DS 1, corrigé

Ex. 1. $A = \begin{pmatrix} -1 & 2 & 0 \\ -2 & -4 & 3 \\ -1 & 0 & 5 \end{pmatrix}$ $\quad B = \begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix}$

$$AB = \begin{pmatrix} -1 & 2 & 0 \\ -2 & -4 & 3 \\ -1 & 0 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix} = \begin{pmatrix} -1 \\ -11 \\ -16 \end{pmatrix}$$

Ex. 2.

$$\left(\begin{array}{ccc|ccc} -2 & 3 & -2 & 1 & 0 & 0 \\ 2 & -1 & -1 & 0 & 1 & 0 \\ 1 & -2 & 3 & 0 & 0 & 1 \end{array}\right) \begin{array}{c} III \\ II \\ I \end{array} \left(\begin{array}{ccc|ccc} 1 & -2 & 3 & 0 & 0 & 1 \\ 2 & -1 & -1 & 0 & 1 & 0 \\ -2 & 3 & -2 & 1 & 0 & 0 \end{array}\right)$$

$$\begin{array}{c} \\ II-2I \\ III+2I \end{array} \left(\begin{array}{ccc|ccc} 1 & -2 & 3 & 0 & 0 & 1 \\ 0 & 3 & -7 & 0 & 1 & -2 \\ 0 & -1 & 4 & 1 & 0 & 2 \end{array}\right) \begin{array}{c} \\ -III \\ II \end{array} \left(\begin{array}{ccc|ccc} 1 & -2 & 3 & 0 & 0 & 1 \\ 0 & 1 & -4 & -1 & 0 & -2 \\ 0 & 3 & -7 & 0 & 1 & -2 \end{array}\right)$$

$$\begin{array}{c} \\ \\ III-3II \end{array} \left(\begin{array}{ccc|ccc} 1 & -2 & 3 & 0 & 0 & 1 \\ 0 & 1 & -4 & -1 & 0 & -2 \\ 0 & 0 & 5 & 3 & 1 & 4 \end{array}\right) \begin{array}{c} \\ \\ \frac{1}{5}III \end{array} \left(\begin{array}{ccc|ccc} 1 & -2 & 3 & 0 & 0 & 1 \\ 0 & 1 & -4 & -1 & 0 & -2 \\ 0 & 0 & 1 & \frac{3}{5} & \frac{1}{5} & \frac{4}{5} \end{array}\right)$$

$$\begin{array}{c} I-3III \\ II+4III \\ \\ \end{array} \left(\begin{array}{ccc|ccc} 1 & -2 & 0 & -\frac{9}{5} & -\frac{3}{5} & -\frac{7}{5} \\ 0 & 1 & 0 & \frac{7}{5} & \frac{4}{5} & \frac{6}{5} \\ 0 & 0 & 1 & \frac{3}{5} & \frac{1}{5} & \frac{4}{5} \end{array}\right) \begin{array}{c} I+2II \\ \\ \\ \end{array} \left(\underbrace{\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}}_{1_3} \middle| \underbrace{\begin{array}{ccc} 1 & 1 & 1 \\ \frac{7}{5} & \frac{4}{5} & \frac{6}{5} \\ \frac{3}{5} & \frac{1}{5} & \frac{4}{5} \end{array}}_{M^{-1}} \right)$$

$$\begin{pmatrix} -2 & 3 & -2 \\ 2 & -1 & -1 \\ 1 & -2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ \frac{7}{5} & \frac{4}{5} & \frac{6}{5} \\ \frac{3}{5} & \frac{1}{5} & \frac{4}{5} \end{pmatrix} =$$

$$= \frac{1}{5} \begin{pmatrix} -2 & 3 & -2 \\ 2 & -1 & -1 \\ 1 & -2 & 3 \end{pmatrix} \begin{pmatrix} 5 & 5 & 5 \\ 7 & 4 & 6 \\ 3 & 1 & 4 \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix} = 1_3 \quad \text{ok.}$$

Ex. 3. $A = \begin{pmatrix} -2 & 4 \\ 1 & 1 \end{pmatrix}$

$$p_A(x) = \det \begin{pmatrix} -2-x & 4 \\ 1 & 1-x \end{pmatrix} = -2 + 2x - x + x^2 - 4 = x^2 + x - 6$$

$$x = \frac{-1 \pm \sqrt{1+24}}{2} = \frac{-1 \pm 5}{2} \quad \begin{cases} \lambda_1 = -3 \\ \lambda_2 = 2 \end{cases}$$

A diagonalisable

$\exists\, P$ t.q. $P^{-1}AP = D = \begin{pmatrix} -3 & 0 \\ 0 & 2 \end{pmatrix}$

vecteurs propres de $\lambda_1 = -3$

$$\begin{pmatrix} -2+3 & 4 \\ 1 & 1+3 \end{pmatrix} X = 0 \qquad \begin{pmatrix} 1 & 4 \\ 1 & 4 \end{pmatrix} X = 0 \qquad \begin{cases} x+4y=0 \\ x+4y=0 \end{cases} \qquad v_1 = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

vecteurs propres de $\lambda_2 = 2$

$$\begin{pmatrix} -2-2 & 4 \\ 1 & 1-2 \end{pmatrix} X = 0 \qquad \begin{pmatrix} -4 & 4 \\ 1 & -1 \end{pmatrix} X = 0 \qquad \begin{cases} -4x+4y=0 \\ x-y=0 \end{cases} \qquad v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\Rightarrow \quad P = \begin{pmatrix} 4 & 1 \\ -1 & 1 \end{pmatrix} \qquad P^{-1} = \frac{1}{5}\begin{pmatrix} 1 & -1 \\ 1 & 4 \end{pmatrix}$$

$$\frac{1}{5}\begin{pmatrix} 1 & -1 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} -2 & 4 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 4 & 1 \\ -1 & 1 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 1 & -1 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} -12 & 2 \\ 3 & 2 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} -15 & 0 \\ 0 & 10 \end{pmatrix} = \begin{pmatrix} -3 & 0 \\ 0 & 2 \end{pmatrix} \text{ ok}$$

<u>Ex. 4.</u> $\left(\sin x + \sqrt{x}\right)' = \cos x + \frac{1}{2\sqrt{x}}$

$$\left(\sin\sqrt{x}\right)' = \cos\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$$

$$\left(\sqrt{x}\,\sin x\right)' = \frac{1}{2\sqrt{x}}\sin x + \sqrt{x}\cos x$$

$$\int_0^{\frac{\pi^2}{4}} \frac{\cos(\sqrt{x})}{2\sqrt{x}}\,dx = \left[\sin\sqrt{x}\right]_0^{\frac{\pi^2}{4}} = \sin\frac{\pi}{2} - \sin 0 = 1-0 = 1$$

ou : $$\text{"} \int_0^{\frac{\pi}{2}} \frac{\cos t}{2t}\cdot 2t\,dt = \int_0^{\frac{\pi}{2}} \cos t\,dt = \left[\sin t\right]_0^{\frac{\pi}{2}} = \sin\frac{\pi}{2} - \sin 0 = 1.$$

$\sqrt{x} = t$

$x = t^2$

$dx = 2t\,dt$

<u>Ex. 5.</u> $$\int_1^2 x\ln x\,dx = \left[\frac{x^2}{2}\ln x\right]_1^2 - \int_1^2 \frac{x^2}{2}\cdot\frac{1}{x}\,dx$$

$$= \frac{4}{2}\ln 2 - \underbrace{\frac{1}{2}\ln 1}_{=0} - \frac{1}{2}\int_1^2 x\,dx$$

$$= 2\ln 2 - \frac{1}{2}\left[\frac{x^2}{2}\right]_1^2 = 2\ln 2 - \frac{1}{2}\left(\frac{4}{2} - \frac{1}{2}\right)$$

$$= 2\ln 2 - \frac{3}{4}$$

Q.6. $f(x) = \frac{1}{1+x} + \cos x$

a) $f'(x) = -\frac{1}{(1+x)^2} - \sin x$

$f''(x) = -(-2)\frac{1}{(1+x)^3} - \cos x = \frac{2}{(1+x)^3} - \cos x$

b) $f(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + o(x^2)$

$= 2 - x + \frac{1}{2}x^2 + o(x^2)$

$f(0) = 1 + 1 = 2$

$f'(0) = -1$

$f''(0) = 2 - 1 = 1$