Low regularity solutions for the two-dimensional "rigid body + incompressible Euler" system.

Olivier Glass[*], Franck Sueur[†‡]

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Abstract

In this note we consider the motion of a solid body in an incompressible perfect fluid filling the plane. We prove the global existence of solutions in the case where the initial vorticity belongs to L^p with $p > 1$ and is compactly supported. We do not assume that the energy is finite.

1 Introduction

In this paper, we consider the motion of a solid body in a perfect incompressible fluid which fills the plane, in a low-regularity setting. Let us describe the system under view. Let $\mathcal{S}_0$ be a closed, bounded, connected and simply connected subset of the plane with smooth boundary. We assume that the body initially occupies the domain $\mathcal{S}_0$ and rigidly moves so that at time t it occupies an isometric domain denoted by $\mathcal{S}(t)$. We denote $\mathcal{F}(t) := \mathbb{R}^2 \setminus \mathcal{S}(t)$ the domain occupied by the fluid at time t starting from the initial domain $\mathcal{F}_0 := \mathbb{R}^2 \setminus \mathcal{S}_0$.

The equations modelling the dynamics of the system then read

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u + \nabla p = 0 \quad \text{for} \quad x \in \mathcal{F}(t), \tag{1}$$

$$\operatorname{div} u = 0 \quad \text{for} \quad x \in \mathcal{F}(t), \tag{2}$$

$$u \cdot n = u_{\mathcal{S}} \cdot n \quad \text{for} \quad x \in \partial\mathcal{S}(t), \tag{3}$$

$$\lim_{|x|\to\infty} |u| = 0, \tag{4}$$

$$m h''(t) = \int_{\partial\mathcal{S}(t)} pn \, ds, \tag{5}$$

$$\mathcal{J} r'(t) = \int_{\partial\mathcal{S}(t)} (x - h(t))^\perp \cdot pn \, ds, \tag{6}$$

$$u|_{t=0} = u_0 \quad \text{for} \quad x \in \mathcal{F}_0, \tag{7}$$

$$h(0) = h_0, \quad h'(0) = \ell_0, \quad r(0) = r_0. \tag{8}$$

Here $u = (u_1, u_2)$ and p denote the velocity and pressure fields, $m > 0$ and $\mathcal{J} > 0$ denote respectively the mass and the moment of inertia of the body while the fluid is supposed to be homogeneous of density 1, to simplify the notations. When $x = (x_1, x_2)$ the notation $x^\perp$ stands for $x^\perp = (-x_2, x_1)$, n denotes the unit normal vector pointing outside the fluid, $h'(t)$ is the velocity of the center of mass $h(t) \in \mathbb{R}^2$ of the body and $r(t) \in \mathbb{R}$ denotes the angular velocity of the rigid body at time t. Finally we denote by $u_{\mathcal{S}}$ the velocity of the body:

$$u_{\mathcal{S}}(t, x) = h'(t) + r(t)(x - h(t))^\perp. \tag{9}$$

[*]CEREMADE, UMR CNRS 7534, Université Paris-Dauphine, Place du Maréchal de Lattre de Tassigny, 75775 Paris Cedex 16, FRANCE

[†]CNRS, UMR 7598, Laboratoire Jacques-Louis Lions, F-75005, Paris, France

[‡]UPMC Univ Paris 06, UMR 7598, Laboratoire Jacques-Louis Lions, F-75005, Paris, France

Without loss of generality and for the rest of the paper, we assume from now on that

$$h_0 = 0,$$

which means that the body is centered at the origin at the initial time $t = 0$.

The equations (1) and (2) are the incompressible Euler equations, the condition (3) means that the boundary is impermeable and the equations (5) and (6) are the Newton's balance law for linear and angular momenta.

A quantity which will play a central role in the sequel is the vorticity field:

$$w(t,x) := \operatorname{curl} u(t,x) = \partial_1 u_2(t,x) - \partial_2 u_1(t,x).$$

The main goal of this paper is to prove the global existence of some solutions this system in the case where

$$u_0 \in \tilde{L}^2 := L^2(\mathcal{F}_0) \oplus \mathbb{R} H_0, \tag{10}$$

$$w_0 := \operatorname{curl} u_0 \in L^p_c(\overline{\mathcal{F}_0}), \text{ with } p > 1. \tag{11}$$

Above

$$H_0(x) := \frac{x^\perp}{2\pi|x|^2} \quad \text{for} \quad x \in \mathcal{F}_0, \tag{12}$$

and the notation $L^p_c(\overline{\mathcal{F}_0})$ denotes the subset of the functions in $L^p(\mathcal{F}_0)$ which are compactly supported in the closure of $\mathcal{F}_0$. We underline in particular that u_0 is not necessarily of finite energy. Note that it is important to be able to take into account solutions with infinite energy in the setting of a fluid occupying the whole plane, see for instance DiPerna-Majda [3], Chemin [2, Section 1.3] and Majda-Bertozzi [10, Section 3.1.3].

Our result below extends the result [15] of Wang and Xin where it is assumed that the vorticity w_0 belongs to $L^1 \cap L^p$ with $p \in (\frac{4}{3}, +\infty]$ (but is not necessarily compactly supported) and that the solution has finite energy; it also extends the result of [6] where it is assumed that the vorticity w_0 belongs to L^p with $p \in (2, +\infty]$ and is compactly supported, with possibly infinite kinetic energy. Also, this provides a counterpart of the results by DiPerna and Majda [3] about the global existence of weak solutions in the case of the fluid alone when the vorticity belongs to $L^1 \cap L^p$ with $p > 1$. Let us also mention that the global existence and uniqueness of finite energy classical solutions to the problem (1)–(8) has been tackled by Ortega, Rosier and Takahashi in [12]. This result was extended in [5] to the case of infinite energy.

2 Main result

Before giving the main statement of this paper, let us give our definition of a weak solution. This relies on a change of variable which we now describe.

Since $\mathcal{S}(t)$ is obtained from $\mathcal{S}_0$ by a rigid motion, there exists a rotation matrix

$$Q(t) := \begin{bmatrix} \cos\theta(t) & -\sin\theta(t) \\ \sin\theta(t) & \cos\theta(t) \end{bmatrix},$$

such that the position $\eta(t,x) \in \mathcal{S}(t)$ at the time t of the point fixed to the body with an initial position x is

$$\eta(t,x) := h(t) + Q(t)x.$$

The angle θ satisfies

$$\theta'(t) = r(t),$$

and we choose $\theta(t)$ such that $\theta(0) = 0$.

In order to transfer the equations in the body frame we apply the following isometric change of variable:

$$\begin{cases} v(t,x) = Q(t)^T\, u(t, Q(t)x + h(t)), \\ q(t,x) = p(t, Q(t)x + h(t)), \\ \ell(t) = Q(t)^T\, h'(t). \end{cases}$$

so that the equations (1)-(8) become

$$\frac{\partial v}{\partial t} + \left[(v - \ell - r x^{\perp}) \cdot \nabla\right] v + r v^{\perp} + \nabla q = 0 \quad \text{for} \quad x \in \mathcal{F}_0, \tag{13}$$

$$\operatorname{div} v = 0 \quad \text{for} \quad x \in \mathcal{F}_0, \tag{14}$$

$$v \cdot n = \left(\ell + r x^{\perp}\right) \cdot n \quad \text{for} \quad x \in \partial \mathcal{S}_0, \tag{15}$$

$$m \ell'(t) = \int_{\partial \mathcal{S}_0} q n \, ds - m r \ell^{\perp}, \tag{16}$$

$$\mathcal{J} r'(t) = \int_{\partial \mathcal{S}_0} x^{\perp} \cdot q n \, ds, \tag{17}$$

$$v(0, x) = v_0(x) \quad \text{for} \quad x \in \mathcal{F}_0, \tag{18}$$

$$\ell(0) = \ell_0, \; r(0) = r_0, \tag{19}$$

where the initial data v_0 coincides with u_0. We also define

$$\omega(t, x) := w(t, Q(t)x + h(t)) = \operatorname{curl} v(t, x).$$

Taking the curl of the equation (13) we get

$$\partial_t \omega + \left[(v - \ell - r x^{\perp}) \cdot \nabla\right] \omega = 0 \text{ for } x \in \mathcal{F}_0. \tag{20}$$

Let us now give a global weak formulation of the problem by considering (for the solution as well as for test functions) a velocity field on the whole plane, with the constraint to be rigid on $\mathcal{S}_0$.

We introduce the following space

$$\mathcal{H} := \left\{ \Psi \in L^2_{loc}(\mathbb{R}^2) \Big/ \operatorname{div} \Psi = 0 \text{ in } \mathbb{R}^2 \text{ and } D\Psi = 0 \text{ in } \mathcal{S}_0 \right\}, \quad \text{where} \quad D\Psi := \nabla \Psi + (\nabla \Psi)^T.$$

It is classical (see e.g. [14, Lemma 1.1]) that

$$\mathcal{H} = \left\{ \Psi \in L^2_{loc}(\mathbb{R}^2) \Big/ \operatorname{div} \Psi = 0 \text{ in } \mathbb{R}^2 \text{ and } \exists (\ell_\Psi, r_\Psi) \in \mathbb{R}^2 \times \mathbb{R}, \; \forall x \in \mathcal{S}_0, \; \Psi(x) = \ell_\Psi + r_\Psi x^{\perp} \right\},$$

and the ordered pair (ℓ_Ψ, r_Ψ) above is unique.

Let us also introduce

$$\tilde{\mathcal{H}} := \left\{ \Psi \in \mathcal{H} \Big/ \Psi|_{\overline{\mathcal{F}_0}} \in C^1_c(\overline{\mathcal{F}_0}) \right\},$$

where $\Psi|_{\overline{\mathcal{F}_0}}$ denotes the restriction of Ψ to the closure of the fluid domain. We also introduce for $T > 0$,

$$\tilde{\mathcal{H}}_T := C^1([0, T]; \tilde{\mathcal{H}}). \tag{21}$$

When $(\overline{u}, \overline{v}) \in \mathcal{H} \times \tilde{\mathcal{H}}$, we denote by

$$(\overline{u}, \overline{v})_\rho := \int_{\mathbb{R}^2} (\rho \xi_{\mathcal{S}_0} + \xi_{\mathcal{F}_0}) \, \overline{u} \cdot \overline{v} \, dx = m \, \ell_u \cdot \ell_v + \mathcal{J} r_u \, r_v + \int_{\mathcal{F}_0} u \cdot v \, dx,$$

where the notation ξ_A stands for the characteristic function of the set A, $u \in L^2_{loc}(\mathcal{F}_0)$ denotes the restrictions of $\overline{u}$ to $\mathcal{F}_0$ and ρ denotes the density in the solid body. Note that this density is related to m and $\mathcal{J}$ by

$$m = \int_{\mathcal{S}_0} \rho(x) \, dx \quad \text{and} \quad \mathcal{J} = \int_{\mathcal{S}_0} \rho(x) |x|^2 \, dx.$$

It follows that the scalar product $(\cdot, \cdot)_\rho$ depends on ρ merely through m and $\mathcal{J}$.

Our definition of a weak solution is the following.

Definition 1 (Weak Solution)**.** *Let us be given* $\overline{v}_0 \in \mathcal{H}$ *and* $T > 0$*. We say that* $\overline{v} \in C([0,T];\mathcal{H} - w)$ *is a weak solution of* (13)–(19) *in* $[0,T]$ *if for any test function* $\Psi \in \tilde{\mathcal{H}}_T$*,*

$$(\Psi(T,\cdot), \overline{v}(T,\cdot))_\rho - (\Psi(0,\cdot), \overline{v}_0)_\rho = \int_0^T \left(\frac{\partial \Psi}{\partial t}, \overline{v} \right)_\rho dt$$
$$+ \int_0^T \int_{\mathcal{F}_0} v \cdot \left(\left(v - \ell_v - r_v x^\perp \right) \cdot \nabla \right) \Psi \, dx \, dt - \int_0^T \int_{\mathcal{F}_0} r_v \, v^\perp \cdot \Psi \, dx \, dt - \int_0^T m r_v \, \ell_v^\perp \cdot \ell_\Psi \, dt. \quad (22)$$

We say that $\overline{v} \in C([0,+\infty);\mathcal{H} - w)$ *is a weak solution of* (13)–(19) *in* $[0,+\infty)$ *if it satisfies* (22) *for all* $T > 0$*.*

Definition 1 is legitimate since a classical solution of (13)–(19) in $[0,T]$ is also a weak solution. This follows easily from an integration by parts in space which provides

$$(\partial_t \overline{v}, \Psi)_\rho = \int_{\mathcal{F}_0} v \cdot \left(\left(v - \ell_v - r_v x^\perp \right) \cdot \nabla \right) \Psi \, dx - \int_{\mathcal{F}_0} r_v \, v^\perp \cdot \Psi \, dx - m r_v \, \ell_v^\perp \cdot \ell_\Psi, \quad (23)$$

and then from an integration by parts in time.

In the sequel we will often drop the index of ℓ_v and r_v and we will therefore rather write ℓ and r. We will equivalently say that (ℓ, r, v) is a weak solution of (13)–(19).

One has the following result of existence of weak solutions for the above system, the initial position of the solid being given.

Theorem 1. *Let* $p > 1$*. For any* $v_0 \in \tilde{L}^2$*,* $(\ell_0, r_0) \in \mathbb{R}^2 \times \mathbb{R}$*, such that:*

$$\operatorname{div} v_0 = 0 \text{ in } \mathcal{F}_0 \quad \text{and} \quad v_0 \cdot n = (\ell_0 + r_0 x^\perp) \cdot n \text{ on } \partial \mathcal{S}_0, \quad (24)$$

and $\omega_0 := \operatorname{curl} v_0$ *satisfies* (11)*, there exists a weak solution of the system such that* $\omega \in L^\infty(\mathbb{R}^+; (L^1 \cap L^p)(\mathcal{F}_0))$*.*

Remark 1. *Uniqueness of such solutions is an open problem, as in the case of a fluid alone.*

The rest of the paper is devoted to the proof of Theorem 1. From now on, we suppose that $p < 2$. In the end of the proof, we will explain how the case $p \geqslant 2$ can be obtained.

3 Representation of the velocity in the body frame

In this section, we study the elliptic div/curl system which allows to pass from the vorticity to the velocity field, in the body frame. We do not claim any originality here, but rather collect some well-known properties which will be useful in the sequel. We refer here for instance to [4, 8, 11, 7, 6].

3.1 Harmonic field

In order to take the velocity circulation around the body into account, the following vector field will be useful. There exists a unique solution H vanishing at infinity of

$$\begin{cases} \operatorname{div} H = 0 \quad \text{for } x \in \mathcal{F}_0, \\ \operatorname{curl} H = 0 \quad \text{for } x \in \mathcal{F}_0, \\ H \cdot n = 0 \quad \text{for } x \in \partial \mathcal{S}_0, \\ \int_{\partial \mathcal{S}_0} H \cdot \tau \, ds = 1. \end{cases} \quad (25)$$

See e.g. [7] and [8]. This solution is smooth and we have (see e.g. [6, Section 2.2]):

$$H(x) - H_0(x) = \mathcal{O}\left(\frac{1}{|x|^2} \right), \ \nabla H(x) - \nabla H_0(x) = \mathcal{O}\left(\frac{1}{|x|^3} \right) \text{ at infinity}, \quad (26)$$

$$\nabla H \in L^\infty(\mathcal{F}_0) \text{ and } H(x) - H_0(x), \ \nabla H, \ H^\perp - x^\perp \cdot \nabla H \in L^2(\mathcal{F}_0). \quad (27)$$

From the complex-analytic viewpoint, one writes

$$H_1(z) - iH_2(z) = \frac{1}{2i\pi z} + \mathcal{O}\left(\frac{1}{z^2}\right), \tag{28}$$

where we identify $(x, y) \in \mathbb{R}^2$ and $x + iy \in \mathbb{C}$.

Let us stress in particular that the second estimate above yields that

$$\tilde{L}^2 = L^2(\mathcal{F}_0) \oplus \mathbb{R}H.$$

Therefore in the sequel we will use this last decomposition rather than (10).

3.2 From vorticity to velocity

In this section, we consider the operator passing from the vorticity field ω to the velocity field v. We denote p^* the Sobolev index associated with p, that is

$$p^* := \frac{2p}{2-p}.$$

Since $p \in (1, 2)$, one has $p^* \in (2, \infty)$.

Lemma 1. *There exists $C > 0$ (depending only on $\mathcal{F}_0$ and on p) such that for any $\omega \in L^p_c(\overline{\mathcal{F}_0})$ and $\gamma \in \mathbb{R}$, there exists a unique $u \in L^{p^*}(\mathcal{F}_0)$ with $\nabla u \in L^p(\mathcal{F}_0)$, such that*

$$\begin{cases} \operatorname{div} u = 0, & \text{for } x \in \mathcal{F}_0, \\ \operatorname{curl} u = \omega & \text{for } x \in \mathcal{F}_0, \\ u \cdot n = 0 & \text{for } x \in \partial\mathcal{S}_0, \\ \int_{\partial\mathcal{S}_0} u \cdot \tau \, ds = \gamma, \end{cases} \tag{29}$$

and this u satisfies

$$\|u\|_{L^{p^*}(\mathcal{F}_0)} + \|\nabla u\|_{L^p(\mathcal{F}_0)} \leqslant C\big(\|\omega\|_{L^p(\mathcal{F}_0)} + |\gamma|\big). \tag{30}$$

Moreover, one has at infinity,

$$u - \left(\int_{\mathcal{F}_0} \omega \, dx + \gamma\right) H = \mathcal{O}\left(\frac{1}{|x|^2}\right), \tag{31}$$

$$\nabla u - \left(\int_{\mathcal{F}_0} \omega \, dx + \gamma\right) \nabla H = \mathcal{O}\left(\frac{1}{|x|^3}\right). \tag{32}$$

In particular, one has $u \in \tilde{L}^2$.

Let us stress in particular that the constant C does not depend on the support of ω.

Proof of Lemma 1. We deduce from [13, Theorem III.5.4] (see also [1, Remark 2.11]) that there exists a $\psi \in L^p_{loc}(\mathcal{F}_0)$ with $\nabla^2\psi \in L^p(\mathcal{F}_0)$ and $\nabla\psi \in L^{p^*}(\mathcal{F}_0)$ such that

$$\Delta\psi = \omega \text{ in } \mathcal{F}_0, \ \psi = 0 \text{ on } \partial\mathcal{F}_0,$$

and

$$\|\nabla\psi\|_{L^{p^*}(\mathcal{F}_0)} + \|\nabla^2\psi\|_{L^p(\mathcal{F}_0)} \leqslant C\|\omega\|_{L^p(\mathcal{F}_0)}.$$

Taking $u = \nabla^\perp\psi + \lambda H$ for

$$\lambda := \gamma + \int_{\partial\mathcal{F}_0} \partial_n\psi \, ds,$$

yields the existence part of the claim.

Considering $R > 0$ such that $\operatorname{Supp}\omega \subset \overline{B(0, R)}$, we have that ψ is harmonic outside of $\overline{B(0, R)}$. Recalling that $\nabla^2\psi \in L^p(\mathcal{F}_0)$ we deduce from [13, Lemma IV.4.1] that ψ takes the form

$$\psi(x) = a + b\ln(|x|) + c_1x_1 + c_2x_2 + \mu\left(\frac{x}{|x|^2}\right) \text{ outside of } \overline{B(0, R)},$$

where μ is a harmonic function in $B(0,1/R)$ vanishing at 0 and where a, b, c_1 and c_2 are real numbers. Since $\nabla\psi \in L^{p^*}(\mathcal{F}_0)$, we see that $c_1 = c_2 = 0$.

Now we can develop $\partial_1\mu - i\partial_2\mu$ in power series in $B(0,1/R)$,

$$\partial_1\mu - i\partial_2\mu = \sum_{k\geqslant 1} \mu_k z^k \text{ in } B\left(0,\frac{1}{R}\right),$$

so that $u_1 - iu_2$ can be developed in Laurent series outside of $\overline{B(0,R)}$:

$$u_1 - iu_2 = \sum_{k\geqslant 1} \frac{d_k}{z^k} \text{ in } \mathbb{C}\setminus B(0,R).$$

Now by Cauchy's residue theorem, and then Green's identity, one can compute d_1 as follows:

$$2i\pi d_1 = \int_{S(0,R)} u\cdot\tau\, ds + i\int_{S(0,R)} u\cdot n\, ds = \gamma + \int_{\mathcal{F}_0} \omega\, dx.$$

The estimates (31)-(32) follow, using (26).

Finally, we consider the uniqueness issue. We take the difference of two solutions of (29) having the claimed regularity and find w in $L^{p^*}(\mathcal{F}_0)$ with $\nabla w \in L^p(\mathcal{F}_0)$, such that

$$\begin{cases} \operatorname{div} w = \operatorname{curl} w = 0 \quad \text{for } x \in \mathcal{F}_0, \\ w\cdot n = 0 \quad \text{for } x \in \partial\mathcal{S}_0, \\ \int_{\partial\mathcal{S}_0} w\cdot\tau\, ds = 0. \end{cases}$$

Hence w is locally the orthogonal gradient of a harmonic function, hence it is smooth, and due to $w\cdot n = 0$ on $\mathcal{S}_0$, it is globally the orthogonal gradient of a harmonic function φ with second derivatives in $L^p(\mathcal{F}_0)$. Using again [13, Lemma IV.4.1], we easily infer that φ has the following form for $|x|$ large:

$$\varphi(x) = a + b\ln(|x|) + c_1x_1 + c_2x_2 + \mu\left(\frac{x}{|x|^2}\right),$$

with μ, a, b, c_1 and c_2 as above. Since w in $L^{p^*}(\mathcal{F}_0)$, one has $c_1 = c_2 = 0$, and using the circulation along $S(0,R)$ with R large, we find that $b = 0$. Hence we deduce that φ converges to the constant a at infinity. From $w\cdot n = 0$, we have that φ is constant along $\partial\mathcal{S}_0$. Using a conformal mapping η sending $\overline{\mathcal{F}_0}\cup\{\infty\}$ to $\overline{B}(0,1)$, we see that $\varphi\circ\eta^{-1}$ is a harmonic function having an artificial singularity at $\eta(\infty)$. Since it is constant on $S(0,1)$, it is constant in $\overline{B}(0,1)$, hence $w = 0$. □

Definition 2. *We will denote $u := K[\omega]$ the vector field obtained from Lemma 1 for $\gamma = -\int_{\mathcal{F}_0}\omega\, dx$.*

Note that one can deduce the linearity of the operator K from the uniqueness in Lemma 1, and that, using (31) and the fact that ω is compactly supported, we can deduce that

$$\int_{S(0,R)} K[\omega]\cdot\tau\, ds = 0 \text{ for } R \text{ large.} \tag{33}$$

Note also that for a constant C depending on $R > 0$ such that $\operatorname{Supp}\omega \subset B(0,R)$, one has

$$\left|\int_{\mathcal{F}_0} \omega\, dx\right| \leqslant C\|\omega\|_{L^p(\mathcal{F}_0)}. \tag{34}$$

Now we have the following estimate on the operator K.

Lemma 2. *Let $R > 0$. There exists $C > 0$ (depending only on $\mathcal{F}_0$, R and p) such that for any $\omega \in L^p_c(\overline{\mathcal{F}_0})$ with*

$$\mathit{Supp}\,\omega \subset \overline{B(0,R)},$$

one has

$$\||x|^2 K[\omega](x)\|_{L^\infty(\mathbb{R}^2\setminus B(0,R+1))} \leqslant C\|\omega\|_{L^p(\mathcal{F}_0)}, \tag{35}$$

$$\|K[\omega]\|_{L^2(\mathcal{F}_0)} \leqslant C\|\omega\|_{L^p(\mathcal{F}_0)}. \tag{36}$$

Proof of Lemma 2. We first deduce from Lemma 1 and (34) that

$$\|K[\omega]\|_{L^{p^*}(B(0,R+2)\setminus B(0,R))} \leqslant \|K[\omega]\|_{L^{p^*}(\mathcal{F}_0)} \leqslant C\|\omega\|_{L^p(\mathcal{F}_0)}. \tag{37}$$

Using the interior elliptic regularity for the Laplace equation we deduce that

$$\|K[\omega]\|_{L^\infty(S(0,R+1))} \leqslant C\|\omega\|_{L^p(\mathcal{F}_0)}. \tag{38}$$

From (33), we deduce that $u_1 - iu_2$ for $u = K[\omega]$ has a decomposition in Laurent series of the form

$$(u_1 - iu_2)(z) = \sum_{k\geqslant 2} \frac{c_k}{z^k}.$$

Integrating $z(u_1 - iu_2)(z)$ over $S(0, R+1)$ we infer that

$$|c_2| \leqslant C\|\omega\|_{L^p(\mathcal{F}_0)}.$$

Applying the maximum principle to $z^2\,(u_1 - iu_2)(z)$ and using again (38), we deduce (35).

Estimate (36) follows from (35) outside $B(0, R+1)$ and from (37) and $p^* > 2$ inside $B(0, R+1)$. □

3.3 Kirchhoff potentials

Now in order to lift harmonically the boundary conditions, we will make use of the Kirchhoff potentials (see e.g. Lamb [9, Paragraph 118]), which are the solutions Φ_i, $i = 1, 2, 3$ of the following problems:

$$-\Delta\Phi_i = 0 \quad \text{for } x \in \mathcal{F}_0, \tag{39}$$

$$\Phi_i \longrightarrow 0 \quad \text{for } x \to \infty, \tag{40}$$

$$\frac{\partial\Phi_i}{\partial n} = K_i \quad \text{for } x \in \partial\mathcal{F}_0, \tag{41}$$

where

$$(K_1,\, K_2,\, K_3) := (n_1,\, n_2,\, x^\perp \cdot n). \tag{42}$$

These functions are smooth and decay at infinity as follows (see e.g. [6, Section 2.3]):

$$\nabla\Phi_i = \mathcal{O}\left(\frac{1}{|x|^2}\right) \quad \text{and } \nabla^2\Phi_i = \mathcal{O}\left(\frac{1}{|x|^3}\right) \quad \text{as } x \to \infty.$$

In particular the functions $\nabla\Phi_i$ belong to $L^2(\mathcal{F}_0)$.

3.4 Velocity decomposition

Using the functions defined above, we deduce the following proposition.

Proposition 1. *Let $p > 1$. Let us be given ω in $L^p_c(\overline{\mathcal{F}_0})$, ℓ in $\mathbb{R}^2$, r and γ in $\mathbb{R}$. Then there is a unique solution v in $\tilde{L}^2 \cap L^{p^*}(\mathcal{F}_0)$, with ∇v in $L^p(\mathcal{F}_0)$, of*

$$\begin{cases} \operatorname{div} v = 0, & \text{for } x \in \mathcal{F}_0, \\ \operatorname{curl} v = \omega & \text{for } x \in \mathcal{F}_0, \\ v \cdot n = \left(\ell + r x^\perp\right) \cdot n & \text{for } x \in \partial\mathcal{S}_0, \\ v \longrightarrow 0 & \text{as } x \to \infty, \\ \int_{\partial\mathcal{S}_0} v \cdot \tau \, ds = \gamma. \end{cases}$$

Moreover v is given by

$$v = K[\omega] + (\gamma + \alpha)H + \ell_1\nabla\Phi_1 + \ell_2\nabla\Phi_2 + r\nabla\Phi_3, \text{ with } \alpha := \int_{\mathcal{F}_0} \omega\, dx.$$

Finally, there exists $C > 0$ (depending only on $\mathcal{F}_0$ and on p) such that for any $\omega \in L^p_c(\overline{\mathcal{F}_0})$,

$$\|\nabla v\|_{L^p(\mathcal{F}_0)} \leqslant C(\|\omega\|_{L^p(\mathcal{F}_0)} + |\ell| + |r| + |\gamma|).$$

4 Regularization of the initial data

A general strategy for obtaining a weak solution is to regularize the initial data so that one gets a sequence of initial data which generate some classical solutions, and then to pass to the limit with respect to the regularization parameter in the weak formulation of the equations. We first benefit from the previous section to establish the following approximation result which will be applied to the initial data in the sequel. Let (ℓ_0, r_0, v_0) as in Theorem 1. According to Proposition 1, Lemma 2 and Section 3.3, the function

$$\tilde{v}_0 := v_0 - \beta H, \text{ where } \beta := \gamma + \alpha,$$

belongs to $L^2(\mathcal{F}_0)$.

Proposition 2. *There exists $(v_0^n)_n$ in $\tilde{L}^2 \cap C^\infty(\overline{\mathcal{F}_0})$, satisfying (24) with the same right hand side, such that*

$$\int_{\partial \mathcal{S}_0} v_0^n \cdot \tau \, ds = \int_{\partial \mathcal{S}_0} v_0 \cdot \tau \, ds, \tag{43}$$

$$v_0^n - \beta H \longrightarrow \tilde{v}_0 \quad in \ L^2(\mathcal{F}_0), \tag{44}$$

such that the functions $\omega_0^n := \operatorname{curl} v_n^0$ have a common compact support in $\mathcal{F}_0$, and such that

$$\omega_0^n \longrightarrow \omega_0 \quad in \quad L^p(\mathcal{F}_0). \tag{45}$$

Proof. Let us consider η a smooth nonnegative compactly supported function on $\mathbb{R}^2$ such that $\int_{\mathcal{F}_0} \eta \, dx = 1$. We define the sequence of smooth mollifiers $(\eta_n)_{n>0}$ by $\eta_n(x) = n^2 \eta(x/n)$. This sequence of functions is supported in a fixed compact subset of $\mathbb{R}^2$ and satisfies $\int_{\mathcal{F}_0} \eta_n \, dx = 1$. Let us denote by $\overline{\omega}_0$ the extension of ω_0 by 0 in $\mathcal{S}_0$. We consider the functions $\hat{\omega}_0^n := (\eta_n * \overline{\omega}_0)|_{\mathcal{F}_0}$ obtained by restriction to $\mathcal{F}_0$ of the convolution product $\eta_n * \overline{\omega}_0$. These functions $\hat{\omega}_0^n$ are smooth, supported in a compact K (not depending on n) of $\overline{\mathcal{F}}_0$ and $(\hat{\omega}_0^n)_n$ converges to ω_0 in L^p.

Now we introduce a function $\chi \in C_0^\infty(\mathcal{F}_0)$ with $\int_{\mathcal{F}_0} \chi \, dx = 1$. Defining

$$\omega_0^n := \hat{\omega}_0^n - \left(\int_{\mathcal{F}_0} (\hat{\omega}_0^n - \omega_0) \, dx \right) \chi,$$

we deduce that ω_0^n are also smooth, supported in a compact (independent on n) of $\overline{\mathcal{F}}_0$ and that $(\omega_0^n)_n$ converges to ω_0 in L^p. Moreover one has

$$\int_{\mathcal{F}_0} (\omega_0^n - \omega_0) \, dx = 0.$$

Then we define

$$v_0^n := K[\omega_0^n] + \beta H + \ell_1 \nabla \Phi_1 + \ell_2 \nabla \Phi_2 + r \nabla \Phi_3.$$

Using (33), we see that (43) is satisfied. The convergence (44) is a consequence of Lemma 2. □

5 A priori estimates

In this section we consider a smooth solution (ℓ, r, v) of the problem (13)–(19) with, for any $t \in [0, T]$, supp $\omega(t, \cdot)$ lying in a compact of $\overline{\mathcal{F}_0}$. Our goal is to derive some a priori bounds satisfied by any such solution.

5.1 Vorticity

Due to the equation of vorticity (20) and since we are considering regular solutions, the following quantities are conserved as time proceeds: for any $t > 0$,

$$\|\omega(t, \cdot)\|_{L^p(\mathcal{F}_0)} = \|w_0\|_{L^p(\mathcal{F}_0)}, \quad \|\omega(t, \cdot)\|_{L^1(\mathcal{F}_0)} = \|w_0\|_{L^1(\mathcal{F}_0)}. \tag{46}$$

One has also the following conservations:

$$\gamma = \int_{\partial \mathcal{S}_0} v(t,\cdot) \cdot \tau \, ds = \int_{\partial \mathcal{S}(t)} u(t,\cdot) \cdot \tau \, ds = \int_{\partial \mathcal{S}_0} u_0 \cdot \tau \, ds, \tag{47}$$

$$\alpha = \int_{\mathcal{F}_0} \omega(t,x) \, dx = \int_{\mathcal{F}(t)} w(t,x) \, dx = \int_{\mathcal{F}_0} w_0(x) \, dx. \tag{48}$$

We introduce

$$\tilde{v} := v - \beta H, \tag{49}$$

where as before $\beta = \alpha + \gamma$. It follows, using Proposition 1, Lemma 1 and Section 3.3, that one has for all t:

$$\tilde{v} = \mathcal{O}\left(\frac{1}{|x|^2}\right) \quad \text{and } \nabla \tilde{v} = \mathcal{O}\left(\frac{1}{|x|^3}\right) \quad \text{as } x \to \infty. \tag{50}$$

The results of this section will be applied in the sequel to the classical solutions generated by the approximation sequence of the previous section.

5.2 Energy-like estimate

Despite the fact that we are not considering finite-energy solutions, we have the following result.

Proposition 3. *There exists a constant $C > 0$ (depending only on $\mathcal{S}_0$, m and $\mathcal{J}$) such that for any solution (ℓ, r, v) of the problem (13)–(19) on the time interval $[0,T]$, with $(\ell, r) \in C^1([0,T];\mathbb{R}^3)$ and $v \in C^1([0,T]; \tilde{L}^2 \cap C^\infty(\overline{\mathcal{F}_0}))$, and with compactly supported vorticity, the energy-like quantity defined by:*

$$E(t) := \frac{1}{2}\left(m|\ell(t)|^2 + \mathcal{J} r(t)^2 + \int_{\mathcal{F}_0} \tilde{v}(t,\cdot)^2 dx \right),$$

satisfies the inequality

$$E(t) \leqslant E(0) e^{C|\beta| t}.$$

Remark 2. *This is consistent with the fact that for $\beta = 0$, that is, when the solution is of finite energy, the energy is conserved.*

Proof. We start by recalling that a classical solution satisfies (23), and we use the decomposition (49) in the left hand side and an integration by parts for the first term of the right hand side to get that for any test function $\Psi \in \tilde{\mathcal{H}}_T$,

$$m \, \ell' \cdot \ell_\Psi + \mathcal{J} r' r_\Psi + \int_{\mathcal{F}_0} \partial_t \tilde{v} \cdot \Psi \, dx = - \int_{\mathcal{F}_0} \Psi \cdot \left(\left(v - \ell - r x^\perp \right) \cdot \nabla \right) v \, dx - \int_{\mathcal{F}_0} r v^\perp \cdot \Psi \, dx - m r \ell^\perp \cdot \ell_\Psi.$$

Then, using a standard regular truncation process, we obtain that the previous identity is still valid for the test function Ψ defined by $\Psi(t,x) = \tilde{v}(t,x)$ for (t,x) in $[0,T] \times \mathcal{F}_0$ and $\Psi(t,x) = \ell(t) + r(t) x^\perp$ for (t,x) in $[0,T] \times \mathcal{S}_0$. Hence we get:

$$\begin{aligned} E'(t) &= - \int_{\mathcal{F}_0} \tilde{v} \cdot \left(\left(v - \ell - r x^\perp \right) \cdot \nabla \right) v \, dx - \int_{\mathcal{F}_0} r v^\perp \cdot \tilde{v} \, dx \\ &= - \int_{\mathcal{F}_0} \tilde{v} \cdot \left(\left(v - \ell - r x^\perp \right) \cdot \nabla \tilde{v} \right) dx - \beta \int_{\mathcal{F}_0} \tilde{v} \cdot ((\tilde{v} \cdot \nabla) H) \, dx + \beta \int_{\mathcal{F}_0} \tilde{v} \cdot ((\ell \cdot \nabla) H) \, dx \\ &\quad - \beta r \int_{\mathcal{F}_0} \tilde{v} \cdot (H^\perp - (x^\perp \cdot \nabla) H) \, dx - \beta^2 \int_{\mathcal{F}_0} \tilde{v} \cdot ((H \cdot \nabla) H) \, dx \\ &=: I_1 + I_2 + I_3 + I_4 + I_5. \end{aligned}$$

Integrating by parts we infer that $I_1 = 0$, since $v - \ell - r x^\perp$ is a divergence free vector field, tangent to the boundary. Let us stress that there is no contribution at infinity because of the decay properties of the various fields involved, see (50).

On the other hand, using (27), we get that there exists $C > 0$ depending only on $\mathcal{F}_0$ such that

$$|I_2| + |I_3| + |I_4| \leqslant C|\beta| \left(\int_{\mathcal{F}_0} \tilde{v}^2 \, dx + |\ell|^2 + r^2 \right).$$

Let us now turn our attention to I_5. We first use that H being curl free, we have

$$\int_{\mathcal{F}_0} \tilde{v} \cdot ((H \cdot \nabla) H) \, dx = \frac{1}{2} \int_{\mathcal{F}_0} (\tilde{v} \cdot \nabla) |H|^2 \, dx,$$

and then an integration by parts and (15) to obtain

$$\int_{\mathcal{F}_0} \tilde{v} \cdot ((H \cdot \nabla) H) \, dx = \frac{1}{2} \int_{\partial \mathcal{S}_0} (\tilde{v} \cdot n) |H|^2 \, ds = \frac{1}{2} \ell \cdot \int_{\partial \mathcal{S}_0} |H|^2 n \, ds + \frac{1}{2} r \int_{\partial \mathcal{S}_0} |H|^2 x^\perp \cdot n \, ds.$$

We will make use of the following classical Blasius' lemma (cf. e.g. [6, Lemma 5])

Lemma 3. *Let $\mathcal{C}$ be a smooth Jordan curve, $f := (f_1, f_2)$ and $g := (g_1, g_2)$ two smooth tangent vector fields on $\mathcal{C}$. Then*

$$\int_{\mathcal{C}} (f \cdot g) n \, ds = i \left(\int_{\mathcal{C}} (f_1 - i f_2)(g_1 - i g_2) \, dz \right)^*,$$

$$\int_{\mathcal{C}} (f \cdot g)(x^\perp \cdot n) \, ds = Re \left(\int_{\mathcal{C}} z (f_1 - i f_2)(g_1 - i g_2) \, dz \right).$$

where $(\cdot)^*$ denotes the complex conjugation.

Therefore using (28) and Cauchy's residue Theorem gives that $I_5 = 0$.

Collecting all these estimates it only remains to use Gronwall's lemma to conclude. □

5.3 Bound of the body acceleration

The aim of this section is to prove the following a priori estimate of the body acceleration.

Proposition 4. *There exists a constant $C > 0$ depending only on $\mathcal{S}_0$, m, $\mathcal{J}$, β and $E(0)$ such that any classical solution of (13)–(19) satisfies the estimate*

$$\|(\ell', r')\|_{L^\infty(0,T)} \leqslant C.$$

Proof. Again, after a regular truncation procedure, we can use (23) with, as test functions, the functions $(\Psi_i)_{i=1,2,3}$ defined by $\Psi_i = \nabla \Phi_i$ in $\mathcal{F}_0$ and $\Psi_i = e_i$, for $i = 1, 2$ and $\Psi_3 = x^\perp$ in $\mathcal{S}_0$. We observe that

$$\big((\partial_t v, \Psi_i)_\rho \big)_{i=1,2,3} = \mathcal{M}_1 \begin{pmatrix} \ell_1 \\ \ell_2 \\ r \end{pmatrix}' (t) + (A_i)_{i=1,2,3}(t),$$

where

$$\mathcal{M}_1 := \begin{bmatrix} m \, \mathrm{Id}_2 & 0 \\ 0 & \mathcal{J} \end{bmatrix} \quad \text{and} \quad A_i := \int_{\mathcal{F}_0} \partial_t v \cdot \nabla \Phi_i \, dx. \tag{51}$$

Let us examine A_i for $i = 1, 2, 3$. As $\operatorname{div} v = 0$ for all time, using (39)-(40) we deduce that

$$A_i = \int_{\partial \mathcal{S}_0} \partial_t v \cdot n \, \Phi_i \, ds.$$

Now using the boundary condition (15) and Green's formula we obtain

$$(A_i)_{i=1,2,3} = \mathcal{M}_2 \begin{pmatrix} \ell_1 \\ \ell_2 \\ r \end{pmatrix}' \quad \text{where } \mathcal{M}_2 := \left[\int_{\mathcal{F}_0} \nabla \Phi_a \cdot \nabla \Phi_b \, dx \right]_{a,b \in \{1,2,3\}}.$$

This encodes the phenomenon of added mass, which, loosely speaking, measures how much the surrounding fluid resists the acceleration as the body moves through it. We also introduce the matrix

$$\mathcal{M} := \mathcal{M}_1 + \mathcal{M}_2,$$

which is symmetric and positive definite. The equations given by (23) are now recast as follows:

$$\mathcal{M} \begin{bmatrix} \ell \\ r \end{bmatrix}' = \big[B_i + C_i + D_i \big]_{i\in\{1,2,3\}},$$

where

$$B_i := \int_{\mathcal{F}_0} v \cdot \left[((v - \ell - rx^\perp)\cdot\nabla)\nabla\Phi_i \right] dx, \quad C_i := -\int_{\mathcal{F}_0} rv^\perp \cdot \nabla\Phi_i \, dx \;\text{ and }\; D_i := \begin{bmatrix} -mr\ell^\perp \\ 0 \end{bmatrix}.$$

Let us now make use of the decomposition (49). We have

$$\begin{aligned} B_i &= \int_{\mathcal{F}_0} \tilde{v}\cdot[(\tilde{v}\cdot\nabla)\nabla\Phi_i]\,dx + \beta\int_{\mathcal{F}_0} H\cdot[(\tilde{v}\cdot\nabla)\nabla\Phi_i]\,dx + \beta\int_{\mathcal{F}_0}\tilde{v}\cdot[(H\cdot\nabla)\nabla\Phi_i]\,dx \\ &\quad +\beta^2\int_{\mathcal{F}_0} H\cdot[(H\cdot\nabla)\nabla\Phi_i]\,dx - \int_{\mathcal{F}_0}\tilde{v}\cdot\left[\left((\ell+rx^\perp)\cdot\nabla\right)\nabla\Phi_i\right]dx - \beta\int_{\mathcal{F}_0} H\cdot\left[((\ell+rx^\perp)\cdot\nabla)\nabla\Phi_i\right]dx, \\ -C_i &= \int_{\mathcal{F}_0} r\tilde{v}^\perp\cdot\nabla\Phi_i\,dx + \beta\int_{\mathcal{F}_0} rH^\perp\cdot\nabla\Phi_i\,dx. \end{aligned}$$

It then suffices to use Proposition 3 and the decaying properties of $\nabla\Phi_i$ and H to obtain a bound of ℓ' and r'.

□

6 Proof of Theorem 1

We start with an initial data (ℓ_0, r_0, v_0) as in Theorem 1. Then we apply Proposition 2 to get a sequence of regularized initial data (ℓ_0^n, r_0^n, v_0^n), keeping $(\ell_0^n, r_0^n) := (\ell_0, r_0)$.

According to [5, Theorem 1], these data generate some global-in-time classical solutions that we denote by (ℓ^n, r^n, v^n). Let us mention that in [5], regarding the regularity in time of the fluid velocity field, these solutions are merely claimed to satisfy $\partial_t v \in L^\infty_{loc}(\mathbb{R}_+; L^2(\mathcal{F}_0))$. However using (20) and Lemma 1 one easily infers that $\partial_t v \in C(\mathbb{R}_+; L^2(\mathcal{F}_0))$. (Alternatively, one could work with test functions which are only Lipschitz in time rather than C^1 as in (21).)

These solutions satisfy uniformly in n the estimates given in Section 5 since these estimates only depends on the geometry and on some norms of the initial data which are bounded uniformly in n according to Proposition 2. We note indeed that for regular solutions, if the initial vorticity is compact, it remains compact for all times as a consequence of (20).

As explained above a classical solution of (13)–(19) in $[0, T]$ is also a weak solution. Therefore for any test function $\Psi \in \tilde{\mathcal{H}}_T$, for any n, one has

$$\begin{aligned} (\Psi(T,\cdot), \overline{v}^n(T,\cdot))_\rho - (\Psi(0,\cdot), \overline{v}_0^n)_\rho &= \int_0^T \left(\frac{\partial\Psi}{\partial t}, \overline{v}^n\right)_\rho dt + \int_0^T\int_{\mathcal{F}_0} v^n\cdot(v^n\cdot\nabla)\,\Psi\,dx\,dt \\ &\quad - \int_0^T\int_{\mathcal{F}_0} v^n\cdot\left(\left(\ell^n + r^n x^\perp\right)\cdot\nabla\right)\Psi\,dx\,dt - \int_0^T\int_{\mathcal{F}_0} r^n (v^n)^\perp\cdot\Psi\,dx\,dt - m\int_0^T r^n(\ell^n)^\perp\cdot\ell_\Psi\,dt. \end{aligned} \tag{52}$$

Let us now explain how to pass to the limit. The main step consists in extracting a subsequence to (ℓ^n, r^n, v^n) by using the above a priori estimates.

- Using Proposition 4 and Ascoli's theorem yields that some subsequences of (ℓ^n) and of (r^n) (which we still call (ℓ^n) and (r^n)) are converging strongly in $C_{loc}([0, +\infty))$.
- Using Propositions 1, 2 and 3 and (46)-(48), we obtain uniform bounds on $\|\tilde{v}_n\|_{L^2}$ and $\|\nabla v^n\|_{L^p(\mathcal{F}_0)}$ in $L^\infty(0, T)$ for each $T > 0$.

- Now we use again (23) to get that for any $\xi \in C_c^\infty(\mathcal{F}_0;\mathbb{R})$ to get a bound of $\xi \partial_t v^n$ in $L^2(0,T;H^{-2}(\mathcal{F}_0))$ for any $T>0$. Using this temporal estimate and the Aubin-Lions lemma we deduce that some subsequence of (v^n) converges strongly in $C_{loc}([0,+\infty);L^2_{loc})$.

Now it is elementary to check that these convergences are sufficient to pass to the limit in (52). This achieves the proof of Theorem 1 when $p<2$.

Now, when $p \geqslant 2$ and given an initial data $v_0 \in \tilde{L}^2$ with ω_0 in $L^p_c(\overline{\mathcal{F}_0})$, we have in particular that $\omega_0 \in L^{3/2}_c(\overline{\mathcal{F}_0})$, so that there exists a solution (ℓ, r, v) constructed by the process above, with $\omega \in L^\infty(\mathbb{R}^+;(L^1 \cap L^{3/2})(\mathcal{F}_0))$. But the corresponding approximations (ℓ^n, r^n, v^n) satisfy

$$\|\omega^n\|_{L^\infty(\mathbb{R}^+;L^p(\mathcal{F}_0))} \leqslant C,$$

for some constant $C>0$ independent of n.

Hence by the lower semi-continuity of the $L^\infty(\mathbb{R}^+;L^p(\mathcal{F}_0))$ norm for the weak-$*$ convergence, the solution constructed in this process satisfies $\omega \in L^\infty(\mathbb{R}^+;L^p(\mathcal{F}_0))$.

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