FAST – (Harder Better) FAster STronger Cryptography

2020/02/19 – Bordeaux

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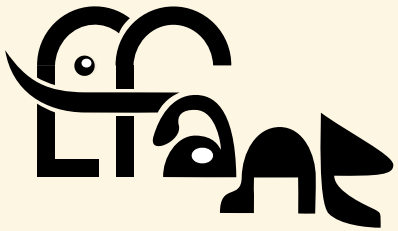


Goal

Cryptology:

- Encryption;
- Authenticity;
- Integrity.

Public key cryptology is based on a one way (trapdoor) function ⇒ asymmetric encryption, signatures, zero-knowledge proofs...

Goal: Improve and extend elliptic curve cryptography to

- Secure the Internet of Things;
- Prepare the next generation of cryptosystems able to resist to quantum computers.

Organisation

- Joint team between LFANT (Lithe and fast algorithmic number theory) https://lfant.math.u-bordeaux.fr/ and PREMA (the Pole of Research in Mathematics and Applications in Africa) http://prmasi.org/;
- Project coordinators: Tony Ezome, Senior Lecturer/Researcher (Cames), University of Sciences and Technology of Masuku (USTM), and Damien Robert (CR Inria).
- PREMA is a Simon's fundation project involving researchers in Cameroun, Gabon, Madagascar, Sénégal along with members in Cote d'Ivoire, Maroc, South Africa and international collaborators in Canada, France, the Netherlands, Singapore.

Results

- Efficiency
  Improving randomness extractions ([KSC+17; CS17]), pseudo-random generators and pseudo-random functions [MV17b].
  - Improving arithmetic and pairing on elliptic curves [GF18; FD17; Fou19; FPE19; MAF19; FD19].
  - Improving normal basis [ES19]
  - Attribute based credentials [SCN19]
- Post quantum cryptography
  - Pairing based signatures [MV17a]
  - Isogenies: modular polynomials for cyclic isogenies between abelian surfaces [MR17], cyclic isogenies given their kernels [DJR+17].
- Misc
  - Attacks [NF19]
  - Arithmetic progression [CM17a; CM17b]
  - Book chapter “Pairings” of the book “Guide to Pairing-Based Cryptography” [EJ17].
- Work in progress:
  - Computing canonical lift of genus 2 curves;
  - Better isogenies in the Hessian model [LF];

Results

- PhDs
  - T. M. Nountu. “Pseudo-Random Generators and Pseudo-Random Functions: Cryptanalysis and Complexity Measures”. PhD thesis. Paris Sciences et Lettres, 2017
  - Aminatou Pecha Njiahouo. Recherche de primitive pour la cryptographie à base de couplage. PhD thesis, Université Paris 8 (France), December 8, 2017.
  - Upcoming PhD thesis: M. Sall: ”Bases Normales, Groupes algébriques et arithmétiques des corps finis” at university Cheikh Anta Diop de Dakar.
  - Upcoming PhD thesis: A. Maiga ”Canonical lift of genus 2 curves” at university Cheikh Anta Diop de Dakar.

Scientific activities for the years 2016–2020

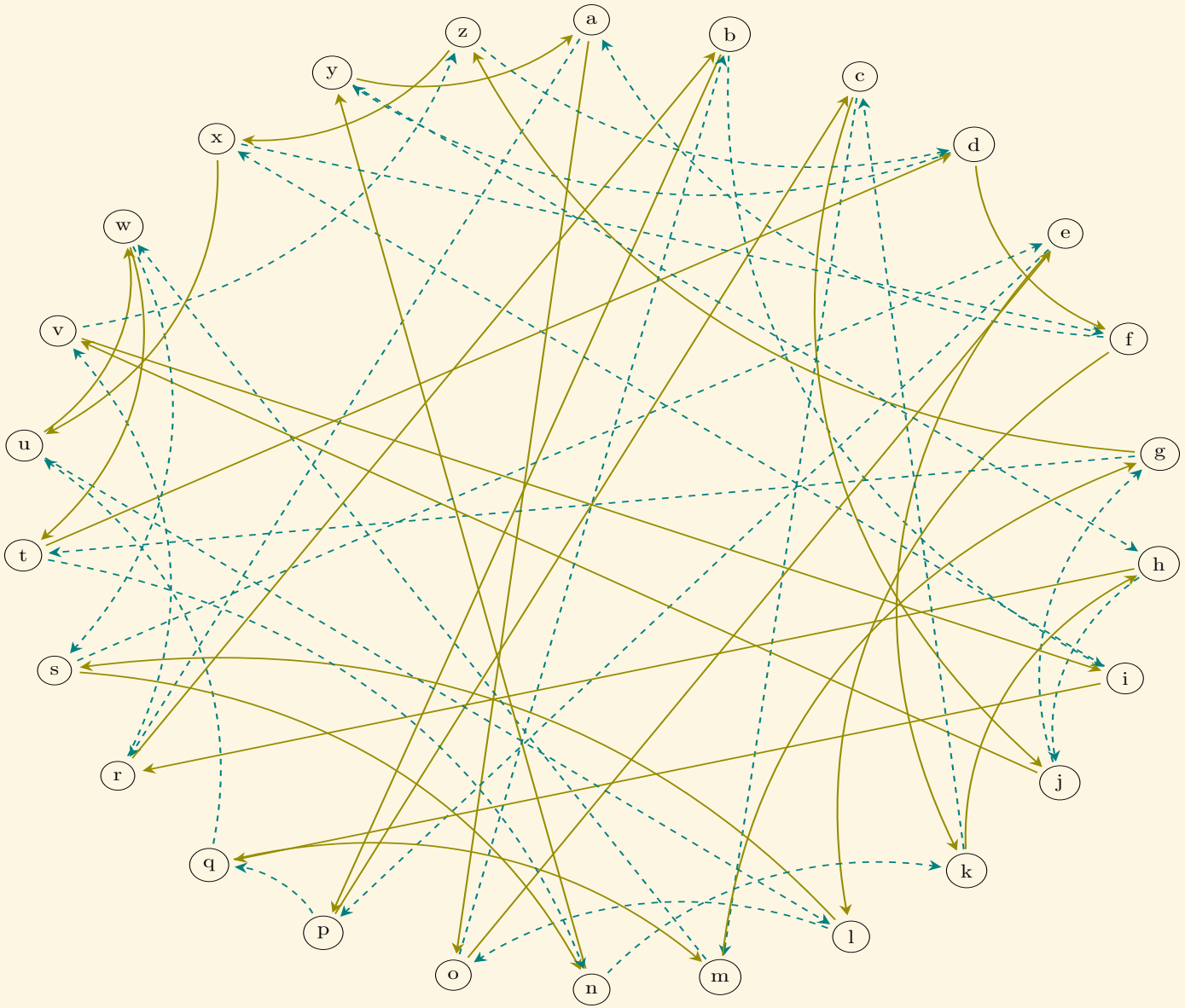
- Lots of short or longer visits;
- One joint workshop at the start;
- Organization of conferences;
- EMA “Mathématiques pour la Cryptographie Post-quantique et Mathématiques pour le Traitement du Signal” at the École Polytechnique de Thiès (Sénégal) from May 10 to May 23 2017 by Djiby Sow and Abdoul Asiz Ciss .
- Ecole Mathématique Africaine (from April 02 to 04 2018 at Franceville), http://prmasi.org/african-mathematical-school-ams-from-april-02-to-april-14-2018 by Tony Ezome;
- Aminatou Pecha organized a CIMPA school (form 2 to 12 July 2019) and the first meeting for women in Mathematics in Central Africa from 13 to 14 July 2019 at AIMS-Cameroon in Limbe.
- Tony Ezome made 4 Teaching stays (two in Burkina Faso, one in The Republic of Congo, and one in Senegal) to introduce Algebraic number theory and algebraic geometry to Master Students.

Assessment and Feedback

- ☺ Lots of teaching and conferences.
- ☺ Four (upcoming) PhD (specific funding for PhD students);
- ☺ Helped opening masters in cryptography in Africa;
- ☹ No industrial collaborations;
- ☹ Not enough visible inter-parternship research collaborations;

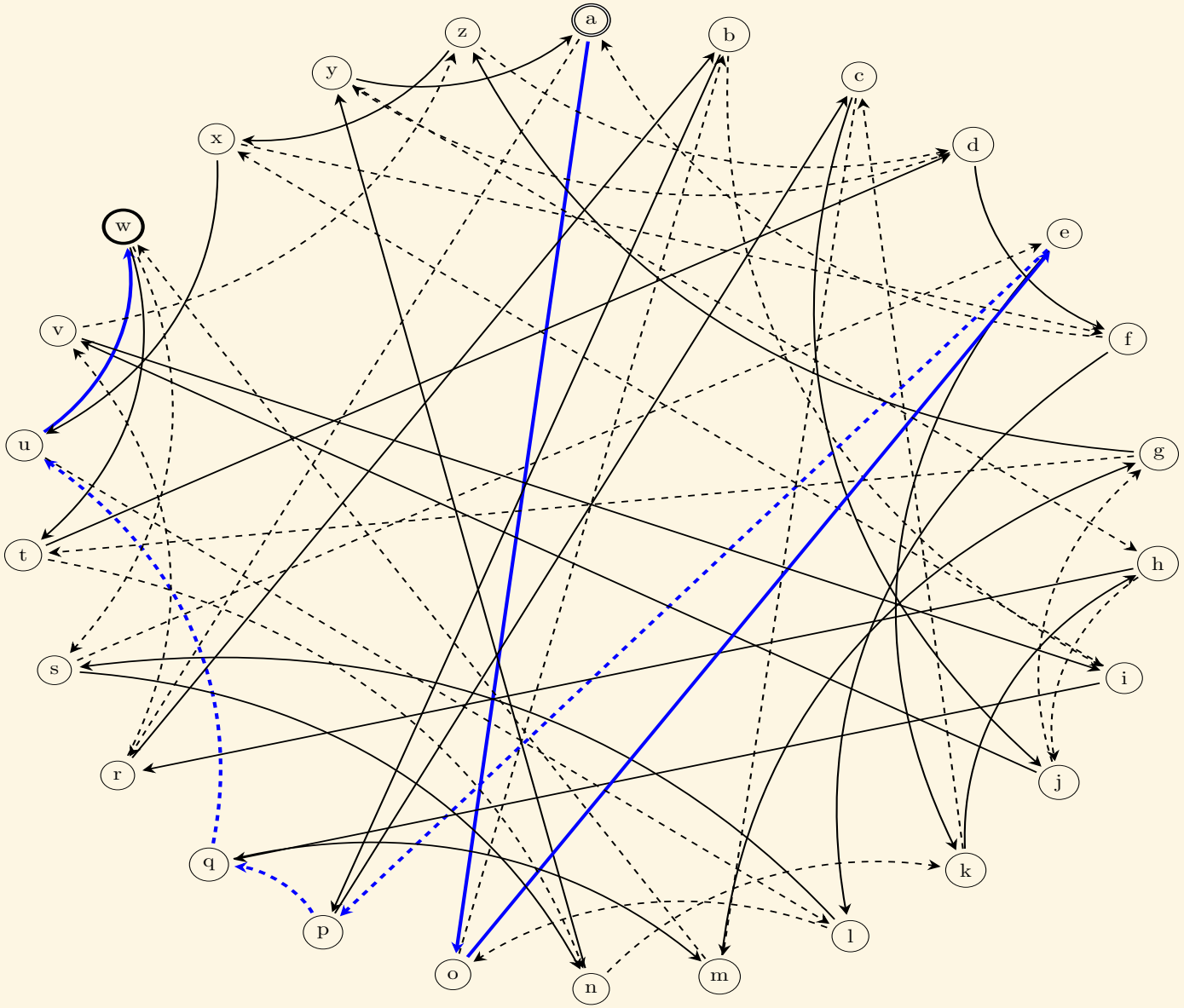
- Lots of Visa problem;
- Administrative burden for long stays;
- Budget cutoff mid December for the last year.

Key exchange on a graph



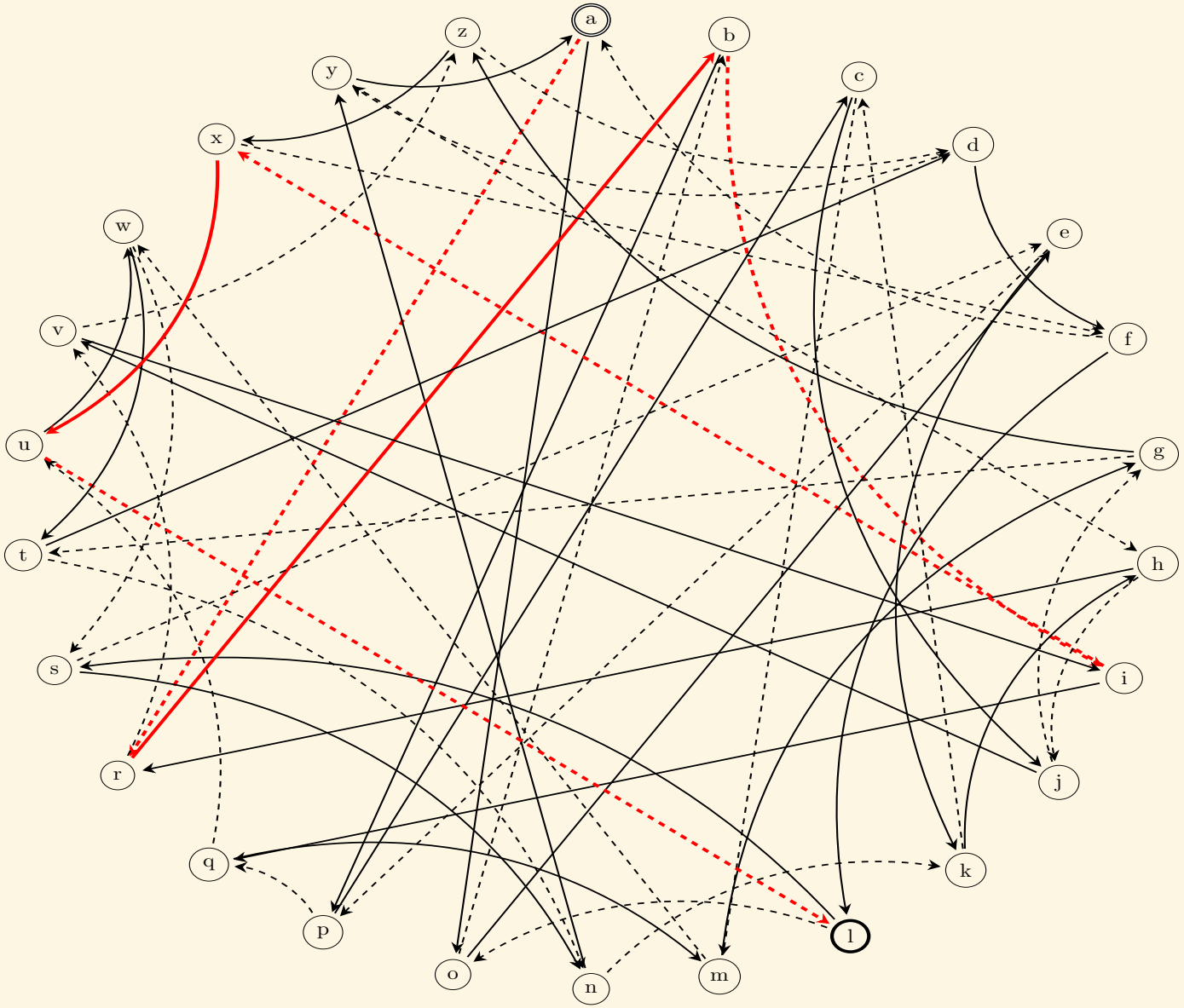
Key exchange on a graph

Alice starts from ‘a’, follow the path 001110, and get ‘w’.



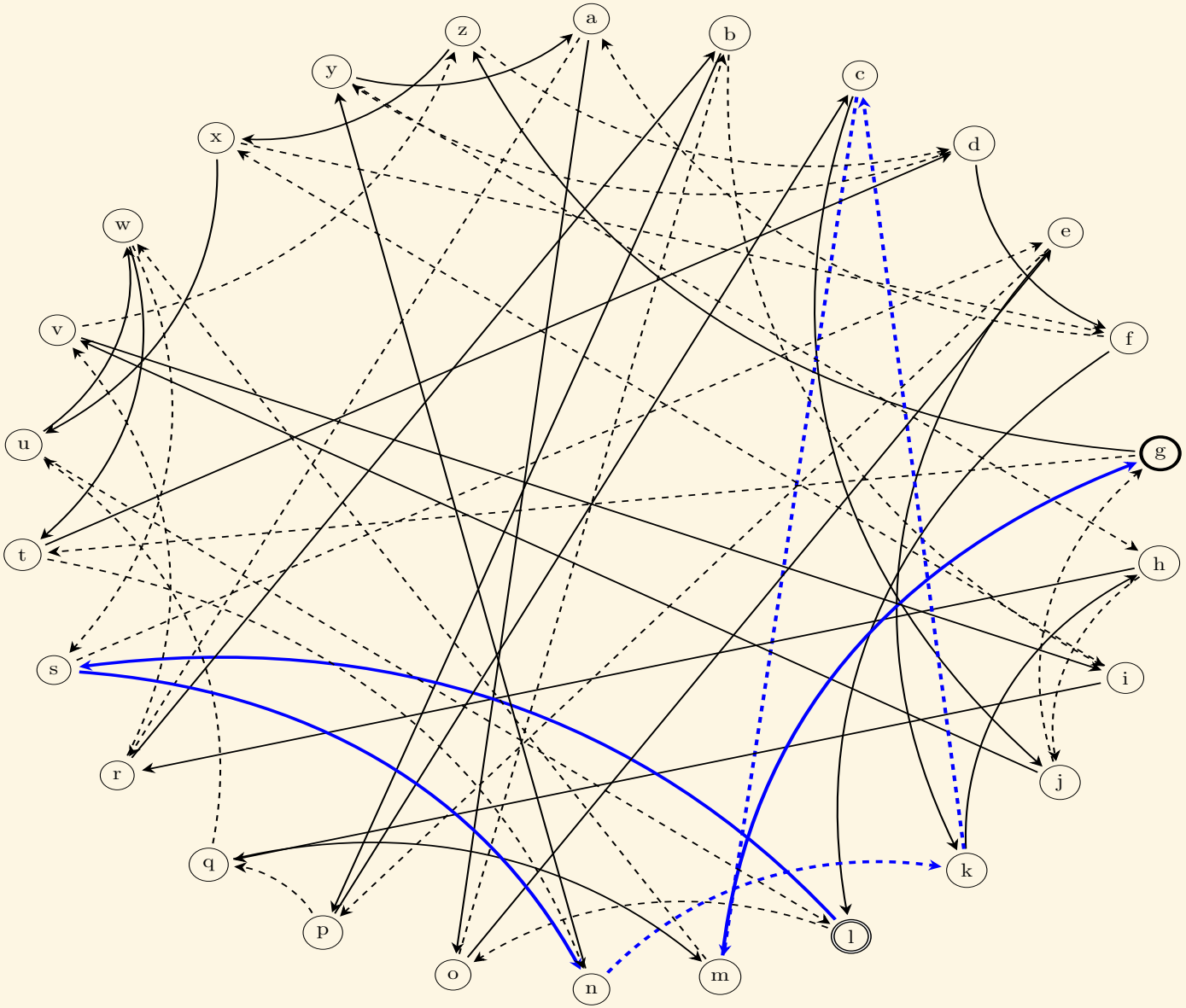
Key exchange on a graph

Bob starts from ‘a’, follow the path 101101, and get ‘l’.



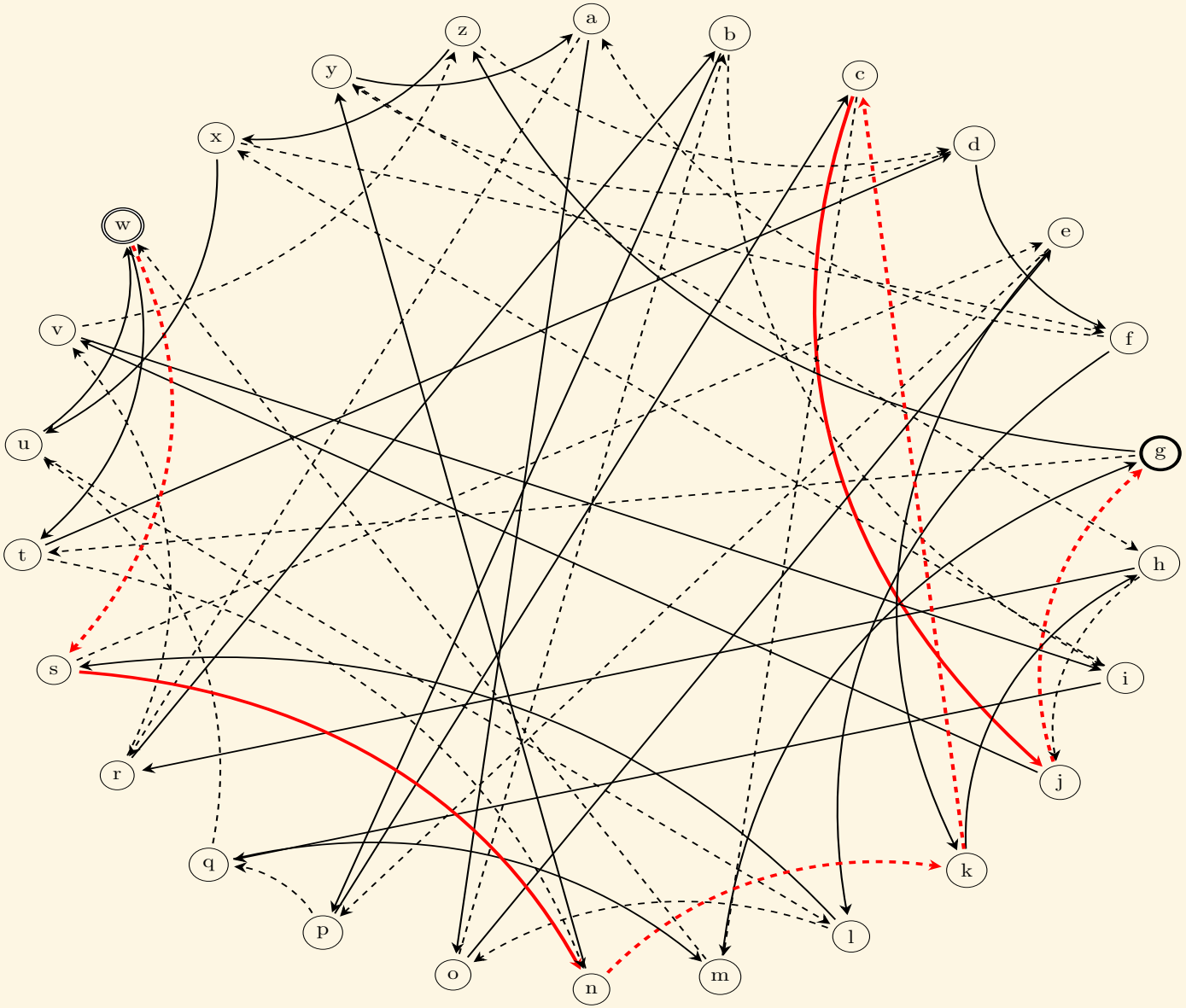
Key exchange on a graph

Alice starts from ‘l’, follow the path 001110, and get ‘g’.



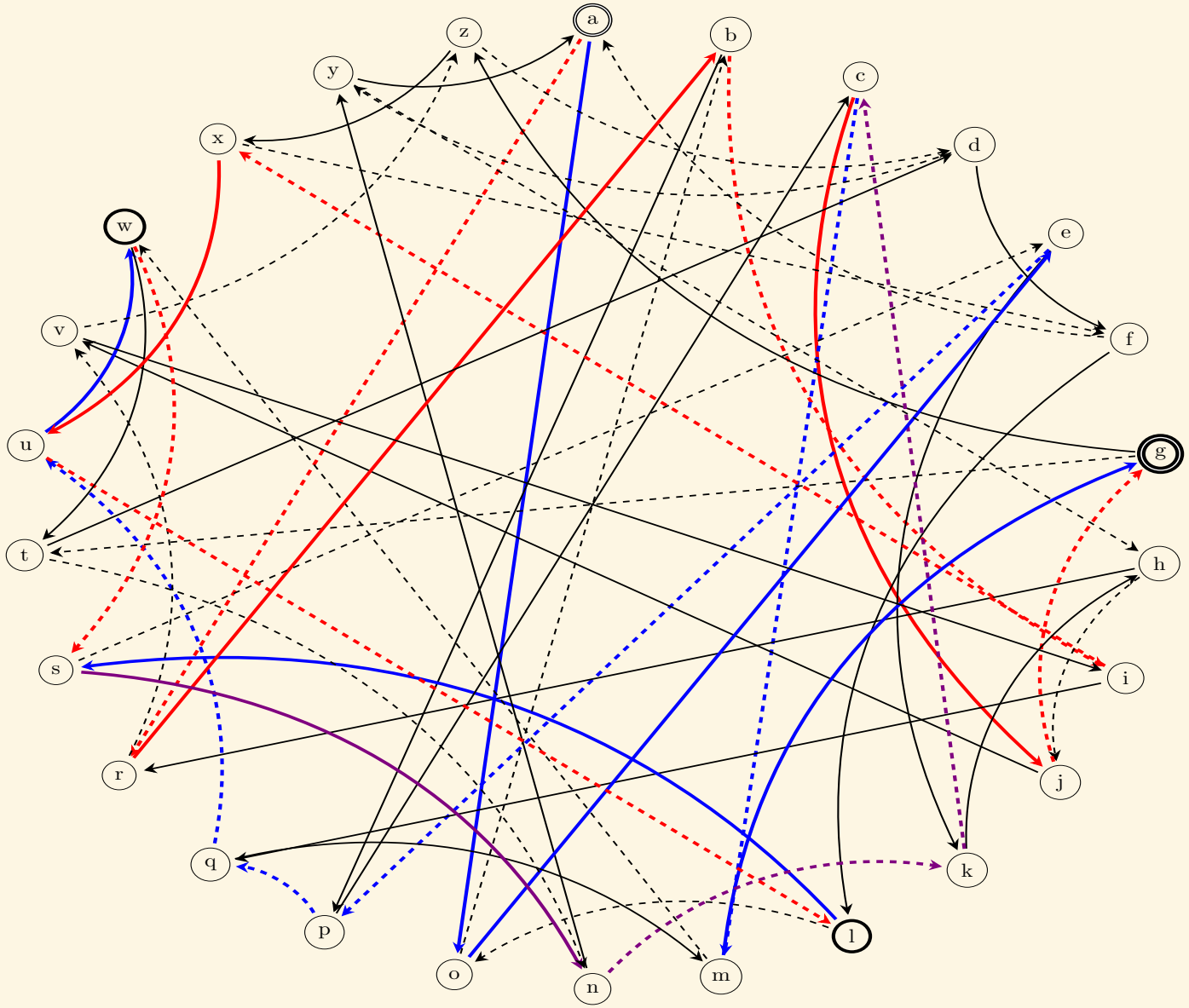
Key exchange on a graph

Bob starts from ‘w’, follow the path 101101, and get ‘g’.



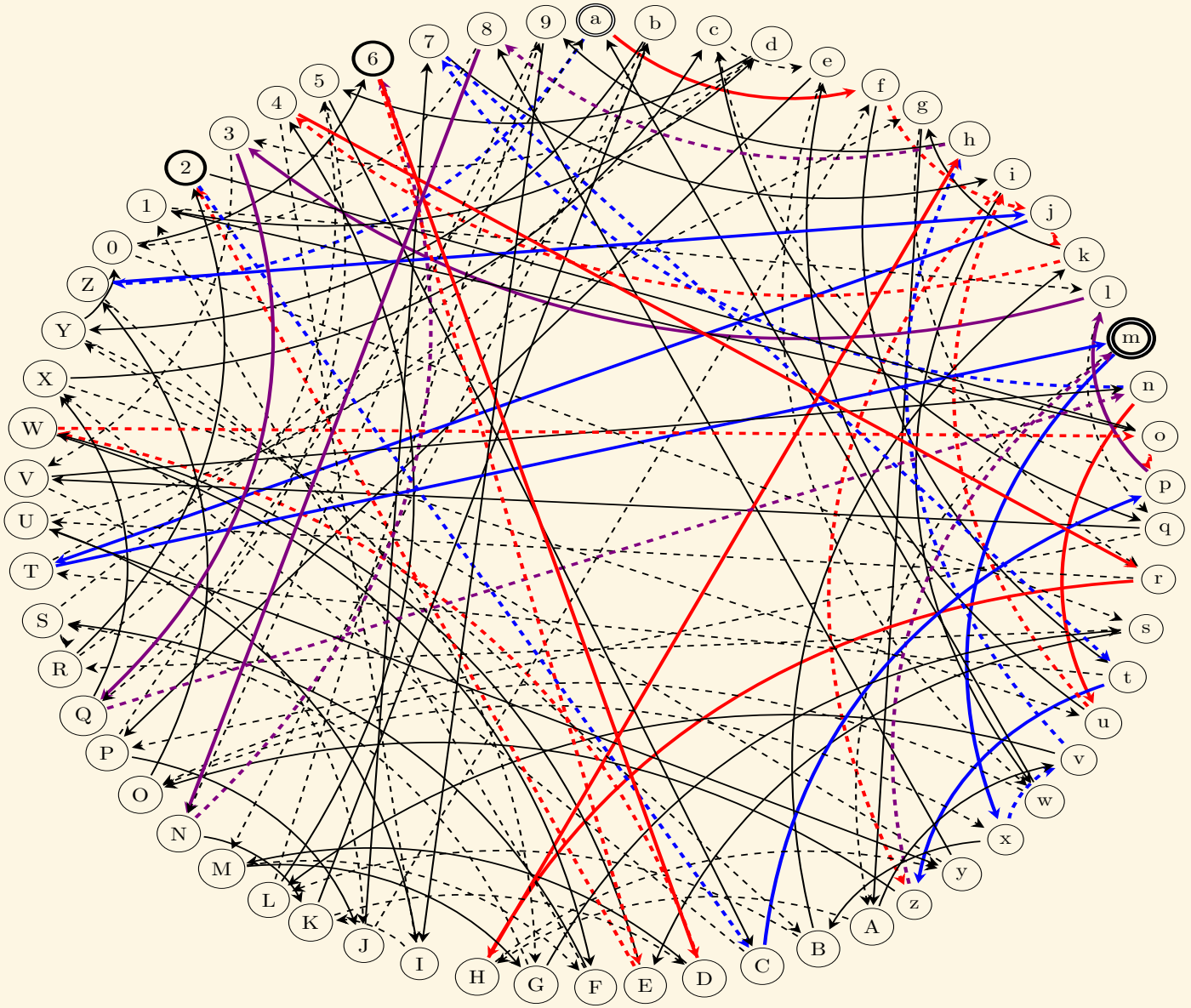
Key exchange on a graph

The full exchange:



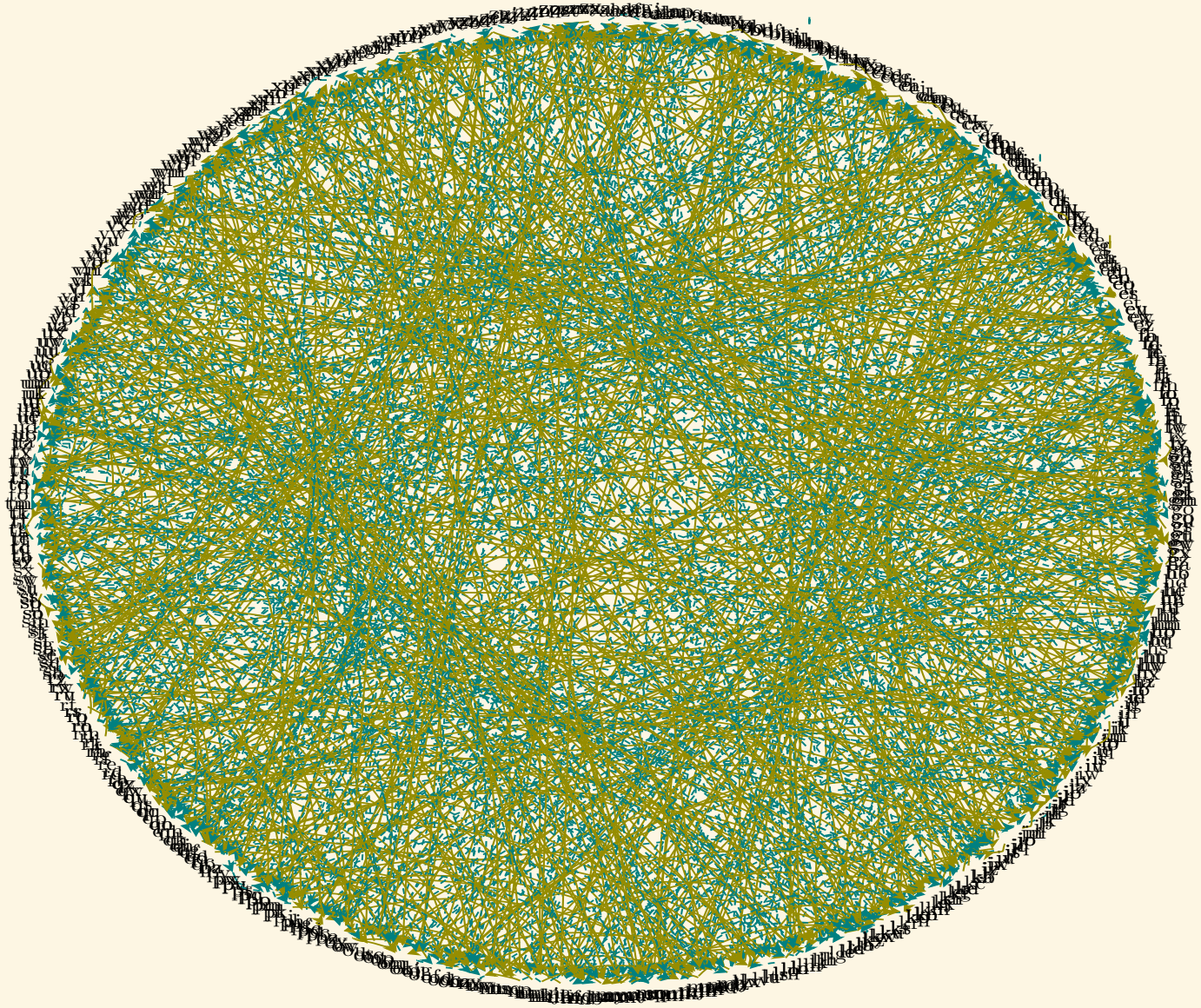
Key exchange on a graph

Bigger graph (62 nodes)



Key exchange on a graph

Even bigger graph (676 nodes)



Elliptic curves isogeny key exchange (Couveignes, Rostovtsev and Stolbunov)

- Use the horizontal isogeny graph of an ordinary elliptic curve E over $\mathbb{F}_q$.
- This is in fact the Cayley graph of the class group of the endomorphism ring of E, which is an imaginary quadratic order.
- For cryptography, choose a curve such that the graph has 2^{256} nodes.

- [LF]: Faster isogenies in the Hessian model of elliptic curves.
- Cost for computing an $\ell = 2s+1$ isogeny in the Hessian model: from $(5s+3)M+4S+8sC$ (Moody2019) to $(3s+3)M+3S+3sC$.

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