

Le baguenaudier

Éric Balandraud

九連環

Plan

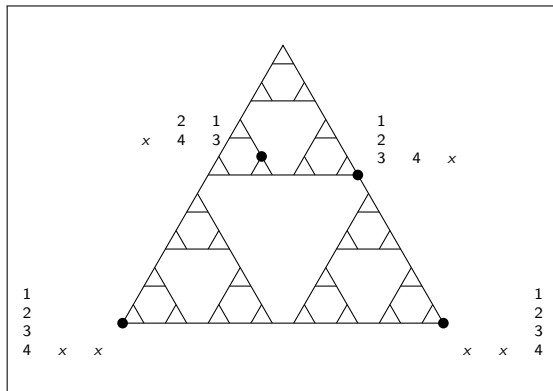
Depuis la chine médiévale...

Le code Gray

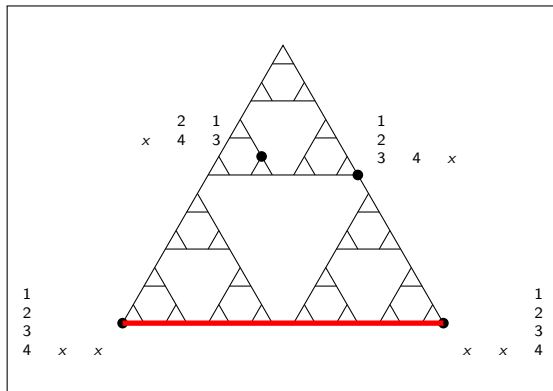
Autres applications

- ▶ En chine médiévale
- ▶ Gerolamo Cardano (1501-1576)
De subtilitate libri XV (1521)
- ▶ Édouard Lucas (1842-1891)
Récréations mathématiques Tome 1 (1892)

Les tours de Hanoï



Les tours de Hanoi



Le code Gray

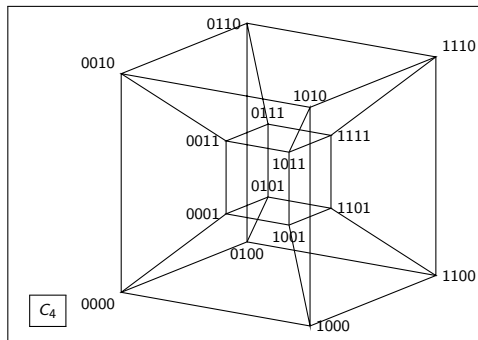
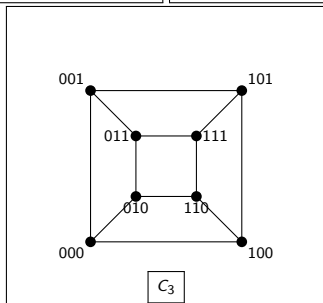
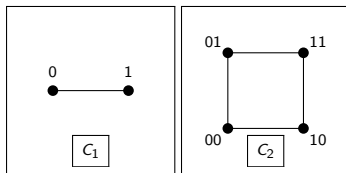
Écrire les entiers sans “phénomène de retenue”
On commence par 000 !!

Le code Gray

n	Code Gray	Binaire	$A(n)$
0	0000	0000	$\emptyset$
1	000 <u>1</u>	0001	$\{0\}$
2	00 <u>1</u> 1	0010	$\{0, 1\}$
3	001 <u>0</u>	0011	$\{1\}$
4	0 <u>1</u> 10	0100	$\{1, 2\}$
5	01 <u>1</u> 1	0101	$\{0, 1, 2\}$
6	01 <u>0</u> 1	0110	$\{0, 2\}$
7	01 <u>0</u> 0	0111	$\{2\}$
8	<u>1</u> 100	1000	$\{2, 3\}$
9	11 <u>0</u> 1	1001	$\{0, 2, 3\}$
10	11 <u>1</u> 1	1010	$\{0, 1, 2, 3\}$
11	11 <u>1</u> 0	1011	$\{1, 2, 3\}$
12	1 <u>0</u> 10	1100	$\{1, 3\}$
13	10 <u>1</u> 1	1101	$\{0, 1, 3\}$
14	10 <u>0</u> 1	1110	$\{0, 3\}$
15	100 <u>0</u>	1111	$\{3\}$

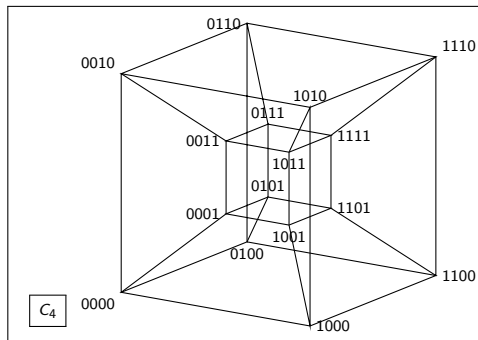
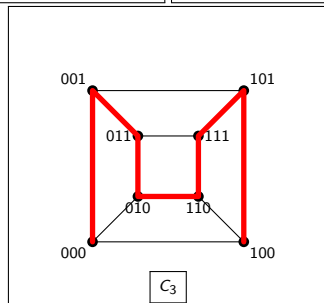
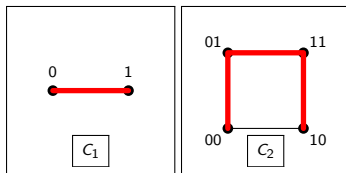
Hypercubes

n -cube: C_n défini par $V = \{0, 1\}^n = \{\epsilon_1 \epsilon_2 \dots \epsilon_n \mid \epsilon_i \in \{0, 1\}\}$,
 $E = \{\{\epsilon, \epsilon'\} \mid d(\epsilon, \epsilon') = 1\}$.



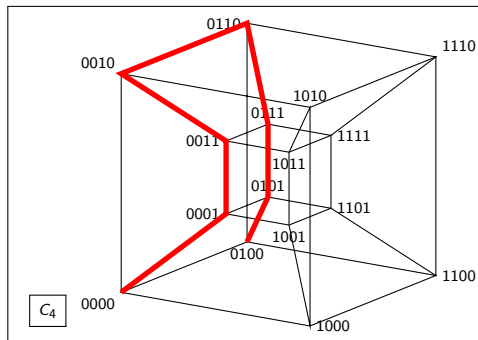
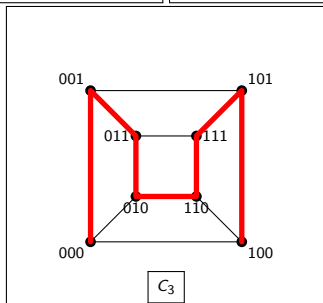
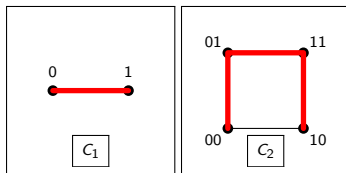
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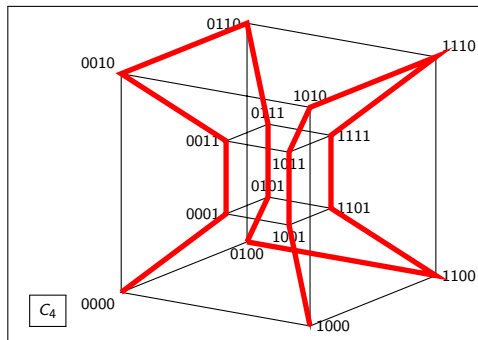
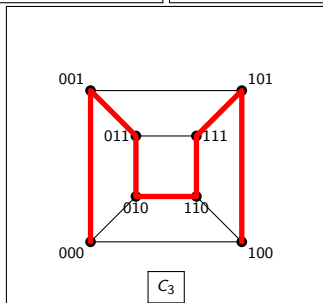
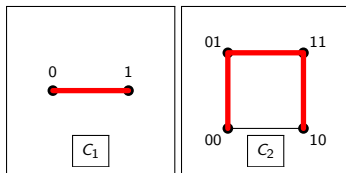
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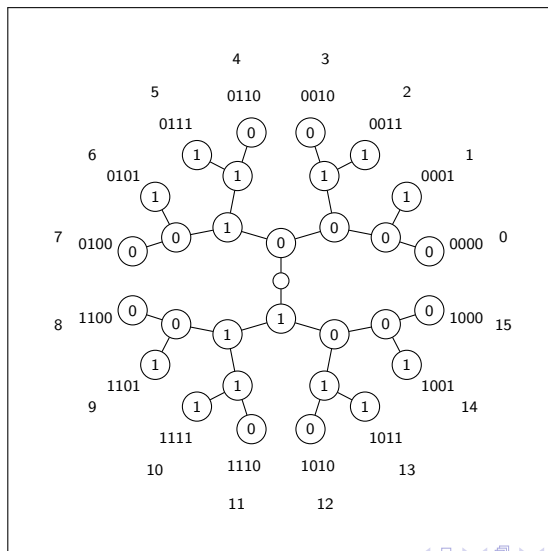


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Codage angulaire



Le permanent

Le déterminant:

$$\det(A) = \sum_{\sigma \in \mathfrak{S}_n} s(\sigma) \prod_{i=1}^n a_{i\sigma(i)}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{aligned} & a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ & - a_{11}a_{23}a_{32} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} \end{aligned}$$

Le permanent:

$$\text{perm}(A) = \sum_{\sigma \in \mathfrak{S}_n} \prod_{i=1}^n a_{i\sigma(i)}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}_p = \begin{aligned} & a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ & + a_{11}a_{23}a_{32} + a_{13}a_{22}a_{31} + a_{12}a_{21}a_{33} \end{aligned}$$

Le permanent

Formule de Ryser:

$$\text{perm}(A) = \sum_{S \subset [1,n]} (-1)^{n-|S|} \prod_{i=1}^n \sum_{j \in S} a_{ij}.$$

$$\begin{aligned} \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}_p &= a_{11}a_{22} + a_{12}a_{21} \\ &= \underbrace{0}_{\emptyset} - \underbrace{a_{11}a_{21}}_{\{1\}} + \underbrace{(a_{11} + a_{12})(a_{21} + a_{22})}_{\{1,2\}} - \underbrace{a_{12}a_{22}}_{\{2\}} \end{aligned}$$

On passe de $O(2^{n-1}n^2)$ à $O(2^{n-1}n)$

Une applet

A vous de jouer:

<https://simonhung.github.io/NineLinkedRings/nineLinkedRings.html>