CIAO — Cryptography, Isogenies and Abelians varieties

Overwhelming

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Elliptic curve cryptography

Currently, the best standard public key cryptography system:

- ☺ Extremely small parameters (256 bits);
- ☺ Extremely fast;
- ☺ More powerful than RSA;
- ☹ Broken by quantum computers.

Goal: Post quantum elliptic cryptography

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Goal: Post quantum elliptic cryptography

Public key exchange

- How to exchange a secret key across a public channel?
- Diffie-Helmann (1976): let $g \in G$ be an element of a group
- Alice uses a random a and sends g^a;
- Bob uses a random b and sends g^b;
- Common secret key: $g^{ab} = g^{a\,b} = g^{b\,a}$
- Attack: Diffie-Helmann problem: recover g^{ab} from (g, g^a, g^b).
- Easy when the Discrete Logarithm Problem (DLP) is easy;
- In a generic group can be reduced to the DLP.

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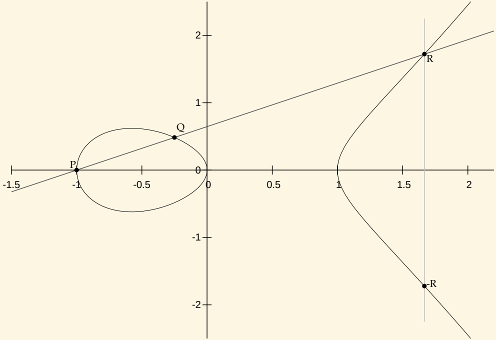
Elliptic curves

Definition (char $k \neq 2,3$)

An elliptic curve is a plane curve

$$y^2 = x^3 + ax + b \qquad 4a^3 + 27b^2 \neq 0.$$



Exponentiation:

$$(\ell, P) \mapsto \ell P$$

DLP:

$$(P, \ell P) \mapsto \ell$$

ECC vs RSA for 128 bits of security

- ECC (Curve25519) 256 bits:

AAAAC3NzaC1lZDI1NTE5AAAAIMoNrNYhU7CY1Xs6v4Nm1V6oRHs/FEE8P+XaZ0PcxPzz

- RSA 3248 bits:

MIIHRgIBAAKCAZcAvlGW+b5L2tmqb5bUJMrfLHgr2jga/Q/8IJ5QJqeSsB7xLVT/
ODN3KNSPxyjaHmDNdDTwgsikZvPYeyZWWFLP0B0vgwDqQugUGHVfg4c73ZolqZk6
1nA45XZGHUPt98p4+ghPag5JyvAVsf1cF/VlttBHbu/noyIAC4F3tHP81nn+lOnB
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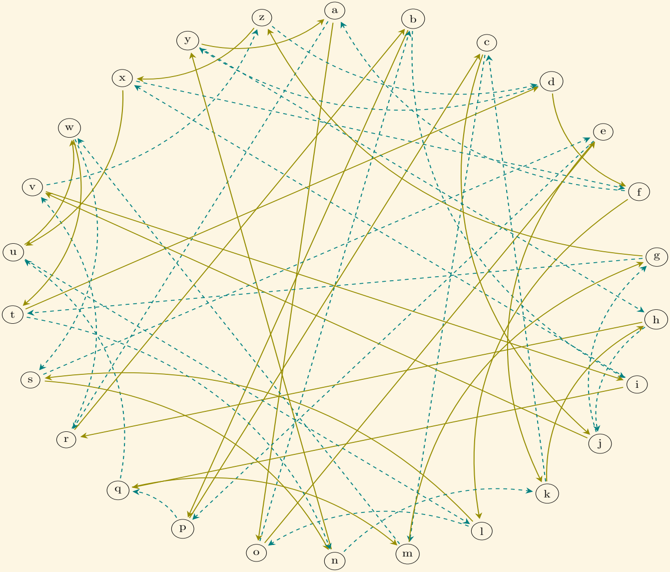
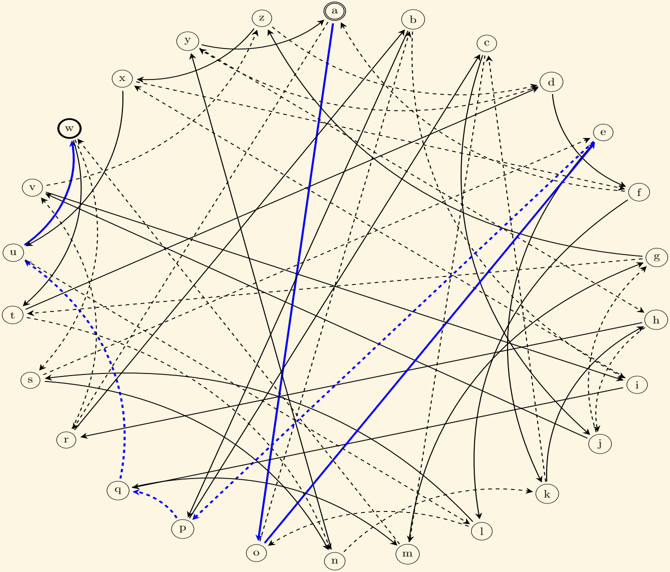
Key exchange on a graph



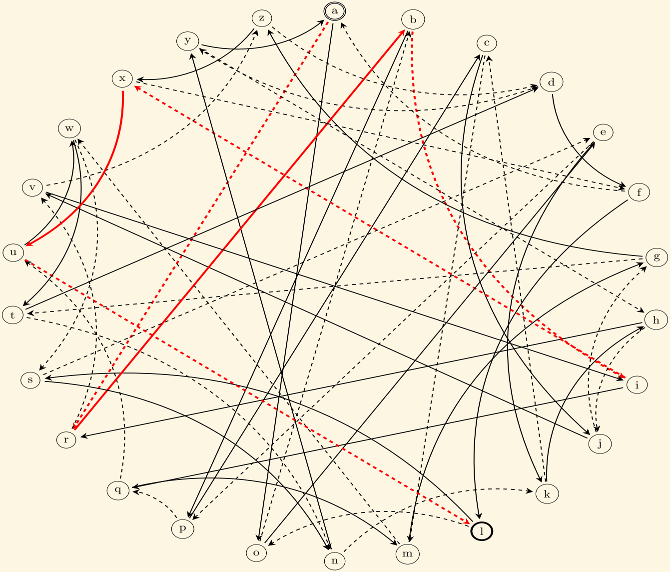
Key exchange on a graph

Alice starts from ‘a’, follows the path 001110, and get ‘w’.



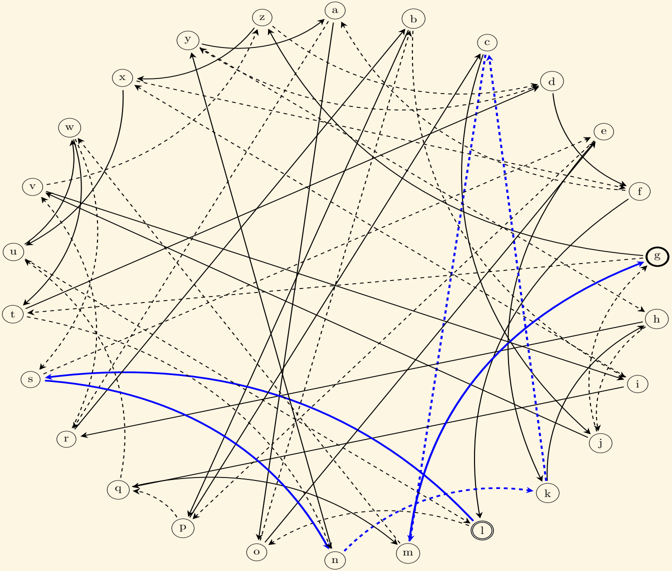
Key exchange on a graph

Bob starts from ‘a’, follows the path 101101, and get ‘l’.

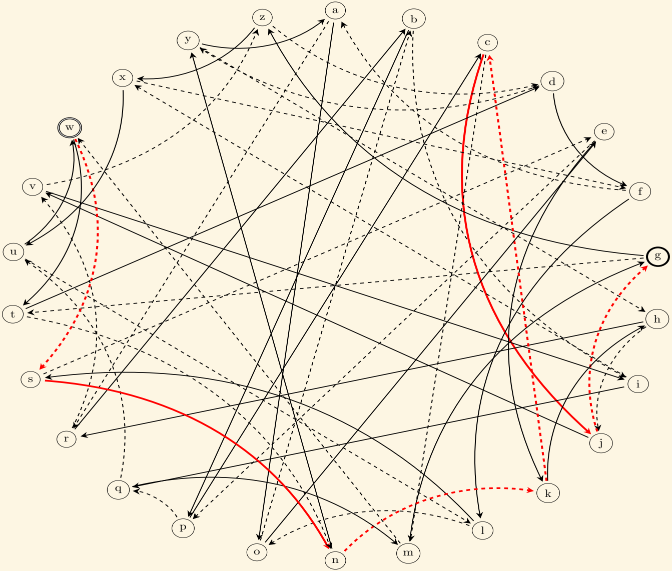


Key exchange on a graph

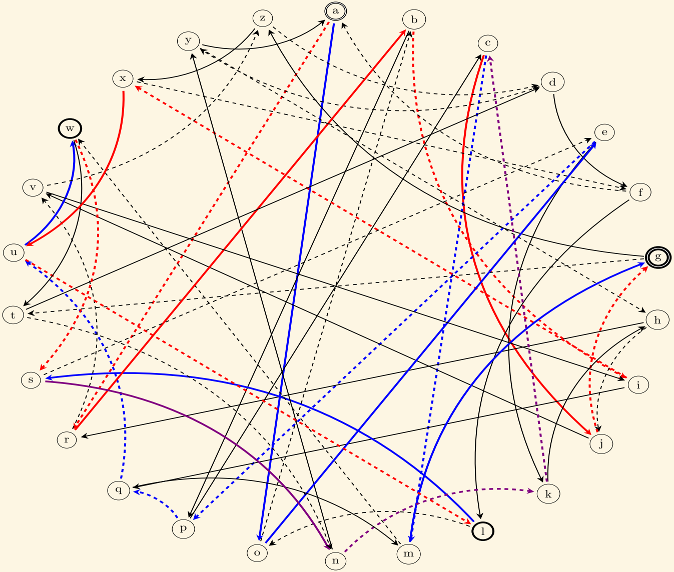
Alice starts from ‘l’, follows the path 001110, and get ‘g’.



Bob starts from ‘w’, follows the path 101101, and get ‘g’.

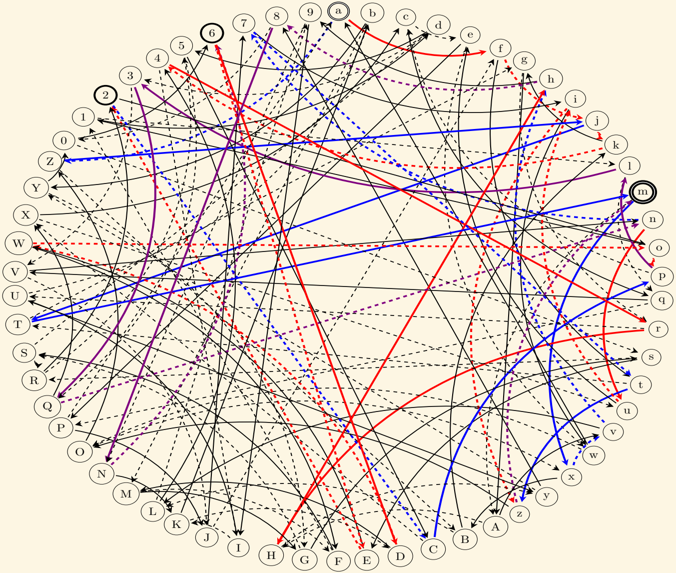


The full exchange:



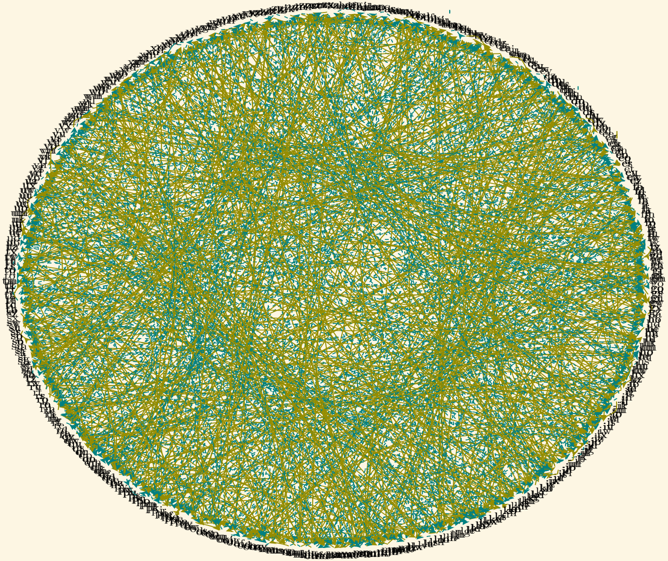
Key exchange on a graph

Bigger graph (62 nodes)



Key exchange on a graph

Even bigger graph (676 nodes)



SIDH: supersingular elliptic curve Diffie-Helmann (De Feo, Jao, Plût)

- Use the isogeny graph of a supersingular elliptic curve E over $\mathbb{F}_{p^2}$.
- There are $O(p)$ nodes and the graph is an expander graph.
- The endomorphism ring is a quaternion algebra (ramified at p and infinity), which is non commutative.
- The isogeny graph is a Cayley graph for the groupoid class group.
- The key exchange can be seen as a pushforward:

$$E/K_A \otimes_E E/K_B = E/(K_A + K_B)$$

⇒ Needs p of 512 bits, and total key size is 2048 bits.

Organisation

- Project coordinator: Damien Robert (CR Inria Bordeaux).
- Members in Bordeaux: Bill Allombert, Jean-Marc Couveignes, Jean Kieffer, Aurel Page
- Exterior members: Luca De Feo, Benjamin Smith (Paris), Cyril Bouvier, Laurent Imbert (Montpellier)
- Temporary members: Jean Kieffer (PhD student), one year postdoc.

Work program

- Computational aspects of isogenies: arithmetic over finite fields, efficient isogenies, models for elliptic curves, implementations.
- Cryptographic protocols related to isogenies: Key exchange and encryption, Signatures and authentication, Verifiable Delay Functions
- Higher dimensional isogenies: isogenies for abelian varieties, moduli spaces, isogeny graphs, Higher dimensional supersingular isogeny Diffie–Hellman
- Security of isogeny-based cryptosystems: security reductions and security parameters, point counting and endomorphism rings computation, security in the wild