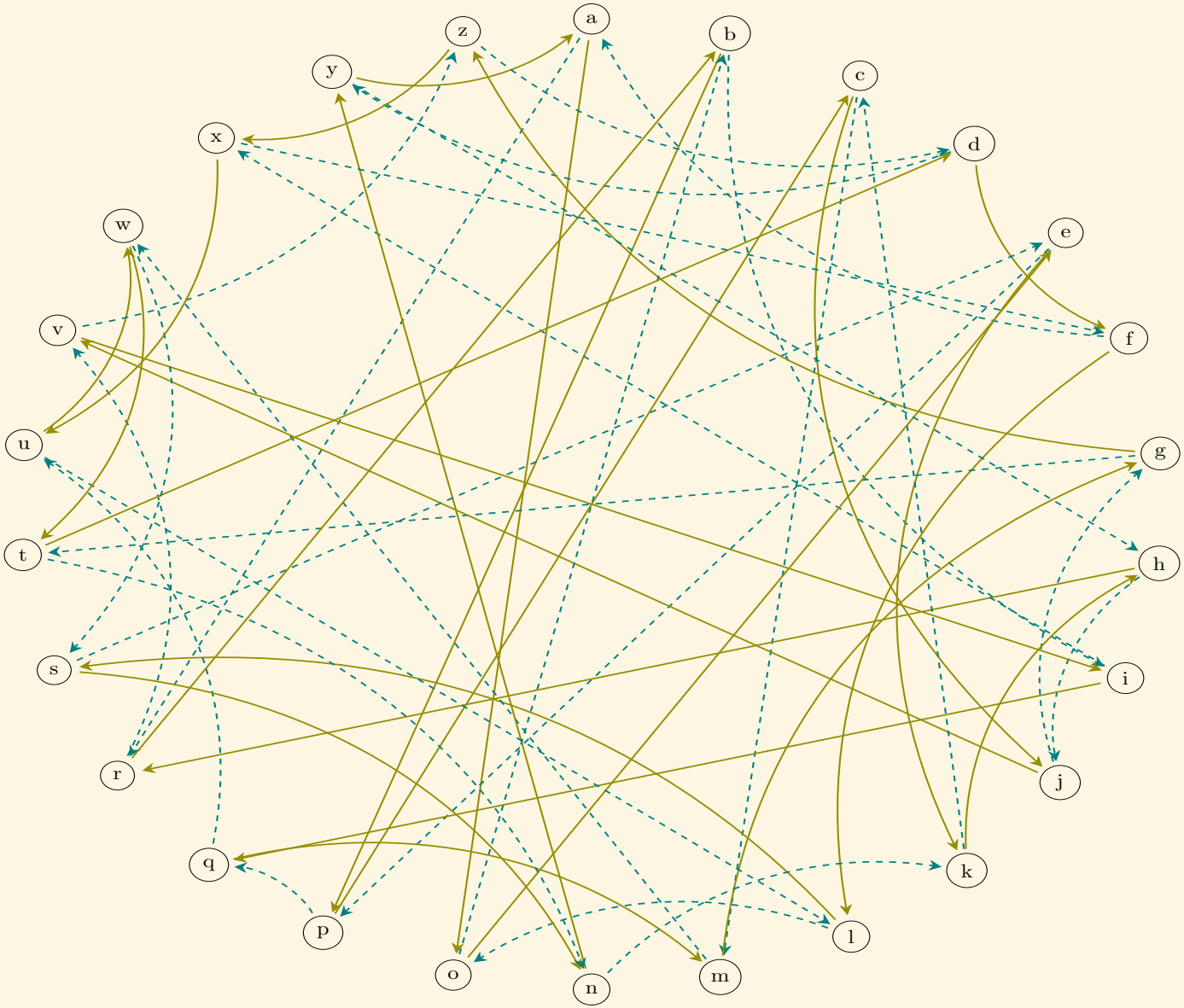
Post-Quantum Cryptography: a survey of isogeny based cryptography

2024/10/08 — Inria-Simula Workshop

Damien Robert

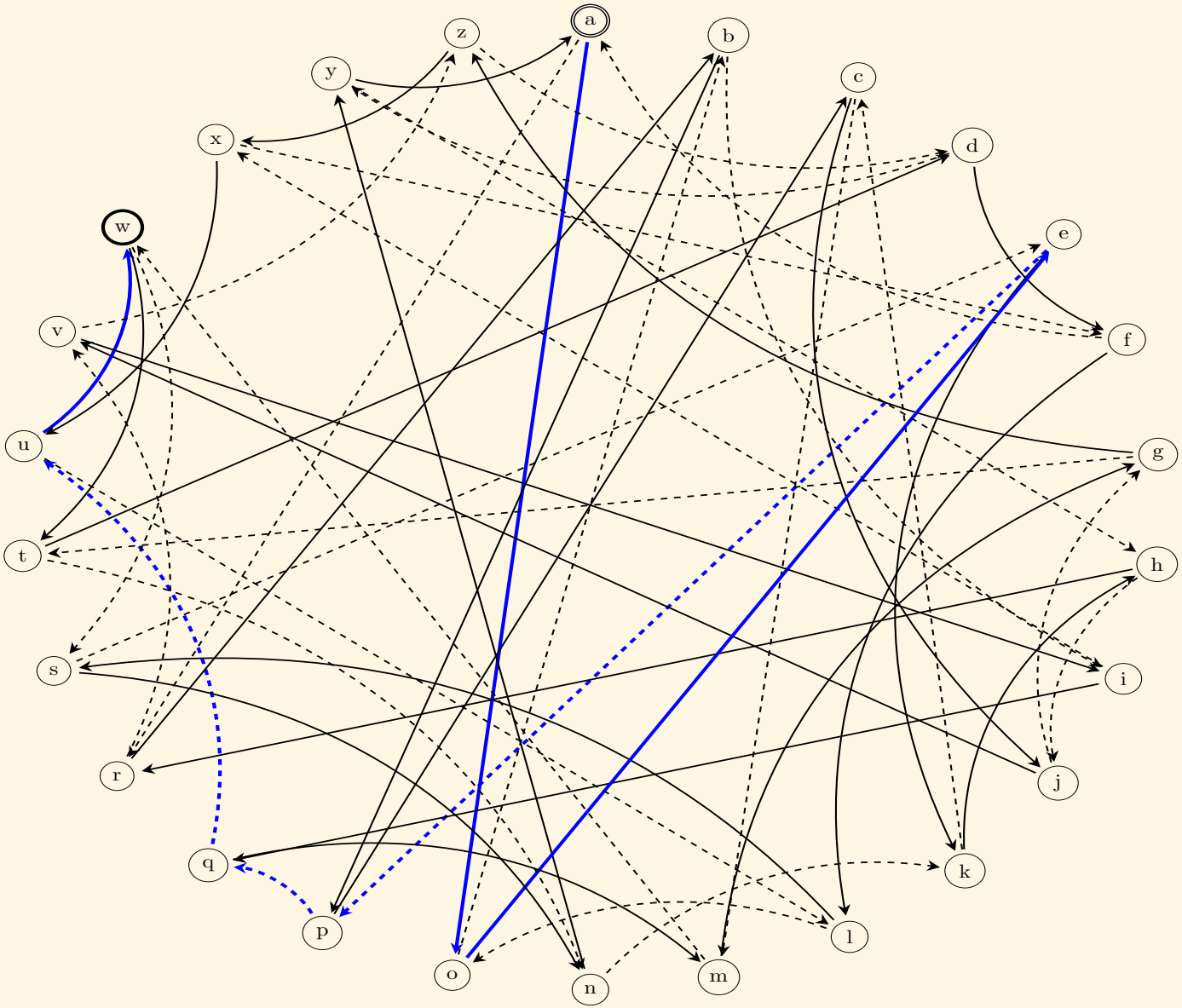
Équipe Canari, Inria Bordeaux Sud-Ouest

Key exchange on a (commutative) graph



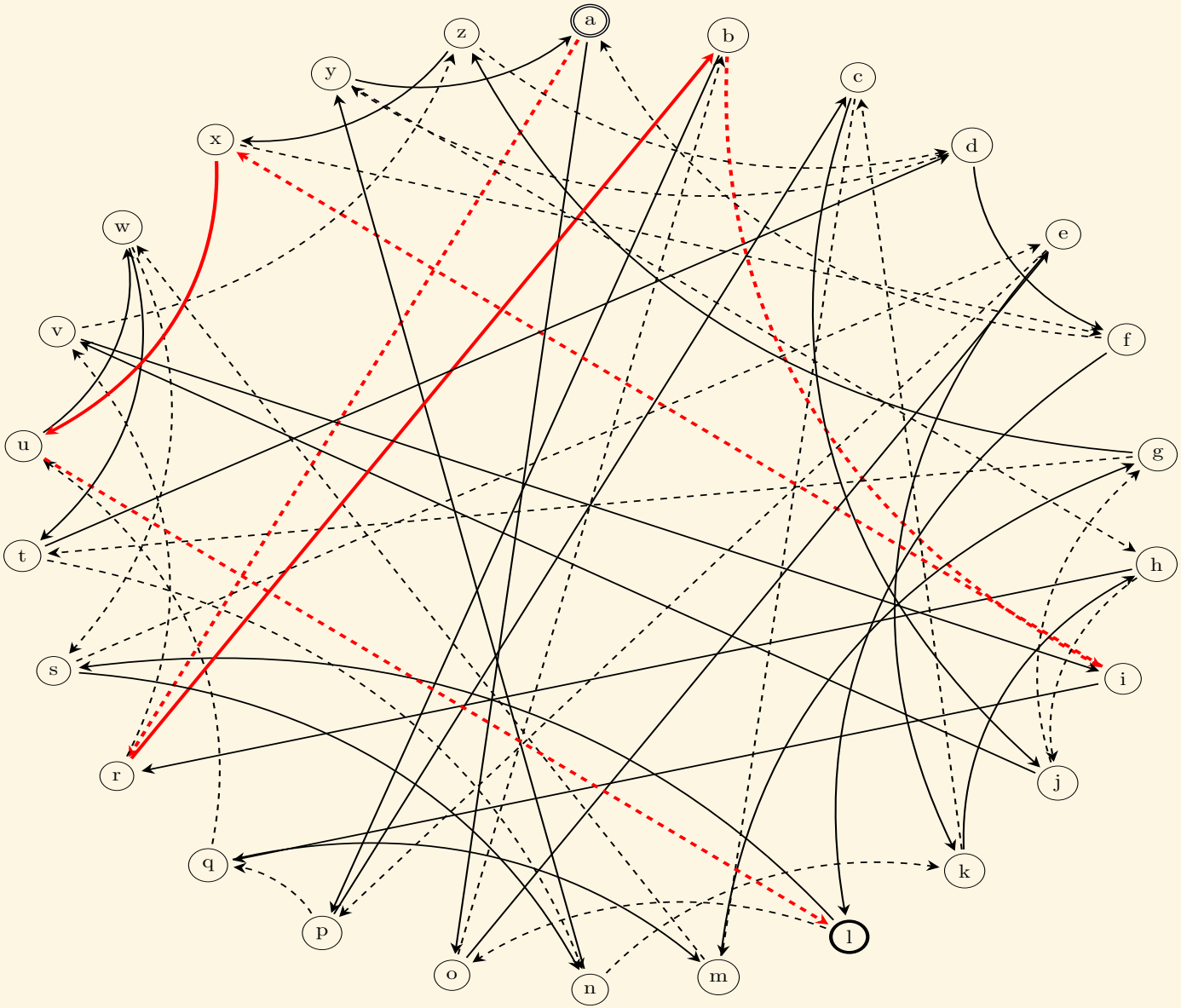
Key exchange on a (commutative) graph

Alice starts from 'a', follows the path 001110, and get 'w'.



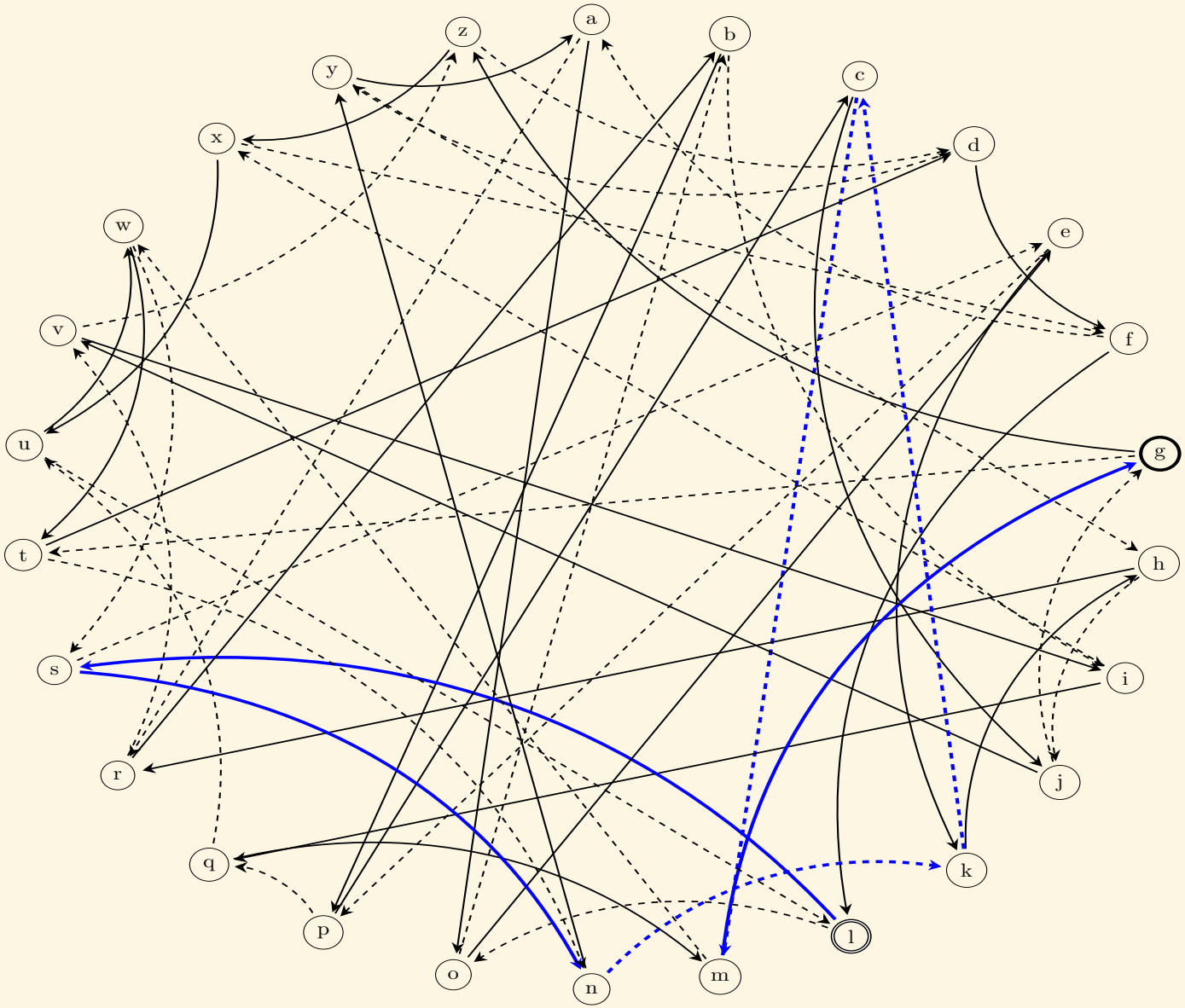
Key exchange on a (commutative) graph

Bob starts from 'a', follows the path 101101, and get 'l'.



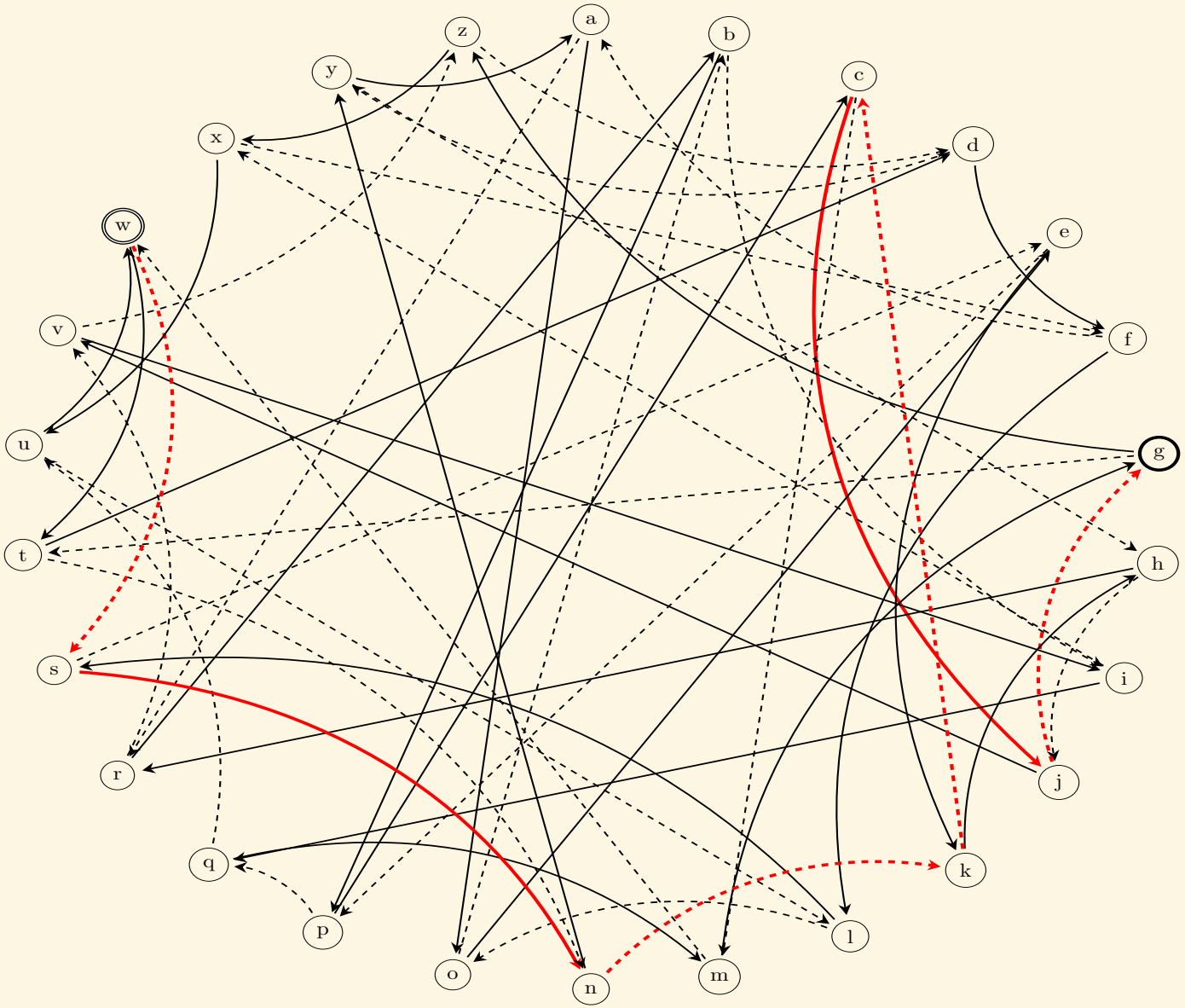
Key exchange on a (commutative) graph

Alice starts from 'l', follows the path 001110, and get 'g'.



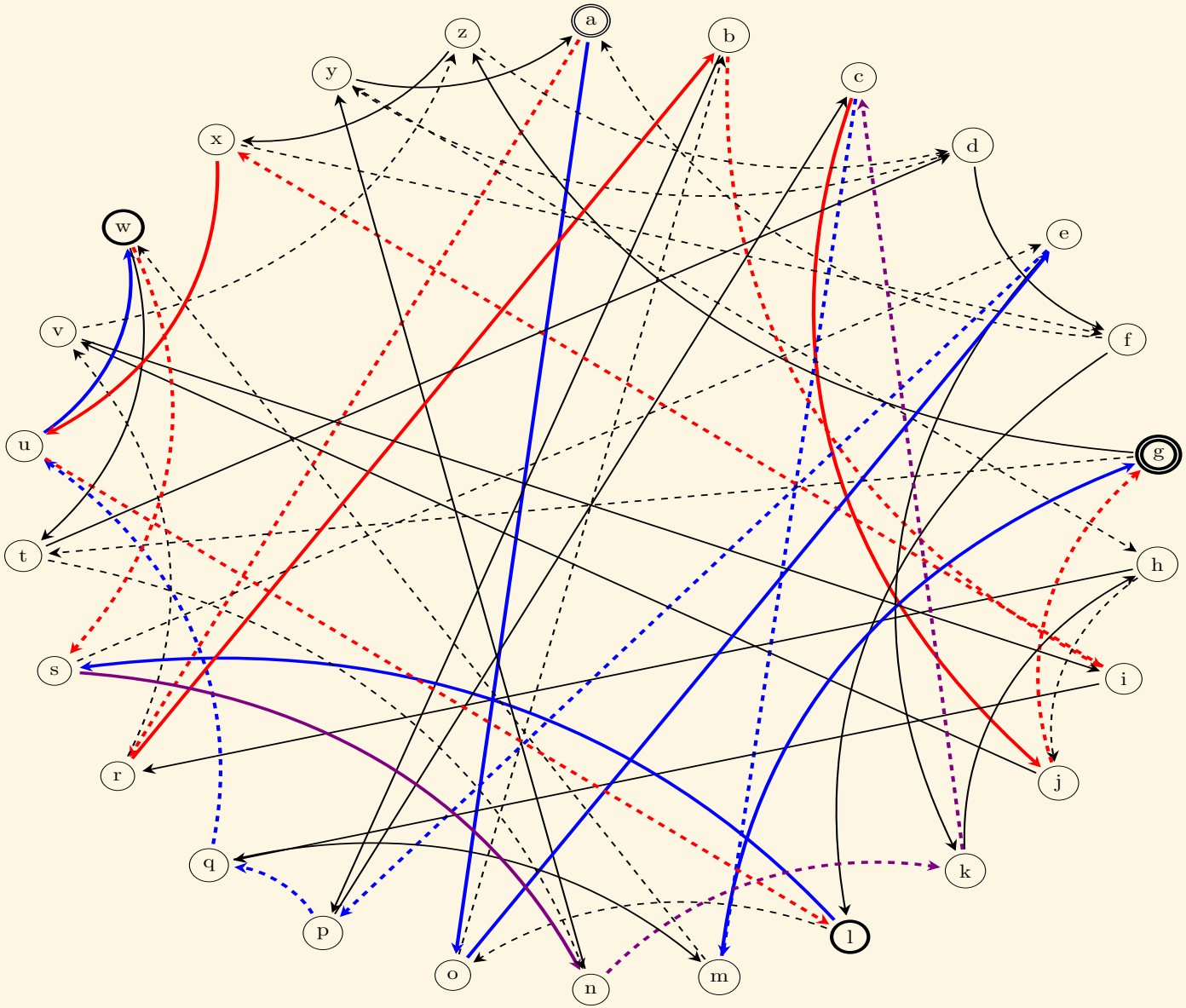
Key exchange on a (commutative) graph

Bob starts from 'w', follows the path 101101, and get 'g'.



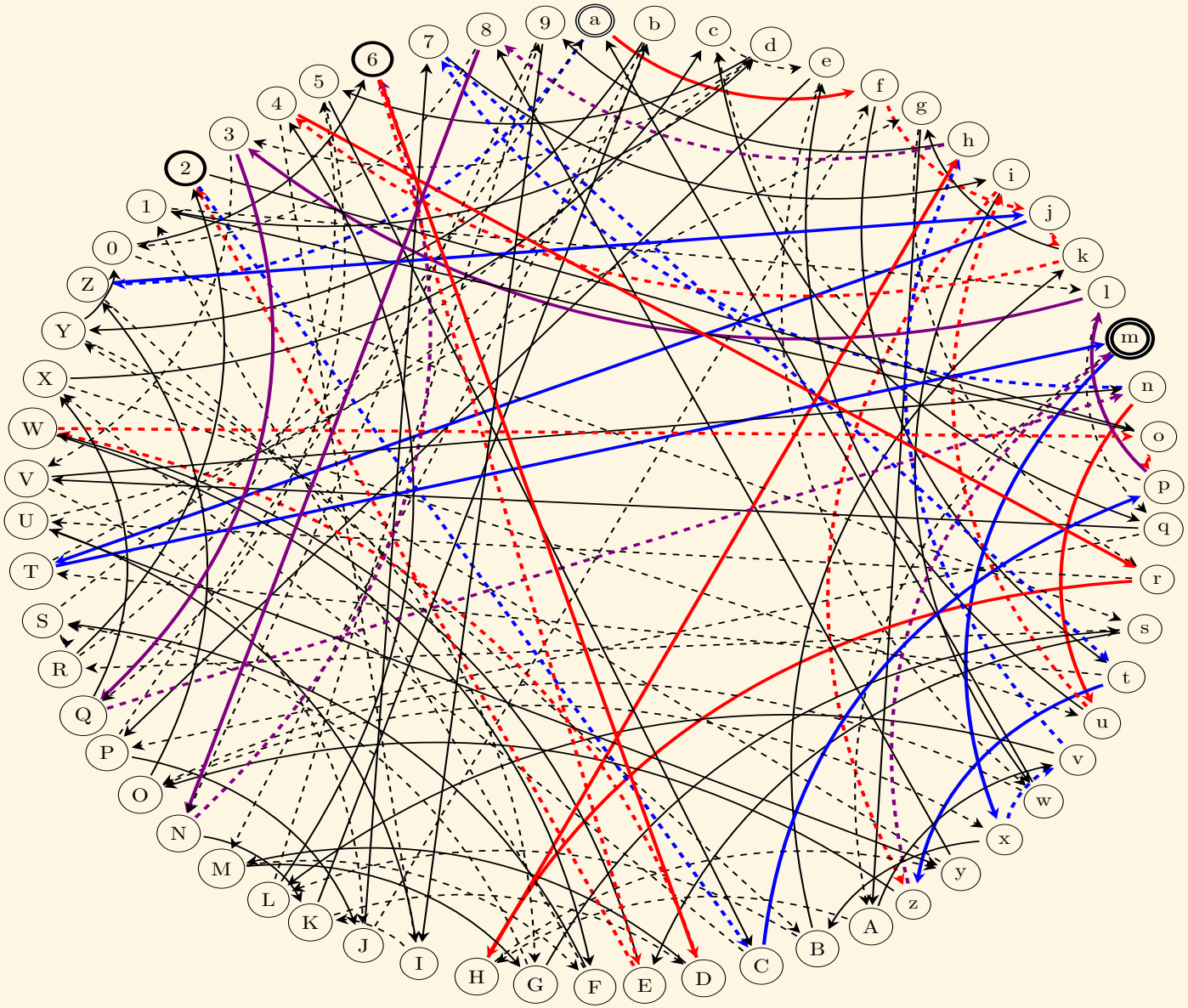
Key exchange on a (commutative) graph

The full exchange:



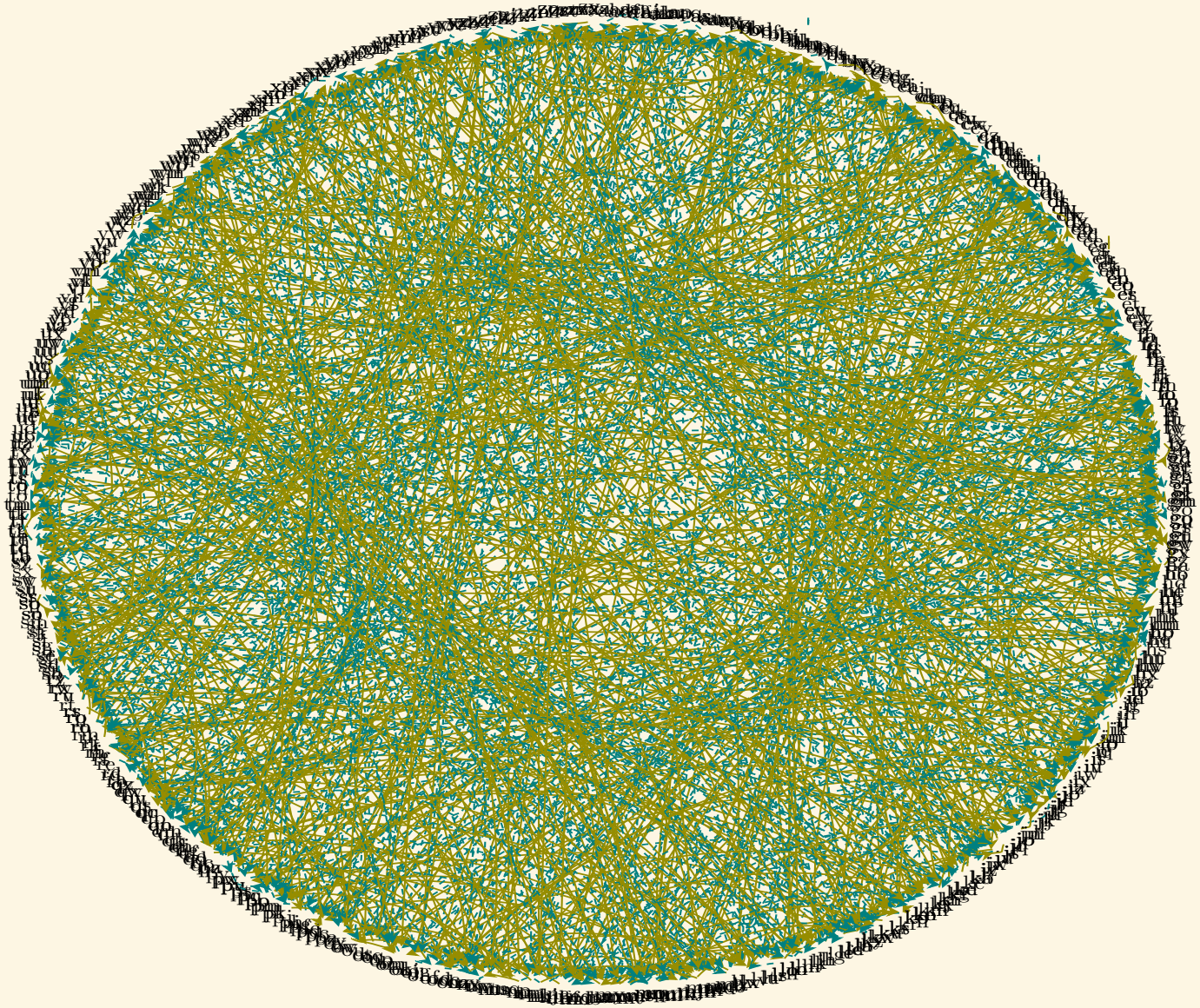
Key exchange on a (commutative) graph

Bigger graph (62 nodes)



Key exchange on a (commutative) graph

Even bigger graph (676 nodes)

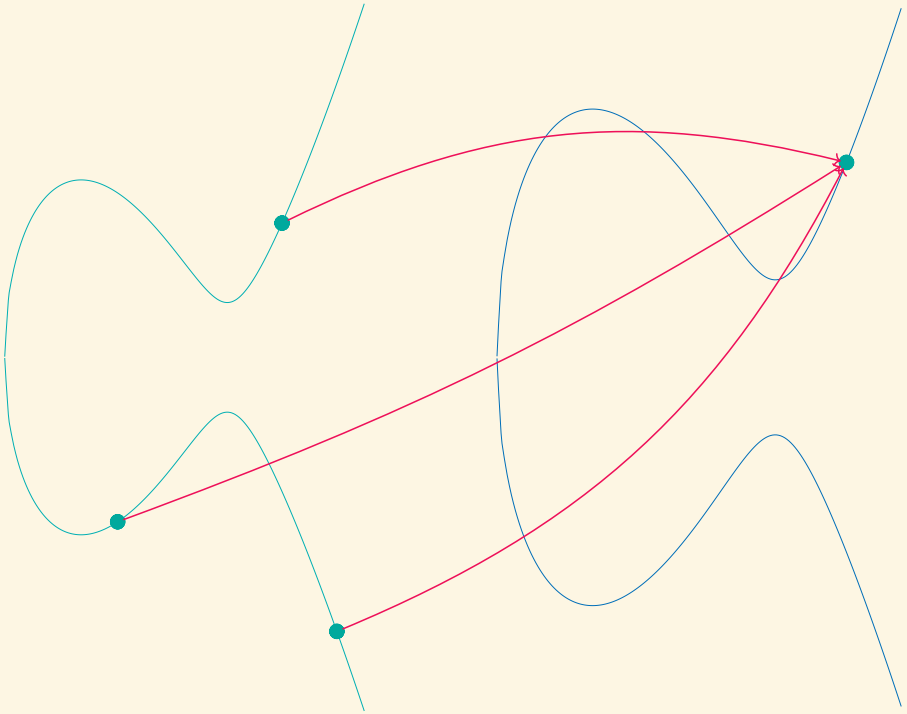


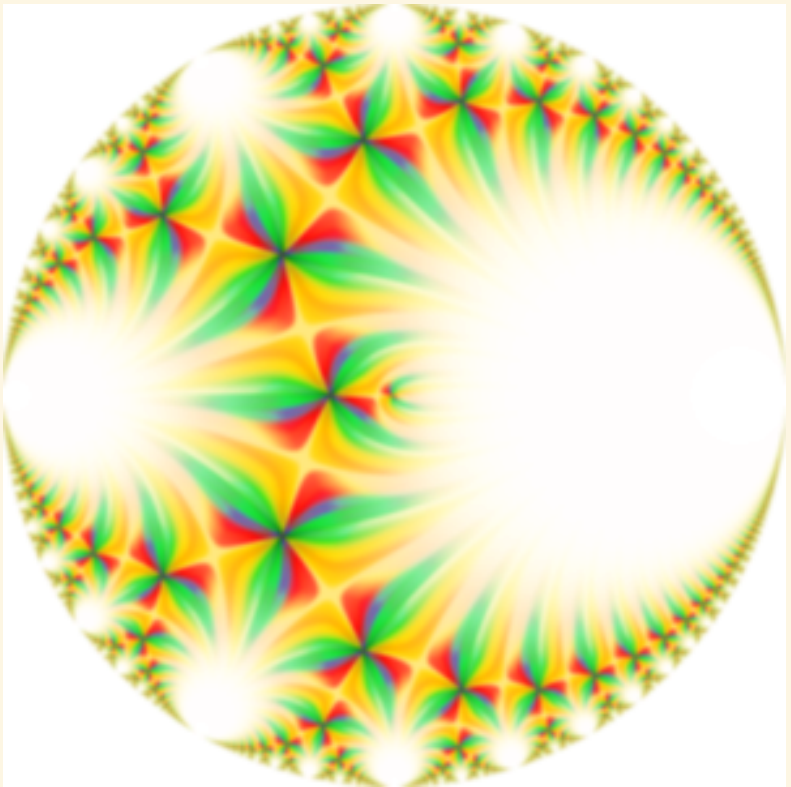
Graphs for key exchange

- Needs a graph with good mixing properties:
  A path of length $O(\log N)$ gives a uniform node $\Rightarrow$ Ramanujan/expander graph.
- The graph does not fit in memory $(N = 2^{256})$.
- Needs an algorithm taking a node as input and giving the neighbour nodes as output.

$\Rightarrow$ Isogeny graphs of elliptic curves

$E_1 : y^2 = x^3 + a_1 x + b_1$ $\qquad$ $E_2 : y^2 = x^3 + a_2 x + b_2$





Isogeny based cryptography

☺ Very compact keys!

- Signature: SQISignHD
  Public key: 64B, signature 109B
  Lattices: 666B–2420B ECDSA: 32B
- NIKE (Non Interactive Key Exchange): $\otimes$-MIKE: 64B
  Lattices: 220KB
- VRF (Verifiable Random Function): DeuringVRF
  Public key 192B, Proof 256B
  Lattices: 2.6KB–39KB

☹ Very slow

- But lot of progress on efficiency recently! (see later)

Isogeny based cryptography brings **diversity** to our other post-quantum schemes like lattice based cryptography or code cryptography who rely on **noisy linear algebra** (over $\mathbb{Z}$ or $\mathbb{F}_2$ respectively)

Graphs in Isogeny based cryptography

Security:

- Random walking in the graph is easy
- Finding a path between two curves is hard
  Even for a quantum computer! ⇒ Post-Quantum Cryptography!

Two kind of isogeny graphs

- Ordinary (or oriented) isogeny graphs
- The graph is commutative
- ☺ Key exchange is easy
- ☹ Path finding easier than for a general graph (Kuperberg's quantum subexponential algorithm)

- Supersingular isogeny graphs
- The graph is non commutative
- ☺ Best known algorithm for path finding is quantum exponential
- ☹ Key exchange is not obvious

Part 1 (1997–2010) — The prehistory: ordinary isogeny graphs

- [Couveignes 1997]: Hard Homogeneous Spaces
  First suggestion to use isogeny graphs of ordinary elliptic curves for key exchange.
  Commutative graphs!
- Rediscovered in [Rostovtsev–Stolbunov (2006)]: Public-key cryptosystem based on isogenies

- [De Feo, Kieffer, Smith 2018]: Towards practical key exchange from ordinary isogeny graphs
  Huge computationally intensive search for "optimised parameters"
  520s for a key exchange (512b=64B)

Part 2 (2011 – 2022) — The history: supersingular isogeny graphs

- [Charles, Goren, Lauter 2006] Cryptographic hash functions from expander graphs
  Switch from commutative ordinary graphs to non commutative supersingular graphs over $\mathbb{F}_{p^2}$

- [De Feo, Jao (2011)],[De Feo, Jao, Plût (2014)]: Towards quantum-resistant cryptosystems from supersingular elliptic curve isogenies
  SIDH: Supersingular Isogeny Diffie-Hellman
  Non commutative graph ⇒ the key exchange needs extra informations
  Not a pure path finding graph problem anymore!
- [Costello, Longa, Naehrig 2016]: Efficient algorithms for supersingular isogeny Diffie-Hellman
  ≈ 50MCycles ≈ 25ms, 564B
- ☹ [Galbraith, Petit, Shani, Ti 2016]: On the security of supersingular isogeny cryptosystems
  Active adaptative attack against SIDH, can no longer be used as a NIKE
  Replaced by SIKE: a PKE (Public Key Encapsulation) scheme
- ☺ NIST PQC Round 4, 5 July 2022: SIKE advances to fourth round

- [Castryck, Lange, Martindale, Panny, Renes 2018]: CSIDH: An Efficient Post-Quantum Commutative Group Action
  Use supersingular graphs over $\mathbb{F}_p$, same commutative properties as ordinary graphs but much easier to find optimised parameters
  ≈ 100MCycles ≈ 50ms, 64B

- [De Feo, Kohel, Leroux, Petit, Wesolowski 2020]: SQISign: compact post-quantum signatures from quaternions and isogenies

Part 3 (2022) — The downfall of isogeny based cryptography

- [Peikert 2019]: He Gives C-Sieves on the CSIDH
  "This strongly invalidates its claimed NIST level 1 quantum security [...] Moreover, under analogous assumptions for CSIDH-1024 and -1792, which target higher NIST security levels [...] even these instantiations fall short of level 1."
  Key size recommendation for CSIDH: from 512b to 2000b–5000b…

- 2022-07-05: SIKE advances to fourth round of the NIST PQC call
- 2022-07-30: [Castryck, Decru], "An efficient key recovery attack on SIDH"
- 2022-08-08: [Maino, Martindale], "An attack on SIDH with arbitrary starting curve"
- 2022-08-10: [R.], "Breaking SIDH in polynomial time"
  Use objects in higher dimension (2, 4 and 8) to break SIDH.

☹ SIDH/SIKE, and many protocols derived from it, are completely dead (full attacks in $\approx$ 1ooms)

- The attacks use the extra SIDH information in the key exchange, the pure path finding graph problem still secure
- Only SQISign remains (and CSIDH with increased parameter sizes)

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Part 4 (2022–) — the higher dimensional renaissance

- The attacks against SIDH exploited higher dimensional isogenies. These add considerable flexibility, and they were quickly exploited constructively:
  [R. 2022]: Evaluating isogenies in polylogarithmic time

- [Dartois, Leroux, R., Wesolowski (2023)]: SQISignHD: New Dimensions in Cryptography
  Improvement of SQISign using dimension 4 for verification
  Best paper award (Eurocrypt 2024)
- [Basso, De Feo, Dartois, Leroux, Maino, Pope, R., Wesolowski (2024)]: SQIsign2D-West: The Fast, the Small, and the Safer
  Improvement of SQISign using dimension 2 for signature and verification

- [R. (October 2024)]: The module action for isogeny based cryptography
  Uses rank 2 modules to build a new NIKE: $\otimes$-MIKE (Module Isogeny Key Exchange), with keys of only 64B

See Castryck's invited talk at Eurocrypt 2024:
"An attack became a tool: Isogeny based cryptography 2.0"

SQISign2d (West) and SQISignHD

	SQIsign	SQIsign2d
Public key	64B	64B
Signatures	177B	148B
Clean security proof	☹	☺
Keygen (Mcycles)	400	60
Sign (Mcycles)	1880	160
Verify (Mcycles)	29	9

SQIsignHDv2: Signature size: 109B, $\approx 5\times$ faster than SQIsign2d
Verification expected $\approx 8\times$ slower