Column Generation based Tactical Planning Method for Inventory Routing

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Vehicle routing
Inventory Routing
Inventory Routing

→ 3 decisions:
1. **when** to visit a site?
2. **how much to collect** from the site?
3. **which tasks are “assigned” to a vehicle** (cluster/routes)?

→ underlying models: **Multi-Period VRP + Inventory manag.**
many variants: # of products - Time horizon - det./stoch demands - stock manag. policy - routing vs clustering

various applications: ammonia shipping, petrol stations, supermarkets

low size of instances: 15 customers, or 1 period, or 2 customers per vehicles

Non exact solution approaches:

- restrictive assumptions: “fixed partition policy” (sets of customers that are serviced together)
  (Bramel and Simchi-Levi 1995)

- hierarchical approach: planning first, routing second
  (Campbell and Savelsbergh 2004)

- Mostly heuristics. Some studies use branch-and-price.
Collecting recycle waste from deposit points

- **Inventory management model:**
  - **order up-to level policy:** empty the bin on each visit
  - **deterministic filling rates:** inventory management $= t_{\text{max}}$ between visits; costs $= \text{transportation}$.
  - $p$–**periodic routes:** $\infty$ horizon, periodic planning constraint periodicities $p \in P = \{1, 2, 3, 4, 5, 6\} \Rightarrow T = LCM(P) = 60$.

- **Routing model:** form compact clusters first (tactical planning), route second (left for operational planning – drivers’ decision)
Challenges

- **problem size:**
  - 260 sites
  - 10 vehicles
  - Frequency of visits: $t_{\text{max}} \in \{1, \ldots, 14\}$
  - 1 vehicle = 10 sites visited on average

- **symmetry drawback:** in selecting periodic solutions

→ we shall model an **average behavior**
Saving vehicles compared to Bin-Packing solution

filling rates = (51, 50, 34, 33, 18), \( W = 100 \Rightarrow V(bpp) = 3 \)
Solution approach & results

- Truncated branch-and-price-and-cut
- Column generation based primal heuristics
- State space relaxation

<table>
<thead>
<tr>
<th></th>
<th>av # of customer visits per week</th>
<th># of vehicles</th>
<th>av travel av distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>our sol</td>
<td>98</td>
<td>9</td>
<td>711 km</td>
</tr>
<tr>
<td>industri. sol</td>
<td>59</td>
<td>10</td>
<td>782 km</td>
</tr>
</tbody>
</table>

+ regional partition of routes
Outline

1. Clustering Model
2. “Compact” Formulation
3. Decomposition into Vehicle Tasks
4. DW reformulations
   - Discrete time
   - Aggregate time
5. Dual bounds
   - Use the aggregate formulation
   - Adding cuts
   - Doing partial branching
6. Primal Bounds: heuristics
   - Use of the Discrete formulation
   - Col Gen based heuristics: Restricted Master, Greedy, Rounding, Local Search
Surrogate transportation costs

\[
c_{ik} = d_{i,k} + \min\{d_{0,i} - d_{0,k}, d_{i,n+1} - d_{k,n+1}\}
\]
Surrogate transportation costs

\[ c_{ik} = d_{i,k} + \min\{d_{0,i} - d_{0,k}, d_{i,n+1} - d_{k,n+1}\} \]

<table>
<thead>
<tr>
<th></th>
<th>routing</th>
<th>cluster</th>
<th>facility loc</th>
</tr>
</thead>
<tbody>
<tr>
<td>routing sol</td>
<td>190.9</td>
<td>214.1</td>
<td>184.5</td>
</tr>
<tr>
<td>cluster sol</td>
<td>196.8</td>
<td>200.4</td>
<td>147.0</td>
</tr>
<tr>
<td>facility loc sol</td>
<td>198.8</td>
<td>208.7</td>
<td>135.5</td>
</tr>
</tbody>
</table>
Compact Formulation

\( x_{i\ell vps} = 1 \) if cust. \( i \) is collected \( \ell \) periods worth of stock by vehicle \( v \), every \( p \) periods, starting in \( s \);
\( y_{vps} = 1 \) if vehicle \( v \) ... \( z_{ikvps} = 1 \) if cust. \( i \) is in a cluster of seed \( k \) ...

\[
\min \ V_{\text{max}} + \alpha \sum_{v,p,s} \frac{1}{p} \left( \sum_k f_k z_{kkvps} + \sum_{i,k:i\neq k} c_{ik} z_{ikvps} \right) \tag{1}
\]

\[
\sum_{\ell,v,p,s} \theta_{t}^{\ell p s} x_{i\ell vps} = 1 \quad \forall i \in N', t = 1, \ldots, T \tag{2}
\]

\[
\sum_{k \in N'} z_{ikvps} = \sum_{\ell} x_{i\ell vps} \quad \forall i \in N', v, p, s \tag{3}
\]

\[
z_{ikvps} \leq z_{kkvps} \quad \forall i \in N', k \in N', v, p, s \tag{4}
\]

\[
\sum_{i \in N', \ell} \ell r_i x_{i\ell vps} \leq W y_{vps} \quad \forall v, p, s \tag{5}
\]

\[
\sum_{v,p,s} \delta_{t}^{ps} y_{vps} \leq V_{\text{max}} \quad \forall t = 1, \ldots, T \tag{6}
\]
Decomposition into Vehicle Tasks

A Complete planning can be decomposed in periodic vehicle tasks:

3 vehicle tasks of periodicity 3 → 1 vehicle needed

A periodic vehicle task $q$ is defined by:

- its pickup pattern $x^q$: $x^q_{il} = 1$ if $l$ period worth of stock is pickup from cust $i$.
- its cluster cost $c^q$
- its first occurrence (starting time) $s^q$
- its periodicity $p^q$
Each column defines a periodic vehicle task: \( \{(c^q, s^q, p^q, x^q)\} \subseteq Q \)

The master models inventory planning

\[
[DM] \equiv \min V_{\text{max}} + \alpha \sum_{q \in Q} \frac{c^q}{p^q} \lambda_q 
\]

\[
\sum_{q} \theta_{it}^q \lambda_q \geq 1 \quad \forall i \in N', t = 1, \ldots, T 
\]

\[
\sum_{q} \delta_t^q \lambda_q \leq V_{\text{max}} \quad \forall t = 1, \ldots, T 
\]

\[
\lambda_q \in \{0, 1\} \quad \forall q \in Q 
\]


Pricing Subproblem

For each starting time $s$ periodicity $p$ cluster center $k$, solve a multiple choice knapsack problem:

$$\text{max } \sum_{i,\ell} g_{i\ell} x_{i\ell}$$

(12)

$$\sum_{\ell} x_{k\ell} = 1$$

(13)

$$\sum_{\ell} x_{i\ell} \leq 1 \quad \forall i \neq k$$

(14)

$$\sum_{i,\ell} \ell r_i x_{i\ell} \leq W$$

(15)

$$x_{i\ell} \in \{0, 1\} \quad \forall i, \ell.$$ 

(16)
State Space Relaxation

Aggregate columns that differ only by their starting dates:

$$\{(c^q, s^q, p^q, x^q)\}_{q \in Q} \rightarrow \{(c^r, p^r, x^r)\}_{r \in R}.$$ 

Summing over $t$, leads to a master modeling an average behavior:

$$Z^A = \min \; V_{\text{aver}} + \alpha \sum_{r \in R} \frac{c^r}{p^r} \lambda_r$$

$$\sum_{r \in R, \ell} \frac{\ell}{p^r} x_{i\ell}^r \lambda_r \geq 1 \quad \forall \; i \in N'$$

$$\sum_{r \in R} \frac{1}{p^r} \lambda_r \leq V_{\text{aver}}$$

$$\lambda_r \in \mathbb{N} \quad \forall \; r \in R$$

$$V \geq V_{\text{aver}} \in \mathbb{N}.$$
Discrete versus Aggregate Time Formulation

- **LP equivalence:**
  \[
  \lambda_r = \sum_{q \in Q(r)} \lambda_q ,
  \]
  \[
  \lambda_q = \frac{1}{pr} \lambda_r , \quad q \in Q(r)
  \]

- **No IP equivalence**
- **≠ LP computing time**

<table>
<thead>
<tr>
<th>Instance</th>
<th>discrete form.</th>
<th>aggregate form.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Col</td>
<td>Time</td>
</tr>
<tr>
<td>IND8</td>
<td>71</td>
<td>1.75s</td>
</tr>
<tr>
<td>IND27</td>
<td>-</td>
<td>&gt;1h</td>
</tr>
<tr>
<td>RAND100</td>
<td>-</td>
<td>&gt;1h</td>
</tr>
</tbody>
</table>
Discrete ≠ Aggregate Time IP Formulation

<table>
<thead>
<tr>
<th>aggreg. sol. $\lambda_r$</th>
<th>discrete sol. $\lambda_q$</th>
<th>t1</th>
<th>t2</th>
<th>t3</th>
<th>t4</th>
<th>t5</th>
<th>t6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_A = 1$</td>
<td>$\lambda_{A_1} = \frac{1}{2}$ $s_{A_1} = 1$</td>
<td>■</td>
<td>✓</td>
<td>■</td>
<td>■</td>
<td>■</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>$\lambda_{A_2} = \frac{1}{2}$ $s_{A_2} = 2$</td>
<td>✓</td>
<td>■</td>
<td>✓</td>
<td>■</td>
<td>✓</td>
<td>■</td>
</tr>
<tr>
<td>$\lambda_B = 1$</td>
<td>$\lambda_{B_1} = \frac{1}{3}$ $s_{B_1} = 1$</td>
<td>■</td>
<td>✓</td>
<td>✓</td>
<td>■</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>$\lambda_{B_2} = \frac{1}{3}$ $s_{B_2} = 2$</td>
<td>✓</td>
<td>■</td>
<td>✓</td>
<td>✓</td>
<td>■</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>$\lambda_{B_3} = \frac{1}{3}$ $s_{B_3} = 3$</td>
<td>✓</td>
<td>✓</td>
<td>■</td>
<td>✓</td>
<td>✓</td>
<td>■</td>
</tr>
</tbody>
</table>

- Task A: $0 - 1(2) - 3$ with $p_A = 2$,
- Task B: $0 - 2(3) - 3$ with $p_B = 3$,

$V_{\text{aver}} = 1 \geq \frac{1}{2} + \frac{1}{3}$ while $V_{\text{max}} = 2$
Discrete ≠ Aggregate Time IP Formulation

- task A: $0 - \cdots - 1(3) - \cdots - n + 1$ with $p_A = 6$, and
- task B: $0 - \cdots - 1(2) - \cdots - n + 1$ with $p_B = 4$.

<table>
<thead>
<tr>
<th></th>
<th>t1</th>
<th>t2</th>
<th>t3</th>
<th>t4</th>
<th>t5</th>
<th>t6</th>
<th>t7</th>
<th>t8</th>
<th>t9</th>
<th>t10</th>
<th>t11</th>
<th>t12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta^A_{1t}$</td>
<td>✓</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>#</td>
<td></td>
<td></td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta^B_{1t}$</td>
<td>✓</td>
<td>#</td>
<td>✓</td>
<td>✓</td>
<td>#</td>
<td></td>
<td></td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

customer pick-up date conflict
Solving the Aggregate Master

- Include artificial columns; their cost defines an UB on dual var.
- Use a dual heuristic to “warm start” the col. gen. procedure.
- Solve multiple choice knapsacks using Dynamic Prog. (Pisinger, 95).
- Partial Pricing: consider
  - periodicities in decreasing order,
  - most attractive seed first.
- Re-optimisation of
  - seed location,
  - periodicity: $p = \max_i \{\ell : x_{i\ell} = 1\}$
Adding Cutting Planes

In Mast LP sol: a pattern that covers \( \frac{5}{6} \) of customer \( i \)'s demand whose \( t_{\text{max}} = 5 \), can be selected 1.2 time.

<table>
<thead>
<tr>
<th>aggreg. sol. ( \lambda_r )</th>
<th>t1</th>
<th>t2</th>
<th>t3</th>
<th>t4</th>
<th>t5</th>
<th>t6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_r = 1.2 )</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>■</td>
</tr>
</tbody>
</table>

To cut this LP solution is:

\[
\sum_{r, \ell: \ell = p^r} x_{i, \ell}^r \lambda_r + \frac{1}{2} \sum_{r, \ell: \ell \neq p^r} x_{i, \ell}^r \lambda_r \geq 1,
\]

• small improvement: 2% gap
• large increase in computing time: 70% of the time in re-opt.
• modifications to the structure of master LP solution: bad for primal heuristic
Branching on $V_{\text{aver}}$

$V_{\text{aver}} \leq v - 1$

$V_{\text{aver}} \geq v$

- $v$ is a lower bound on number of vehicles if N1 infeasible
- Improvement in dual bound in N2: since

$$Z^A = \min V_{\text{aver}} + \alpha \sum_{r \in R} \frac{c_r}{p_r} \lambda_r$$

- Cutting Planes + Partial Branching

<table>
<thead>
<tr>
<th>gaps</th>
<th>gap-root</th>
<th>gap-cut</th>
<th>gap-br</th>
<th>gap-br-cut</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>23.05</td>
<td>21.80</td>
<td>10.63</td>
<td>9.67</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>% time</th>
<th>PP</th>
<th>RM</th>
<th>CP</th>
<th>Sep</th>
<th>N0</th>
<th>N1</th>
<th>N2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10.84</td>
<td>8.14</td>
<td>73.49</td>
<td>4.14</td>
<td>5.87</td>
<td>16.17</td>
<td>77.91</td>
</tr>
</tbody>
</table>
Primal Heuristics

- **Rounding:**
  Round aggregate master solution to construct discrete master solution
  - Round-up mast col and select starting date: greedy selection, \( \text{argmin}_{rs} \left\{ \left( \delta_{rs} + \alpha \frac{c_r}{p_r} \right) / \left( \sum_{i,\ell} \ell x_{i\ell} \right) \right\} \)
  - Solve the residual master: pricing requires enumerating over starting dates
  - Diversify the search: limited backtracking at the root node

- **Local Search:** starting from a master IP sol
  - Remove “worst” col: low load columns + complementary:
    - favour custo. exchange: i.e. close-by clusters;
    - favour vehicle saving: i.e. same \((s, p)\).
  - Solve the residual master using the Rounding heur.
**Computational results**

260 customers, 60 periods, 10 vehicles

- **Dual Bounds:**

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>bound</th>
<th>time</th>
</tr>
</thead>
<tbody>
<tr>
<td>AM LP + br + cut</td>
<td>838.17</td>
<td>5h21m</td>
</tr>
</tbody>
</table>

- **Primal Bounds:**

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>gap (in %)</th>
<th>$V_{\text{max}}$</th>
<th>time</th>
</tr>
</thead>
<tbody>
<tr>
<td>RH + Branch, $P = {1, 2, 3, 4, 5, 6}$</td>
<td>9.25</td>
<td>9</td>
<td>4h29m</td>
</tr>
<tr>
<td>RH + LS, $P = {1, 2, 3, 4, 5, 6}$</td>
<td>8.92</td>
<td>9</td>
<td>3h40m</td>
</tr>
<tr>
<td>RH + Branch, $P = {1, 2, 3} + \text{post-opt}$</td>
<td>6.67</td>
<td>9</td>
<td>1h49m</td>
</tr>
<tr>
<td>RH + LS, $P = {1, 2, 3} + \text{post-optim}$</td>
<td>6.23</td>
<td>9</td>
<td>2h53m</td>
</tr>
</tbody>
</table>
Summary: Planning for Inventory Routing

- Column generation + surrogate relaxation to avoid symmetries
- Rounding heuristic implicitly carried in discrete time formulation
- Solutions with quality warranty: $LB \leftrightarrow 6\% \leftrightarrow SOL \leftrightarrow 10\% \leftrightarrow INDUST$