

OUTPUT DATA OF THE ALGORITHM OF ENUMERATION OF CONFIGURATIONS FOR A GIVEN STRATUM $\mathcal{Q}(\alpha)$

The algorithm written in the file *configurations.sws* (in sage) just follows the idea of Masur and Zorich formulated in Appendix B of [MZ]. It enumerates naively all configurations for a given stratum $\mathcal{Q}(\alpha)$ of quadratic differentials. Since I'm not a programmer, the realisation of this algorithm is costly, bad-written and should be improved. But it works for small genus and for small number of zeroes.

1. OUTPUT DATA

We explain here how to read the output data of the main program "configurations" (at the end of the worksheet *configurations-quad.sws*). For this, you need to be familiar with graphs on Figure 3 and 6 of [MZ], that are reproduced at the end of this document, with some conventions added. In particular arrows in the copy of Figure 6 show how to turn your head in order to see the pictures as a part of pictures in Figure 3.

First look at (the copy of) Figure 3. See that all graphs representing configurations are made of chains of surfaces \oplus (which may contain cylinders that are not shown in the pictures), denoted by Ci : one chain for the graphs a and b , two chains for the graphs c and e , and three chains for the graphs d ; and "special" vertices at the boundary of these chains, denoted by Ai : two vertices \ominus in the graph a , one vertex \ominus in the graph b , one vertex \oplus and one vertex \ominus in $c1$, one vertex \ominus and one vertex \circ in $c2$, two vertices (\oplus or \circ) in $d1$, $d2$, $d3$, one vertex (\oplus or \circ) in $e1$, $e2$.

The input data for the main program "configurations" (at the very end of the worksheet *configurations-quad*) is the genus g and the number of poles n . The output is a list of length the number of strata of quadratic differentials with genus g and n poles. Each element is a list which looks like $[strata, config1, config2, \dots, configk]$ where $strata$ is a list $[d_1, \dots, d_m]$ and represent the stratum $\mathcal{Q}(d_1, \dots, d_m)$, and $config$ is a list organized as follows:

[type of the graph, vertices and chains, number of cylinders]. The "type of the graph" is given by letter 'a', 'b', 'c1', 'c2', 'd1', 'd2', 'd3', 'e1', 'e2', that determines the graph as in Figure 3.

The "number of cylinders" is an integer, the number of \hat{h} homologous cylinders.

The "vertices and chains" correspond to some lists that represent vertices and chains in the graph, in the order determined in Figure 3, as follows:

A vertex A is represented by a list $[[strata], (sub-type), k \text{ or } [k_i]]$. For each type of graph, some distinguished vertices are given (cf Figure 6). Examples:

- if the graph is of type 4, then $[[], 11, [1,0,1,0]]$ means vertex \oplus associated to a surface of boundary stratum $\mathcal{H}(0)$, of type 11 (i.e. $+4.1a$ on notations of Masur-Zorich, cf Figure 6) with boundary singularities $k_1 = 1$, $k_2 = 0$, $k_3 = 1$, $k_4 = 0$.
- if the graph is of type 1, then $[[1,1,-1,-1], [1]]$ means a vertex \ominus associated to a surface of boundary stratum $\mathcal{Q}(1^2, -1^2)$ with boundary singularity of order 1 (so there are three interior singularities, $\{1^2, -1\}$).

A chain C is represented by a list [cylinder: 0 or 1, boundary stratum, type: 1 or 2, boundary singularities, cylinder: 0 or 1...]. Examples:

- [0, [1,1], 1, [1,3], 1] is a chain composed by one surface \oplus of boundary stratum $\mathcal{H}(1,1)$ of subtype 1 with boundary singularities of order $k_1 = 1$ and $k_2 = 3$ followed by a cylinder.
- [1,[],1, [1,1], 0, [2,2], 2, [6,6], 1] is a chain composed by one cylinder, followed by a surface in $\mathcal{H}(0)$ of subtype 1 with boundary singularities of order 1 and 1, followed by a surface in $\mathcal{H}(2,2)$ of subtype 2 with singularities of order 6 and 6 followed by a cylinder.

So to resume, here is what *config* looks like depending of the type of the graph:

- [‘a’, A1, C, A2, number of cylinders]
- [‘b’, A, C, number of cylinders]
- [‘c1’, A1, C1, A2, C2, number of cylinders]
- [‘c2’, A, C1, C2, number of cylinders]
- [‘d1’, C1, A1, C2, A2, C3, number of cylinders]
- [‘d2’, C1, C2, A, C3, number of cylinders]
- [‘d3’, C1, C2, C3, number of cylinders]
- [‘e1’, C1, A, C2, number of cylinders]
- [‘e2’, C1, C2, number of cylinders]

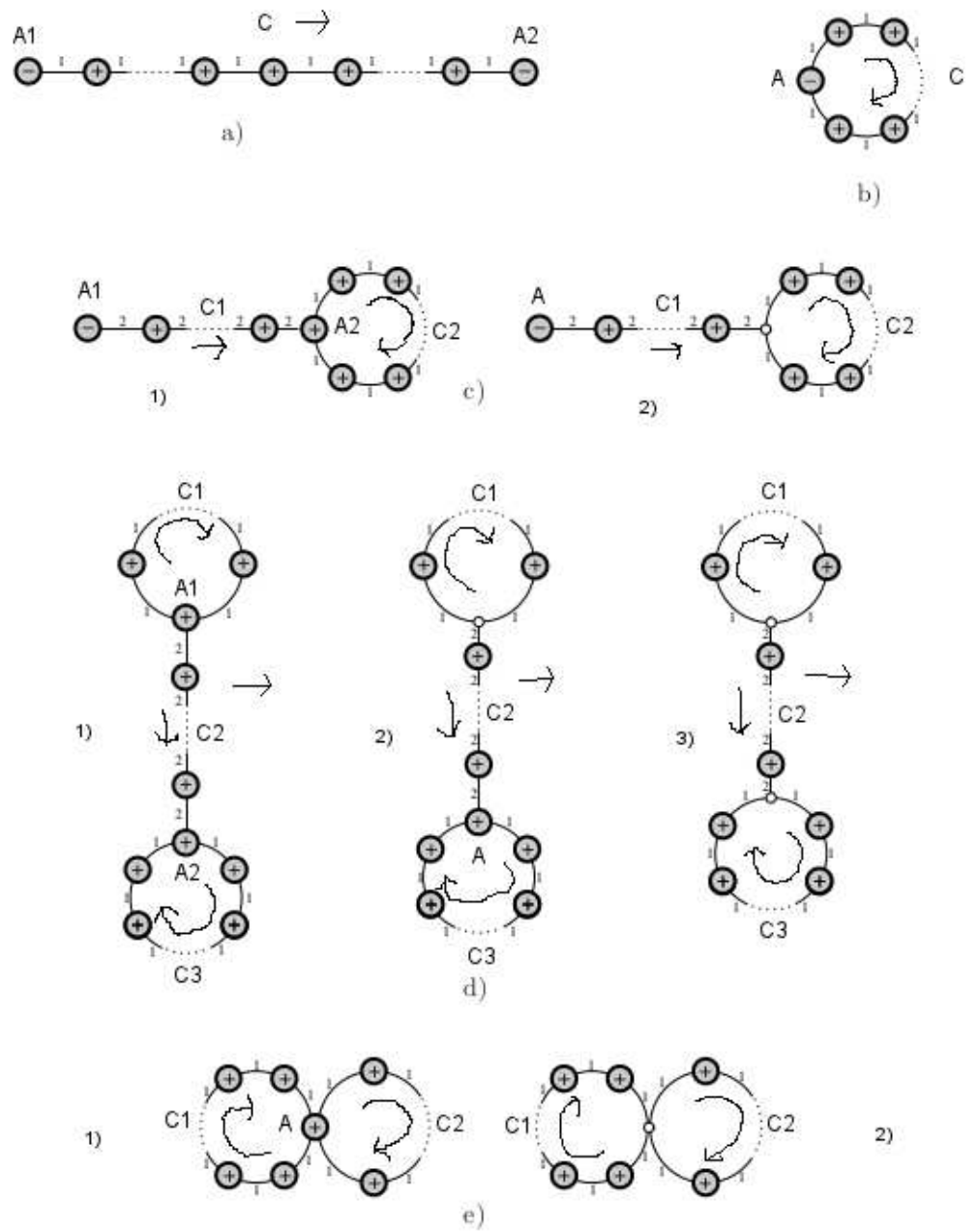


FIGURE 3. Classification of admissible graphs.

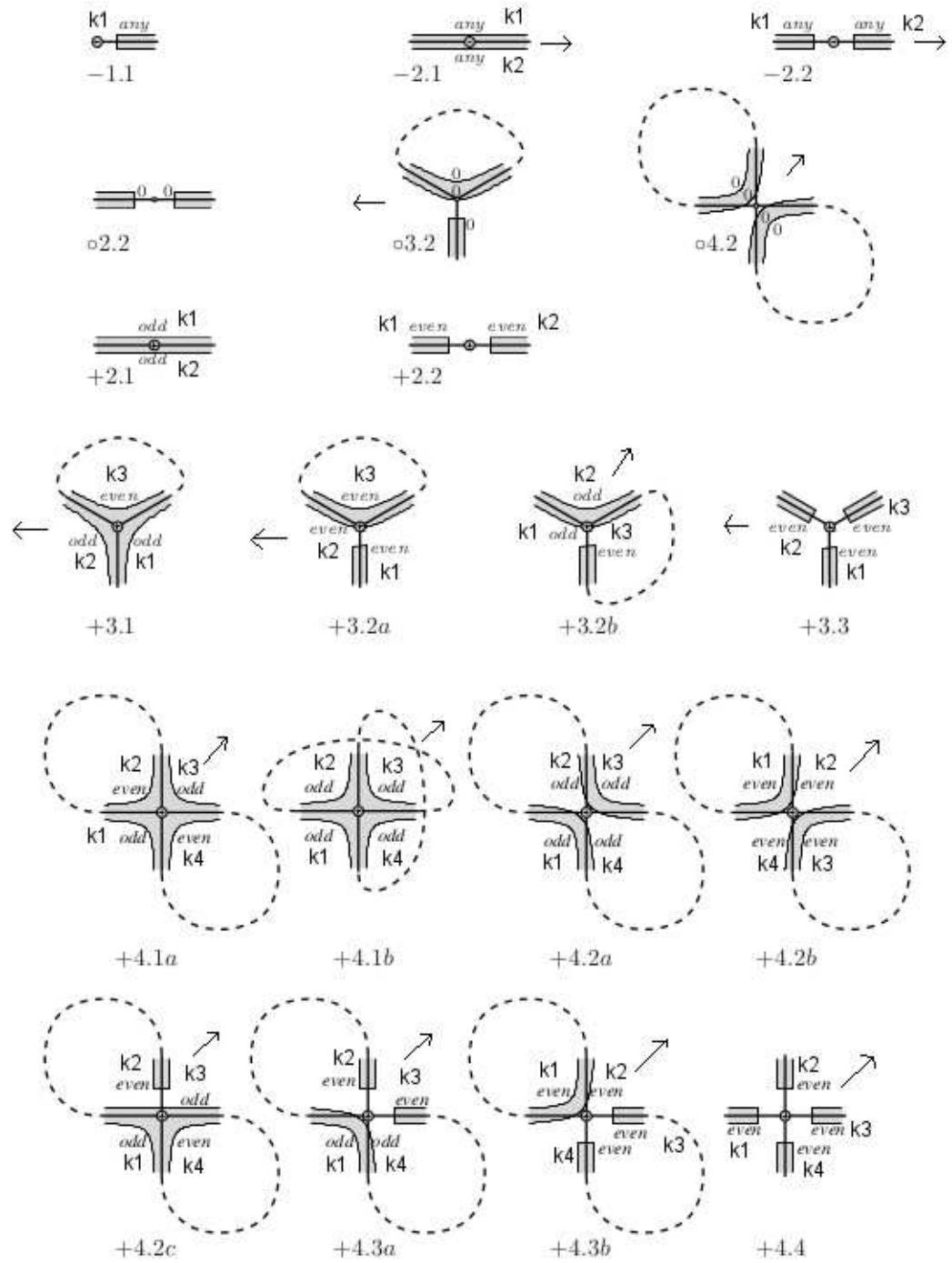


FIGURE 6. Classification of embedded local ribbon graphs

REFERENCES

- [MZ] H. Masur, A. Zorich, *Multiple Saddle Connections on Flat Surfaces and the Principal Boundary of the Moduli Spaces of Quadratic Differentials*, *Geom. Funct. Anal.*, **18:3** (2008), pp. 919–987.