

# Control of vortex shedding around a pipe section using a porous sheath

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## ABSTRACT

The passive control of the flow around a fixed circular cylinder is achieved using a porous layer between the obstacle and the fluid. The various media are easily handled by means of the penalization method. The computational domain is reduced to a close neighbourhood of the body thanks to efficient non-reflective boundary conditions. The porous layer changes the vortex shedding and induces a strong reduction of the vorticity magnitude and of the root mean-square lift coefficient.

**KEY WORDS:** Passive flow control; cylinder shedding.

## INTRODUCTION

In the vicinity of bluff bodies, the shedding of vortices can induce unsteady forces of small amplitude with excitation close to a structural resonant frequency that provoke structural failures (Williamson and Govardhan 2004). Therefore, the study and the control of vortex shedding has a crucial importance in engineering applications like offshore oil industry. In this case, the vortex-induced vibrations (VIV) can affect the risers. As the environmental conditions are given and can not be changed, the only way to reduce the VIV is to use an efficient control technique adapted to the riser framework. Several control methods are already proposed to reduce the drag and lift forces or to regularize the vortex shedding around 2D and 3D circular cylinders (Wong 1979, Williamson and Govardhan 2004). Most of them use the active control strategies (Gatulli and Ghanem 2000, Gillies 1998, Zhijinn 2003) that are very difficult to implement in the riser geometry. In fact, such a geometry needs passive devices which don't need additional energy supply in the system. Some fruitful researches have been already performed in such a case. For example, some authors have added dimples (Bearman and Harvey 1993) or splitter plates (Kwon and Choi 1996) to control and regularize the flow around a circular cylinder. In some other cases, the

control technique is performed using a secondary small cylinder (Mittal and Raghuvanshi 2001) or an appropriately distributed electromagnetic field (Posdziech and Grundmann 2001).

In this paper the passive control is achieved introducing a porous interface between the solid body and the flow in order to reduce the vorticity production of the boundary layer (Bruneau and Mortazavi 2001). In fact, the porous medium changes the no-slip boundary condition into a kind of intermediate Fourier boundary condition. Consequently, the whole vortex shedding mechanism is smoothed, and the flow instabilities, the lift and drag forces and the enstrophy are decreased. Mathematically, it was shown that the change of boundary condition has a significant influence on the boundary layer (Luchini 1995, Achdou et al. 1998). Thus, we have to solve a problem involving three different media, the solid body, the porous interface and the incompressible fluid. This can be very easily handled using the double penalization method. The original penalization method (Angot et al. 1999) is a way to take into account an immersed body in a fluid with two permeability coefficients. The method has already been successfully used to simulate transitional and turbulent flows by an array of cylinders (Bruneau et al. 1999, Kevlahan and Ghidaglia 2001). In the double penalization (Bruneau and Mortazavi 2001, Carbou), three values of the permeability coefficient will represent the bluff body, the porous medium and the fluid. The method was analysed and tested numerically in Bruneau and Mortazavi 2004.

In the present work, we consider a two-dimensional, unsteady and incompressible flow around a fixed circular cylinder. This cylinder corresponds to a section of a three-dimensional riser pipe. Such a study with an appropriate choice of the Reynolds number, can give significant informations on the real flow behaviour even if a responding body should be closer to the reality. Here we focus on the effect of the proposed control strategy on a given geometry. Numerical simulations are performed for transitional and turbulent flows to better understand the effects of the control with respect to the flow regime. In this paper after the description of the computational method a parametric study

is performed to choose an optimal porous layer with the best control properties. Then, the reduction effects of this control approach on different global flow quantities like the enstrophy ( $Z$ ), the lift coefficient root mean square ( $CLrms$ ) and the drag coefficient ( $Cd$ ) is analysed. We shall see that the present method induces a drastic reduction of the  $CLrms$  and consequently of the vortex induced vibrations. Furthermore, instantaneous pressure, velocity and vorticity fields (with and without control) are plotted to show the effect of the control on the flow pattern.

## OUTLINE OF THE METHOD

Let  $\Omega = (0, 5) \times (0, 2)$  be the two-dimensional computational domain around the circular cylinder of diameter 0.16 which the center is located at  $(1.1, 1)$ , we add a porous ring of thickness 0.02 (Figure 1). This ring can be seen as a sheath around the pipe and the thickness is chosen to have significant results on medium meshes. Previous works have shown that efficient results can be obtained for a wide range of layer thicknesses (Bruneau and Mortazavi 2004). To simulate the global flow both in the porous and fluid media, it is necessary to solve simultaneously the Darcy equations in the porous medium and the Navier-Stokes equations in the fluid. This is generally quite difficult to handle. Here, it is easily achieved by the penalization technique that consists in adding a term  $U/K$  in the non-dimensional incompressible Navier-Stokes equation where  $K$  represents the non-dimensional permeability coefficient of the medium. This parameter  $K = (\rho k \Phi \bar{U}) / (\mu D)$  is linked to the intrinsic permeability  $k$  through the Brinkman and Forchheimer-Navier-Stokes equations when the porosity  $\Phi$  of the porous medium is close to one (Nield and Bejan 1999). The variables  $\rho$ ,  $\bar{U}$ ,  $\mu$  and  $D$  denote respectively the density of the fluid, the mean flow, the viscosity of the fluid and the diameter of the pipe.

$$\partial_t U + (U \cdot \nabla) U - \frac{1}{Re} \Delta U + \frac{U}{K} + \nabla p = 0 \text{ in } \Omega_T = \Omega \times (0, T) \quad (1)$$

$$\text{div} U = 0 \text{ in } \Omega_T \quad (2)$$

where  $U = (u, v)$  is the velocity,  $p$  the pressure and  $Re = (\rho \bar{U}) / \mu$  the non-dimensional Reynolds number based on the unit inlet velocity and length. These equations are solved in the whole domain  $\Omega$  including the cylinder which is considered as a porous medium of zero permeability while the fluid is considered as a porous medium of infinite permeability. Numerically, we take  $K = 10^{-8}$  in the solid and not smaller values to avoid numerical instabilities, and  $K = 10^{16}$  in the fluid in order the penalization term vanishes. Furthermore, the porous medium will be defined by an intermediate value to determine by a parametric study (see the next section). The capability of the penalization method to represent the three media is discussed by the authors in a recent paper (Bruneau and Mortazavi 2004). To get a well-posed problem we have to impose boundary conditions at the limits of the domain. On upstream boundary a constant flow  $U = (1, 0)$  is imposed while for other boundaries an efficient artificial boundary condition is implemented (Bruneau and Fabrie 1997). This boundary condition reads

$$\sigma(U, p) \cdot n + \frac{1}{2} (U \cdot n)^- (U - U^{ref}) = \sigma(U^{ref}, p^{ref}) \cdot n \quad (3)$$

where  $\sigma(U, p) = \frac{1}{2} \frac{1}{Re} (\nabla U + \nabla U^t) - pI$  is the stress tensor,  $n$  is the unit normal vector pointing outside of the domain,  $a^- = -\min(a, 0)$  and  $(U^{ref}, p^{ref})$  is the reference flow which is taken equal to the flow on the previous cell. This boundary condition enables the vortices to convey properly outside of the computational domain without any reflection on the artificial frontiers.

As we use the penalization method, it is not necessary to fit the body. So we mesh the domain with an uniform cartesian grid on which finite differences are applied. The discretization is achieved by a second-order Gear scheme in time with explicit treatment of the convection term. All the other terms are considered implicitly. These linear terms are discretized by centered second-order finite differences whereas an upwind third-order Murman-like scheme is used for the convection terms (Bruneau and Saad). To get high performance, a multigrid method is applied with a sequence of grids from a coarse  $25 \times 10$  cells grid to a fine  $1600 \times 640$  cells grid. This whole method is efficient, accurate and stable enough to make 2D direct numerical simulations of complex flows. The code has already been validated for both the benchmark flow in a lid-driven cavity (Bruneau and Saad) and for the flow behind an array of circular cylinders with comparisons to soap film experiments (Bruneau et al. 1999).

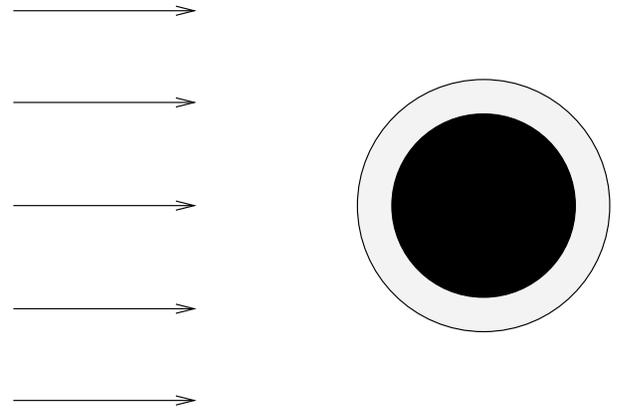


Figure 1: The cylinder with a porous sheath.

## NUMERICAL RESULTS

To a body immersed in a flowing fluid, vortices are undesirable on three major accounts, oscillations induced by the non-symmetric and unsteady shedding of vortices, high pressure drag and wake buffeting of leeward structures. The principal target of this work is to reduce the first and third effects, regularizing the flow in the vicinity of the cylinder. Adding a porous layer between the body and the fluid generates an appropriate interface for such a goal. As we shall see, the present control is much more efficient to reduce the  $CLrms$  than the drag. In this section we first perform a parametric study to choose the most efficient permeability  $K$  of the porous layer and then we present accurate results with careful comparisons between the uncontrolled and controlled flows. To study the control effects on the flow we analyse the vorticity fields, the pressure fields and the velocity fields as well as the global quantities defined by

$$Z = \frac{1}{2} \int_{\Omega} |\omega|^2 dx$$

$$Cd = \frac{Fd}{\rho U_{\infty}^2 R} ; CLrms = \sqrt{\frac{1}{T} \int_0^T CL^2 dt}$$

where  $\omega$  is the vorticity,  $Fd = \int_{body} \frac{u}{K} dx$  as the drag force is computed using the penalization term,  $\rho = 1$ ,  $U_{\infty} = (1, 0)$ ,  $R = 0.08$  and  $CL$

is computed in the same way than  $Cd$  integrating the  $v$  component. The above computation of  $Fd$  and  $FL$  is equivalent to the usual computation integrating the pressure and shear force on the body surface (Caltagirone 1994). Because of the geometrical symmetry of the body the  $CL$  is oscillating around the mean value zero. Therefore the  $CLrms$  gives a better measure of the symmetry, the regularity and the steadiness of the flow.

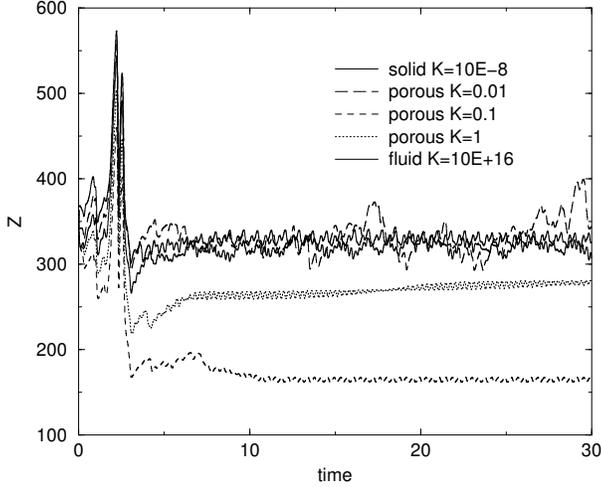


Figure 2: Enstrophy history at non-dimensional  $Re = 15000$ .

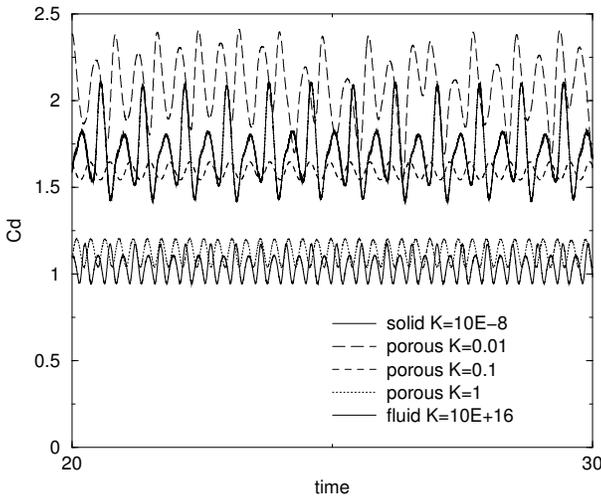


Figure 3: Drag history at non-dimensional  $Re = 15000$ .

## Parametric study

The numerical simulations of this subsection are performed at non-dimensional Reynolds number  $Re = 15000$  on a fine grid  $800 \times 320$  which is fine enough to observe the influence of the permeability parameter  $K$ . The value of  $K$  is taken between  $10^{-3}$  and  $10^3$  in the porous layer, beyond these values the porous medium is either too close to a solid

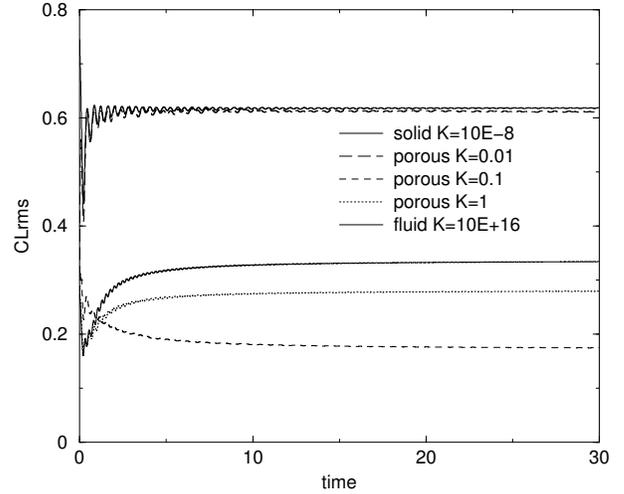


Figure 4:  $CLrms$  history at non-dimensional  $Re = 15000$ .

medium or too close to the fluid state. A ring of thickness 0.02 around the cylinder is used to apply the control strategy (Bruneau and Mortazavi 2004). When  $K = 10^{-8}$  is set in the layer we refer to it as the solid case and when  $K = 10^{16}$  is set in the layer we refer to it as the fluid case. These two cases correspond to a circular cylinder of diameter  $D = 0.2$  and  $D = 0.16$  and consequently to real Reynolds numbers based on the diameter of the cylinders  $Re_D = Re \times D = 3000$  and  $Re_D = 2400$  respectively. The numerical results with various porous layers show that an efficient value is obtained for  $K = 10^{-1}$ . Indeed, we compare the results for three consecutive values of the porous permeability to the fluid and solid cases. To measure quantitatively the vorticity evolution in the computational domain, the enstrophy history is plotted on Figure 2. It appears clearly that the porous layer with the above optimal value  $K = 10^{-1}$  is very efficient as the enstrophy production is almost twice lower and the oscillations are much lower than in the solid and fluid cases that are very close. Although we observe also an improvement for  $K = 1$ , the other values of  $K$  are less efficient.

The drag coefficient and the  $CLrms$  give two different features of the effect of the external forces on the bluff body. Let us point out to the reader that in the solid case the sheath added to the circular cylinder increases the diameter and consequently increases the drag forces. When the sheath is made of a porous medium we expect an intermediate behaviour. However, the Figure 3 shows that the behaviour is more complex. An optimum is reached for  $K = 1$  with a value close to the fluid case, the value for  $K = 10^{-1}$  is a bit lower than the solid case but the value for  $K = 10^{-2}$  is even higher. On Figure 4 we see that the root mean square of the lift coefficient evolution is almost constant and its reduction is drastic for  $K = 10^{-1}$  as it is almost twice smaller than the fluid case. The value for  $K = 1$  is about 20% smaller and the value for  $K = 10^{-2}$  is very close to that of the solid case. Then it appears that the porous medium has to be sufficiently permeable to influence positively the boundary layer characteristics and to decrease the shear effects due to the no-slip boundary condition. Nevertheless the increase of the permeability should not be too high to avoid strong shear effects between the

cylinder and the porous layer. In fact, adding the porous layer is equivalent to impose a mixed boundary condition intermediate between the no-slip and the slip one on the solid boundary (Carbou). Thus, the regularization effects are not due to a change of the Reynolds number but to a change of the shear forces modifying the flow dynamics. In the following, we shall take  $K = 10^{-1}$  as the optimal value because our priority is the regularization effects (strong decrease of  $Z$  and  $CLrms$ ). Nevertheless in an other context, the value  $K = 1$  could be considered optimal as the flow control yields to a good compromise between the regularization and the drag effects. The  $K = 10^{-1}$  value was also found in a previous paper (Bruneau and Mortazavi 2004) for the flow around a square cylinder in a channel for various mesh sizes. This value gives very good results for higher Reynolds numbers with fine grids. Even if it does not correspond to the exact optimal value it gives an efficient order of magnitude for the permeability coefficient.

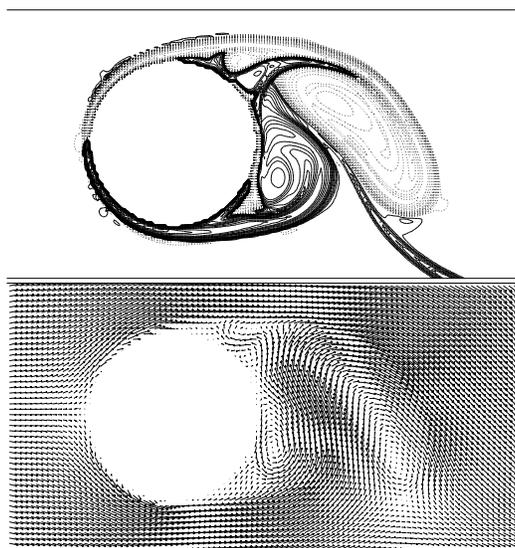


Figure 5: Zoom of the vorticity and pressure fields for a solid layer at non-dimensional  $Re = 15000$  on grid  $3200 \times 1280$ .

## Flow control on a fine grid

As we said in the previous subsection the choice of the Reynolds number  $2400 \leq Re_D \leq 3000$  corresponds to a quasi periodic transitional flow for which the control effect is very clear. This kind of flow is in the range of computed flows in deep water with weak streams (Miliou et al. 2003). For general flows around risers with higher Reynolds numbers a 2D approximation is still relevant as it gives important informations about the flow characteristics. The numerical simulations are performed on three different grids,  $800 \times 320$ ,  $1600 \times 640$  and  $3200 \times 1280$  to get a good representation of vortices and accurate values of the global quantities. The Figure 5 corresponds to a zoom of the vorticity and the velocity fields at  $Re_D = 3000$  on the finest grid and shows that the near wake is streamwise and has the size of the cylinder as pointed out in Braza et al. 1990. For that Reynolds number we can see on Table 1 that the results on the two finest consecutive grids are very close. Therefore,

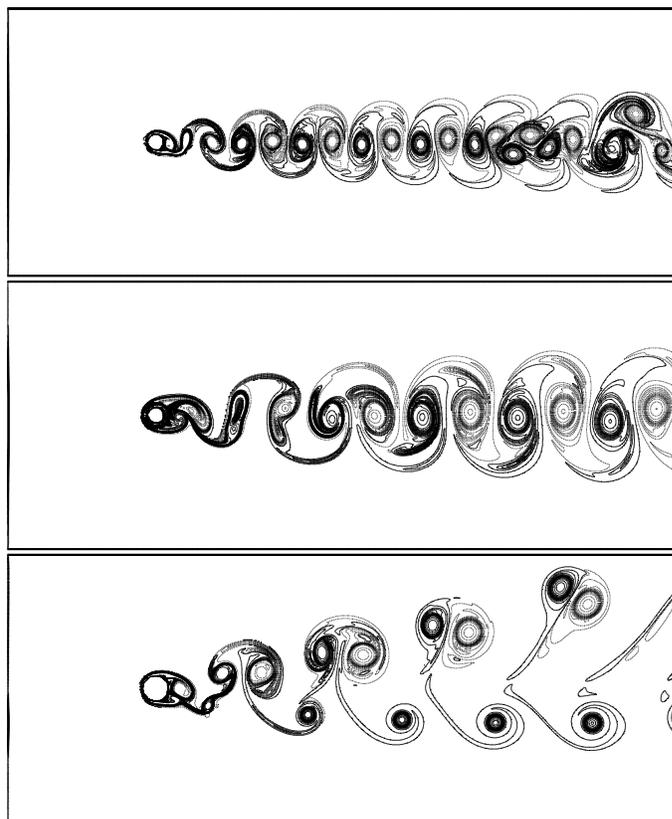


Figure 6: Vorticity field for fluid (top), porous (center) and solid (bottom) layers for the same time at non-dimensional  $Re = 15000$ .

the results of this section are computed on grid  $1600 \times 640$ . On the instantaneous vorticity field (Figure 6) we see that the presence of the sheath around the pipe changes radically the vortex shedding and the wake dynamics. The frequency is lower and is close to the frequency of the solid case as expected because the real body has the same size. In the two first cases there is a clear transitional street with smaller vortices in the fluid case whereas the merging of vortices generates tripolar structures that separate into two dipolar and monopolar diverging streets in the solid case. These observations are confirmed by the pressure and velocity fields (Figures 7 and 8). In addition they both indicate that the wake is rapidly disturbed in the fluid case and become transitional. On the contrary the flow is very regular in the porous case and exhibits low pressure intensity inside the vortices. Let us note that the flow in the fluid case corresponds to a real Reynolds number  $Re_D = 2400$  but is less regular than the flow in the porous case. This regularization property due to the porous medium can be emphasized computing the flow around the small cylinder (fluid case) at a higher Reynolds number. The results of Figure 9 show that we find again the same qualitative solution than the solution around the big cylinder (solid case) of figures 6 and 7 with a higher frequency as the diameter is smaller. This last test is performed at real Reynolds number  $Re_D = 3200$ . The discrepancy in the Reynolds number compare to the solid case is due to the less good representation of a small cylinder on this medium grid with the penalization method as the diameter is under estimated. So the fluid case at  $Re = 15000$

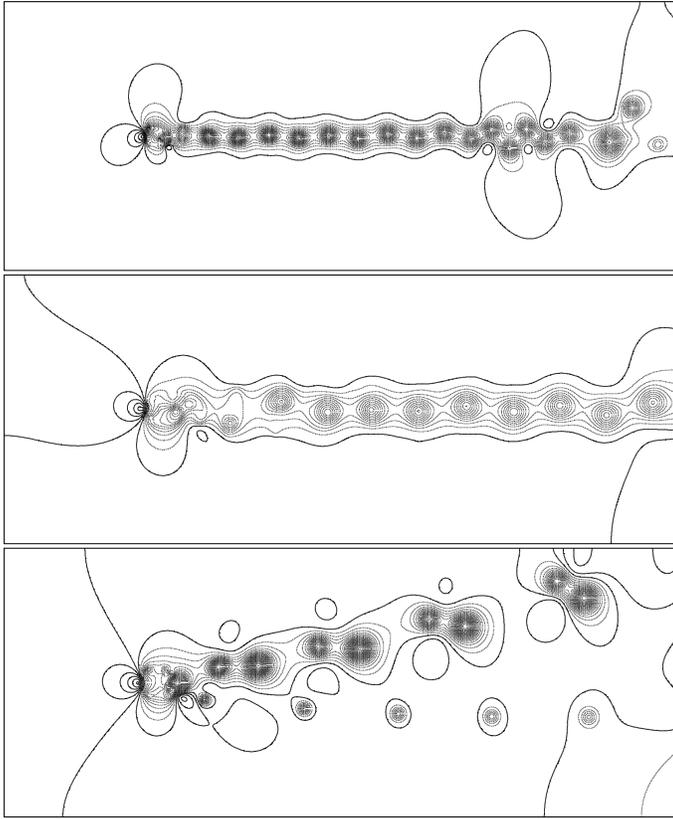


Figure 7: Pressure field for fluid (top), porous (center) and solid (bottom) layers for the same time at non-dimensional  $Re = 15000$ .

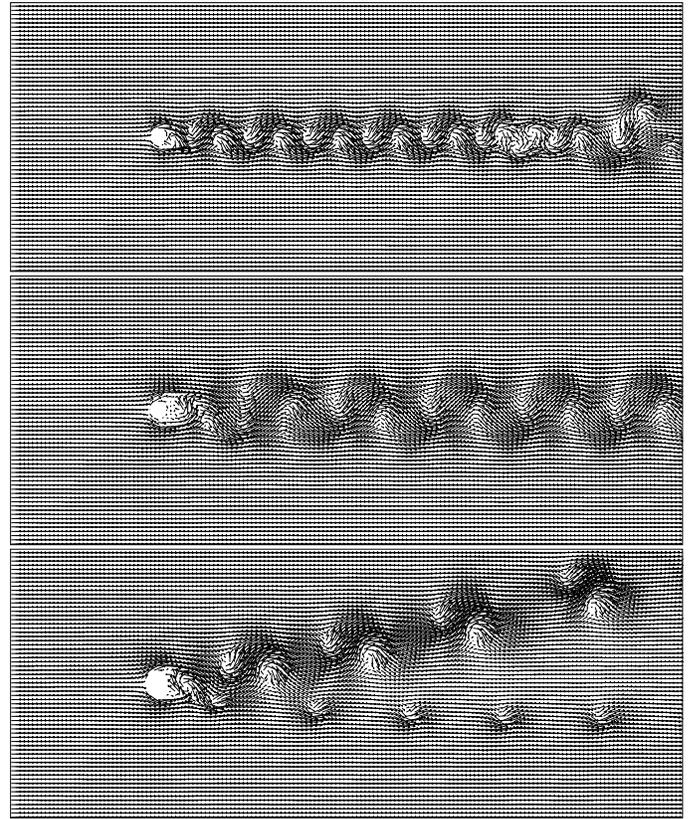


Figure 8: Velocity field for fluid (top), porous (center) and solid (bottom) layers for the same time at non-dimensional  $Re = 15000$ .

corresponds to a real Reynolds number about 2250 showing an even higher efficiency of the porous sheath. We had shown in Bruneau and Mortazavi (2004) that the solution obtained with a porous layer corresponds qualitatively to the solution at a much smaller Reynolds number without control. This is due to the fact that the no-slip boundary condition at the surface is replaced by a Fourier-like condition as shown in Achdou et al. (1998) and Carbou; consequently the shear forces are decreased and the vortex shedding is changed. From another point of view it is shown in Tang (2001) that the velocity of a vortex is decreased in the vicinity of a porous layer implying a regularisation of the flow.

To better analyze the regularity of the flow, we plot on Figures 10 and 11 the horizontal velocity history at two monitoring points which location corresponds to two parts of the flow. The first one, closer to the cylinder, shows an almost periodic flow in the fluid case and a pure periodic flow in the porous case. The second, further in the wake, points out clearly the onset of the instabilities in the wake of the small pipe whereas the flow behind the sheathed pipe is still fully periodic. Therefore we expect a reduction of the VIV in the porous case. To confirm this point we plot on the last three figures the global quantities. Due to the low number of vortices with weak intensity, the enstrophy in the whole domain is much lower in the porous case and due to the periodicity the oscillations are very low (Figure 12). But the drag is larger and closer to the drag of the solid case as the real bodies have the same diameter. Once again the amplitude of the oscillations are much smaller than in the two other

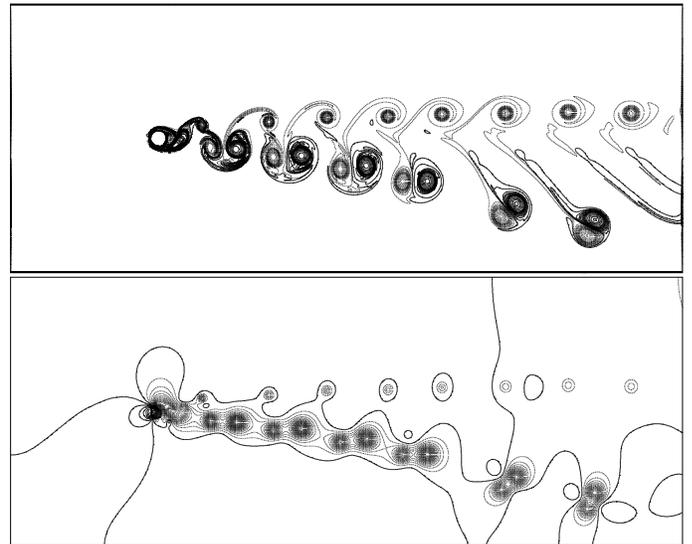


Figure 9: Vorticity and pressure fields for a fluid layer at non-dimensional  $Re = 20000$ .

cases (Figure 13). The best result concerns the drastic reduction of the  $CLrms$  (Figure 14). Indeed the flow is significantly regularized

and the transition is delayed as the shear effects are smoothed by the porous interface. The vortex shedding is controlled and consequently the vortical interactions in the wake are reduced. The meanvalues of the global quantities are summarized in Table 1. The same behaviour is observed for the computation on both grids. On the finest grid, for both the enstrophy and the  $CLrms$ , the values for the controlled pipe are 55% lower than the values for the small pipe. We notice also a tremendous reduction compared to the solid case.

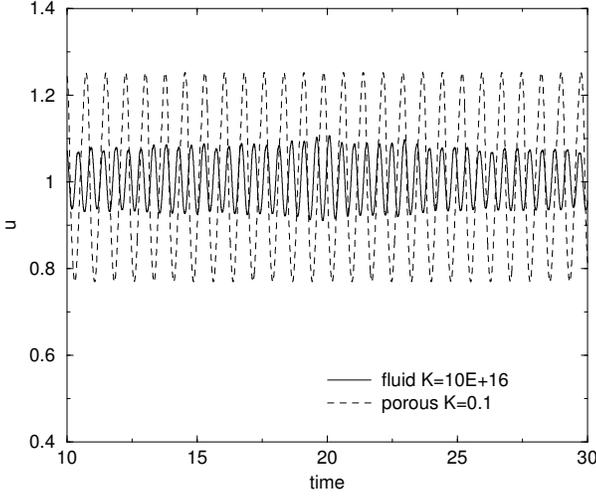


Figure 10: Horizontal velocity history at monitoring point (2.825,0.75).

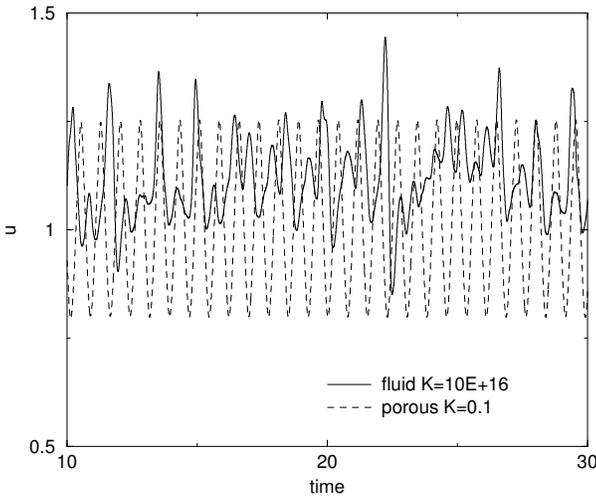


Figure 11: Horizontal velocity history at monitoring point (4.0625,0.75).

### Flow control at high Reynolds number

An interesting point is to observe how the porous layer control behaves

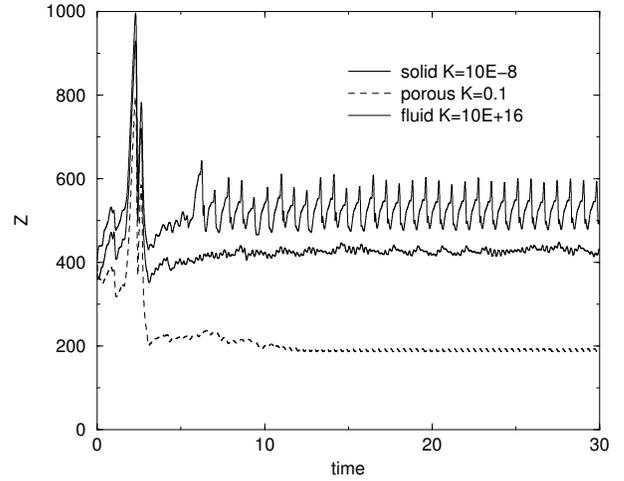


Figure 12: Enstrophy history at non-dimensional  $Re = 15000$ .

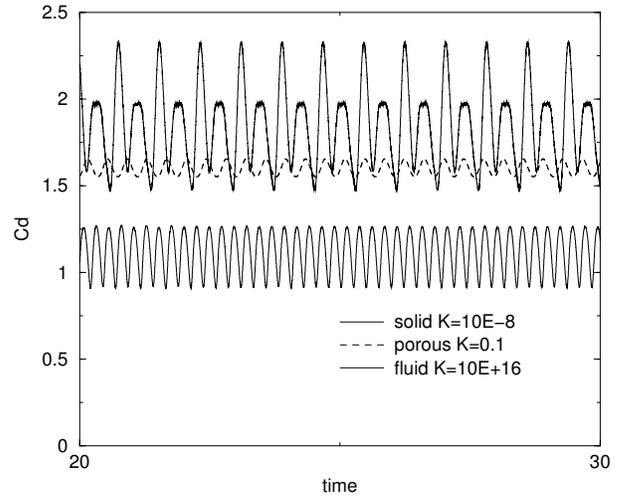


Figure 13: Drag history at non-dimensional  $Re = 15000$ .

for high Reynolds numbers closer to the real marine conditions. Namely we choose the non-dimensional value  $Re = 150000$  and the numerical simulation is performed on a fine  $3200 \times 1280$  grid to get reliable results. The vorticity fields are plotted on Figure 15 for the fluid and the porous cases. A tremendous regularized flow is obtained with the passive control as the flow changes from a chaotic distribution of vortices to a regular Karman street behind the cylinder. It appears that the control is more efficient when the Reynolds number increases. This can be seen also on Figure 16 and Table 2 as the reduction of the enstrophy is larger and the reduction of the  $CLrms$  is more drastic: 55% at  $Re = 15000$  and 72% for this test. The Figure 16 confirms that the vortex shedding is completely controlled and generates a symmetrical periodic flow.

### CONCLUSIONS

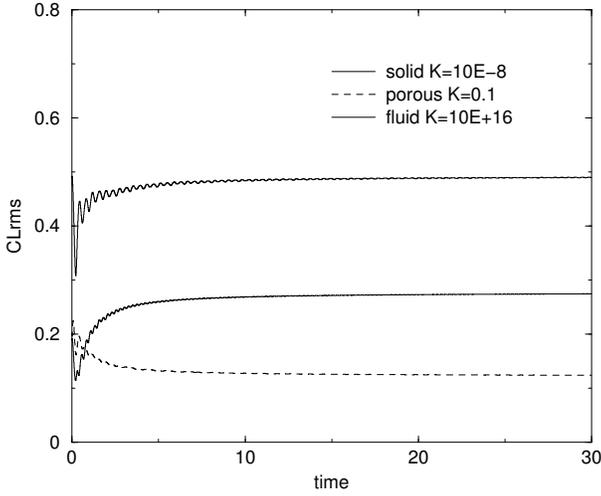


Figure 14: CLrms history at non-dimensional  $Re = 15000$ .

Table 1: Meanvalues at non-dimensional  $Re = 15000$ .

Grid	K	Enstrophy	Drag	CLrms
$800 \times 320$	10E-8	336	1.71	0.616
	10E-1	183	1.60	0.184
	10E+16	322	1.04	0.322
$1600 \times 640$	10E-8	526	1.86	0.489
	10E-1	190	1.60	0.125
	10E+16	428	1.12	0.274
$3200 \times 1280$	10E-8	527	1.88	0.462

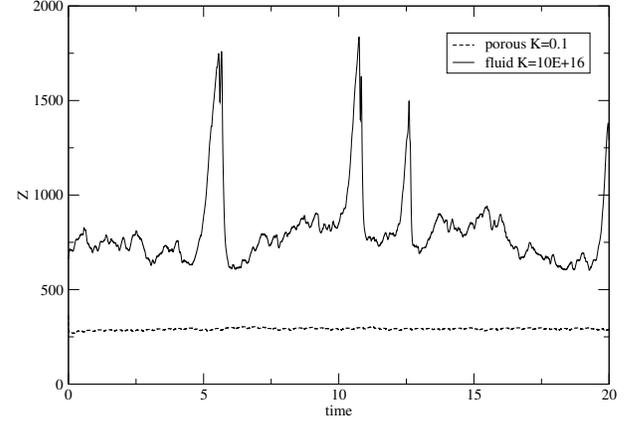


Figure 16: Enstrophy history at non-dimensional  $Re = 150000$ .

Table 2: Meanvalues at non-dimensional  $Re = 150000$ .

Grid	K	Enstrophy	Drag	CLrms
$3200 \times 1280$	10E-1	291	1.56	0.081
	10E+16	810	1.10	0.293

A numerical investigation is performed to understand the effects of a passive control introducing a porous ring around a circular cylinder. The simulations use two main tools. The porous layer is modelised by means of a double penalization method that is especially convenient for porous media and solid body flow simulations. And a non reflective boundary condition is applied to let the vortices convey properly through the artificial frontiers of the computational domain.

The numerical results concern a low transitional flow at non-dimensional Reynolds number  $Re = 15000$  and  $Re = 150000$  equivalent to  $2400 \leq Re_D \leq 3000$  and  $24000 \leq Re_D \leq 30000$  according to the nature of the layer surrounding the cylinder. The presence of the porous layer decreases the shear effects in the boundary layer and beneficially modifies the vortex shedding, reducing the wake instabilities and the damaging effects of the vortex induced vibrations on the body. Indeed, this passive control strategy yields a decrease up to 72% of the  $Clrms$  for the higher value of the Reynolds number close to the natural flow conditions. The resulting flow in the wake of the body is much more regular with low intensity periodic vortices. Therefore this passive control using a porous sheath should be a very efficient tool to prevent VIV around riser pipes. As the modeling is valid for a porosity of the porous sheath close to one, the medium must be chosen out of highly fibrous or railed materials. Forthcoming works with responding cylinders to the flow could be useful.

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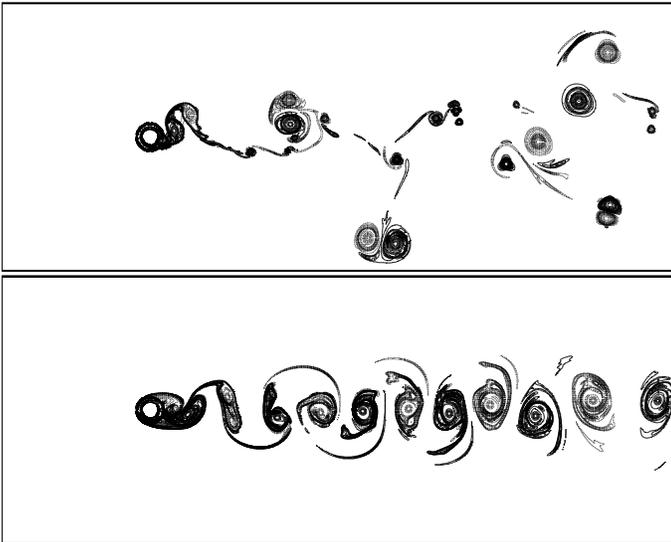


Figure 15: Vorticity field for fluid (top) and porous (bottom) layers for the same time at non-dimensional  $Re = 150000$ .

works, in particular by Williamson and Govardhan 2004 and Tang 2001.

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