\gamma - 2n < \frac{20}{\alpha^{n-m}}$$



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$$\alpha^{n-m} < 20 \cdot 102 \cdot 9 \times 10^{43}.$$

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$$\implies n - m < 124.$$

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- ▶  $n - m < 124$ .
- ▶  $n - \ell < 122$ .

Finally, in order to obtain a better upper bound on  $n$ , we use consider the relation

$$\Lambda_3 := (a + 1) \log 2 - n \log \alpha + \log \phi(n - m, n - \ell),$$

with  $\phi(x_1, x_2) := \sqrt{2}(1 + \alpha^{-x_1} + \alpha^{-x_2})^{-1}$ .

## Last Computations...

Hence, we use the inequality

$$0 < \left| (a+1) \left( \frac{\log 2}{\log \alpha} \right) - n + \left( \frac{\log \phi(n-m, n-\ell)}{\log \alpha} \right) \right| < \frac{5}{\alpha^n},$$

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- For all possible choices of  $n-m \in [0, 124]$  and  $n-\ell \in [0, 140]$ , use the reduction algorithm

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with  $\phi(x_1, x_2) := \sqrt{2}(1 + \alpha^{-x_1} + \alpha^{-x_2})^{-1}$ .

- ▶ For all possible choices of  $n - m \in [0, 124]$  and  $n - \ell \in [0, 140]$ , use the reduction algorithm
- ▶ Find if  $(n, m, \ell, a)$  is a possible solution of the equation  $P_n + P_m + P_\ell = 2^a$ ,



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Hence, we use the inequality

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- ▶ For all possible choices of  $n-m \in [0, 124]$  and  $n-\ell \in [0, 140]$ , use the reduction algorithm
- ▶ Find if  $(n, m, \ell, a)$  is a possible solution of the equation  $P_n + P_m + P_\ell = 2^a$ ,
- ▶ One gets that  $n < 150$ , contradiction.

## Theorem 5 (Bravo, F., Luca, 2017)

*The only solutions  $(n, m, \ell, a)$  of the Diophantine equation*

$$P_n + P_m + P_\ell = 2^a \quad (2)$$

*in integers  $n \geq m \geq \ell \geq 0$  are in*

$(2, 1, 1, 2), (3, 2, 1, 3), (5, 2, 1, 5), (6, 5, 5, 7),$

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”I love mathematics for its own sake, because it allows for no hypocrisy and no vagueness.” Stendhal



***THANKS FOR YOUR ATTENTION !***

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