Parallel mesh adaptation
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Introduction
Extending mesh adaptation algorithms to parallel is a recent trend in the field of mesh generation. Most of today’s finite element solvers are working in parallel. They are able to scale reasonably when they are run on distributed memory clusters. Mesh adaptation should therefore also be used in parallel.

We propose a parallel mesh adaptation procedure that make use of an existing serial mesh adaptation algorithm. In the parallel extension of the algorithm, edges that are on inter-processor boundaries are frozen i.e. are left unchanged during the iteration. At the end of the iteration, the elements that are situated at inter-processor boundaries are migrated so that they can be modified at the next iteration.

The advantage of the approach is its simplicity. Yet, despite of the very naive nature of the principle, we are able to produce very large meshes that are well adapted i.e. that respect well a given metric field. Moreover, a reasonable scaling was obtained up to 64 processors.

Mesh adaptation
Metric definition: the mesh adaptation procedure is based on metric tensor. This metric is a symmetric definite positive matrix:

\[ M = \mathbf{K} \mathbf{A} \mathbf{K}^{-1} \]

with \( \mathbf{A} = \text{diag}(\lambda_1, \lambda_2, \lambda_3), \mathbf{K} \) the eigenvector matrix and \( \lambda_i \) the eigenvalues of \( M \).

The eigenvectors of \( M \) are the directions wished for the mesh edges and its eigenvalues theirs lengths.

Unit mesh: we search to generate an unit mesh: a mesh such that all its edges \( e \) has a length equal (or close) to one in \( \mathbf{M} \).

\[ \langle \mathbf{e}, \mathbf{e} \rangle_M = \sqrt{\mathbf{e}^T \mathbf{M} \mathbf{e}} = 1. \]

Mesh adaptation procedure: Given the mesh metric field defined over the domain, local mesh modification is applied to yield the desired anisotropic mesh. Mesh modification operations include entity (i) split, (ii) collapse, (iii) swap and (iv) reposition. To make any given mesh satisfying the given mesh size field by mesh modifications, we take philosophy as follows:
- identify those mesh entities not satisfying the mesh size field;
- perform appropriate mesh modifications so that local mesh will better satisfy the mesh size field;
- repeat above steps until the mesh size field is satisfied to an acceptable degree.

Since it is not possible to ensure that all mesh edges exactly match the requested lengths, the goal of mesh modifications is to make the transformed length of all mesh edges fall into an interval close to one. Particularly, we choose interval \([0.5, 1.4]\) in the examples which is large enough to avoid oscillations [1, 2].

Examples
2d isotropic case: two Archimede’s spirals.
The following size is defined:

\[ a = 1, b_{\text{max}} = 0.8, b_{\text{min}} = 0.02, h_{\text{max}} = 0.05, h_{\text{min}} = 0.001 \]

with \( \gamma = \sqrt{\rho^2 + (\rho - b_{\text{max}})^2} \) and \( \theta = \frac{\rho}{\rho - b_{\text{max}}} \).

The analytical metric is given by \( \mathbf{K} = \mathbf{R} \mathbf{K} \) and

\[ \lambda = \text{diag} \left( \left( \min \left( \lambda_{\text{max}} \right) \right)^2, \left( \min \left( \lambda_{\text{min}} \right) \right)^2 \right) \]

2d anisotropic case: plane shocks.
We defined the following analytical size field:

\[ M_0 = 0.6 \left( 1 - e^{-\rho} \right) + 0.003 \]

\[ M_1 = 0.6 \left( 1 - e^{-(\rho-0.5)} \right) + 0.003 \]

where \( x_i = -20 + 2 \times i \) and \( y_i = -47 + 5 \times i \).

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References