

ECE 188 – Homework #2

Writing part

1 Exercise 1 (Directionally separable kernels)

Let ν be a 2d convolution kernel whose support is restricted to a $s \times s$ window. Show that if $\nu_{k,l} = \nu_k \nu_l$, then the discrete convolution of y by ν can be performed in linear time. Describe the algorithm.

2 Exercise 2 (Derivative of convolutions)

Recall that the convolution product of two real functions f and g reads as

$$(f * g)(t) = \int_{-\infty}^{\infty} f(u)g(t-u)du .$$

Assuming that the integral exists and is finite for all t , what would be the necessary extra condition on f and g to have

$$f' * g = f * g' ?$$

3 Exercise 3 (Eigenvectors of circulant matrices)

Show that the n eigenvectors, with unit norm, of any circulant matrix H reads as

$$e_k = \frac{1}{\sqrt{n}} \left(1, \exp\left(\frac{2\pi ik}{n}\right), \exp\left(\frac{4\pi ik}{n}\right), \exp\left(\frac{6\pi ik}{n}\right), \dots, \exp\left(\frac{2(n-1)\pi ik}{n}\right) \right)^T$$

for $k = 0$ to $n - 1$, and identify the corresponding eigenvalues λ_k . Here i is the imaginary number. Recall that the n eigenvectors (e_k) of a matrix H with unit norm must satisfy:

$$He_k = \lambda_k e_k, \quad \langle e_k, e_l \rangle = e^* e_l = 0 \quad \text{if } k \neq l \quad \text{and} \quad \|e_k\| = 1 .$$

Here $*$ stands for conjugate transpose.

Hint: you can use the sum of a geometric series

$$\text{For } r \neq 1, \quad \sum_{p=a}^b r^p = \frac{r^a - r^{b+1}}{1 - r}$$

Practical part

The following exercises should be done after completing the Matlab codes of Homework #1 and committing them as

```
git commit -am 'exercises for homework #1 done'
```

Once done switch the the branch `homework2` as follow

```
git fetch
git checkout homework2
```

This version of the code contains updated and new files required for the following exercises. In order to merge them with your own progress from Homework #1, type

```
git merge homework1
```

Read carefully the messages, you may have conflicts. Check conflict status with `git status`. Solve all the conflicts before doing anything else. Once solved

```
git add file1_in_conflict file2_in_conflict ...
git commit -am 'solved conficts after merging'
```

4 Exercise 4 (Separable convolutions)

Complete the file `imconvolve.m` such that its complexity reduces when the convolution kernel is separable. The script `hw2_ex4_imconvolve.m` should produce the following results:

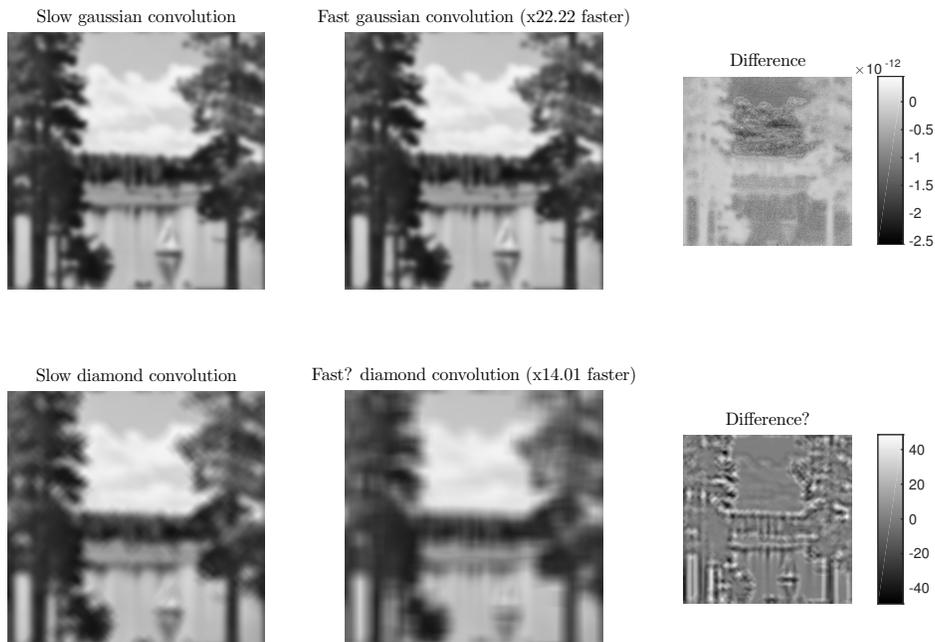


Figure 1: Don't forget to interpret the results!

5 Exercise 5 (Order-statistic filtering)

Order-statistic filters (OSF) are local filters that are based on the ranking of pixel values inside a sliding window. Complete the file `imosf.m` such that it implements the following order-statistic filters

- median: select the median value
- erode: select the min value
- dilate: select the max value
- opening: performs erode and dilate
- closing: performs dilate and erode
- trimmed mean: compute the average of k pixel values after excluding extreme ones
- extreme: choose either the `min` or the `max` whether the central value is closer of the `min` or `max`.

The script `hw2_ex5_imosf.m` should produce the following results:

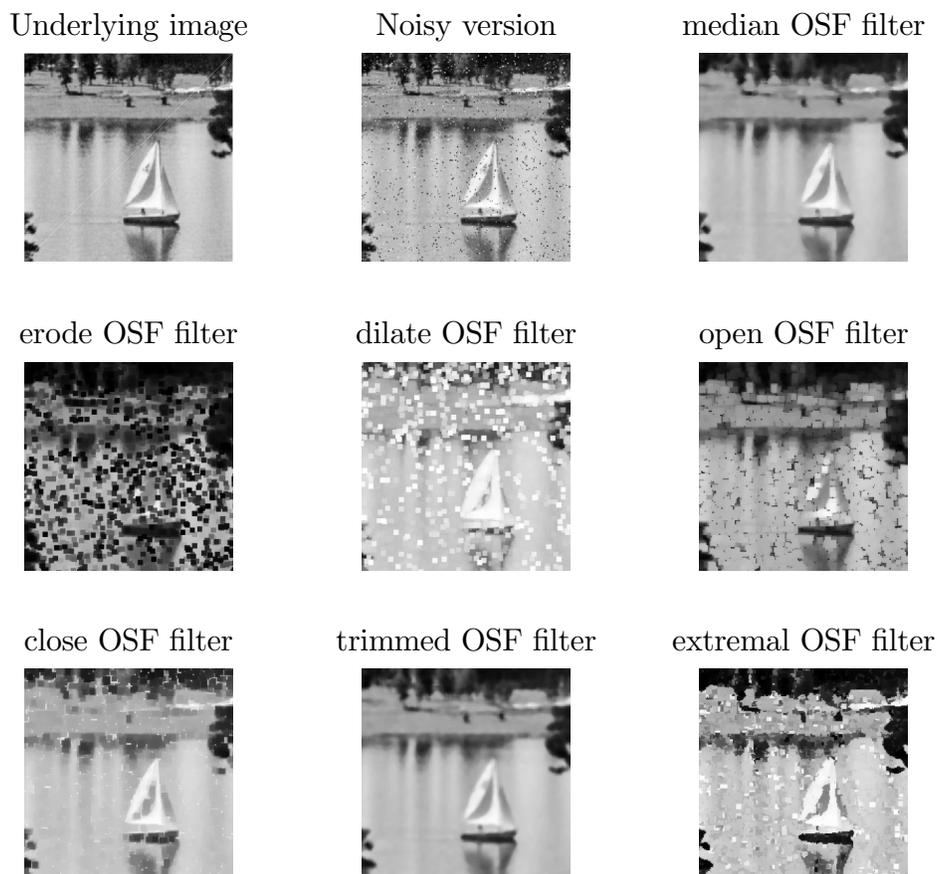


Figure 2: Results of OSF filters

6 Exercise 6 (Bilateral filter)

Complete the file `imbilateral.m` such that it implements the bilateral filtering. The script `hw2_ex6_imbilateral.m` should produce the following results:

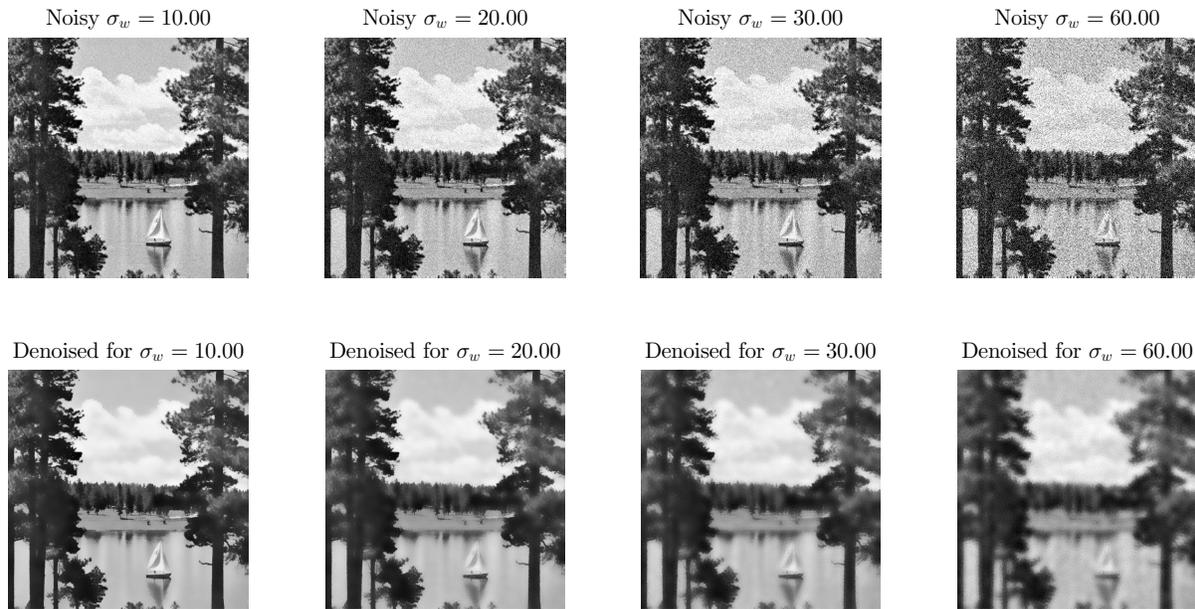


Figure 3: Results of the bilateral filter

Hint: only one line of code per section should be enough.